**Prerequisite Skills**

1. Vertical asymptotes give restrictions on the domain. Horizontal asymptotes give restrictions on the range.

2. a) $x = -2, y = 0$

b) $x = 1, y = 0$

c) $x = 4, y = 0$

3. a) $\{x \in \square\}, \{y \in \square, y \geq -4\}$

b) $\{x \in \square\}, \{y \in R\}$

c) $\{x \in \square, x \geq -3\}, \{y \in \square, y \leq -5\}$
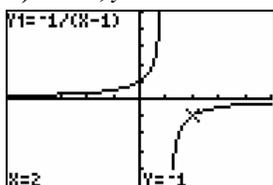
d) $\{x \in \square, x \neq -2\}, \{y \in \square, y \neq 0\}$

4. a) $-\dfrac{2}{3}$ **b)** 2 **c)** $-\dfrac{3}{2}$

d) $-\dfrac{6}{7}$ **e)** $\dfrac{60}{17}$

5. a) 0.5 **b)** -0.22
  c) -2.24 **d)** -1.13

6. a) $(x+4)(x-1)$

b) $(3x+1)(2x-3)$

c) $(4x+3)(3x-2)$

d) $(x+1)(x-3)(4x+1)$

e) $(2x+5)(4x^2-10x+25)$

7. a) $-7, 5$ **b)** $-\dfrac{1}{3}, 4$

c) $\dfrac{1}{2}, \dfrac{4}{3}$ **d)** $-\dfrac{5}{4}, -\dfrac{4}{3}$

8. a) $-1 \pm \sqrt{6}$ **b)** $2 \pm \sqrt{3}$

c) $\dfrac{-1 \pm \sqrt{57}}{4}$ **d)** none

9. a) $-4 < x < 4$ **b)** $x \leq -3$ or $x \geq 5$

c) $-\sqrt{\dfrac{3}{2}} \leq x \leq \sqrt{\dfrac{3}{2}}$ **d)** $x \leq -6$ or $x \geq 1$

e) $x \leq \dfrac{-5 - \sqrt{13}}{2}$ or $x \geq \dfrac{-5 + \sqrt{13}}{2}$

**f)** $x \in \square$

3.1 Reciprocal of a Linear Function

1.

As $x \rightarrow$	$f(x) \rightarrow$
4^+	$+\infty$
4^-	$-\infty$
$+\infty$	0
$-\infty$	0

2. $y = 0, x = -1, y = \dfrac{1}{x+1}$

3. a) $x = 4, y = 0$ **b)** $\dfrac{1}{2}$

4. a) $y = \dfrac{1}{x-2}$ **b)** $y = \dfrac{1}{2x+3}$

5. a) The slope is positive and increasing for $x < -3$. The slope is positive and decreasing for $x > -3$.
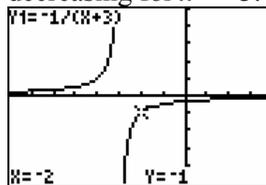
**b)** The slope is negative and decreasing for $x < \dfrac{3}{2}$. The slope is negative and increasing for $x > \dfrac{3}{2}$.
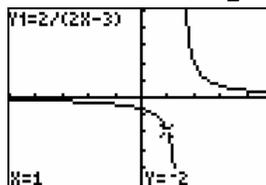
6. a)
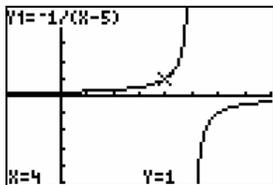
y-intercept $\dfrac{1}{5}$, $\{x \in \mathbb{R}, x \ne 5\}$,

$\{y \in \mathbb{R}, y \ne 0\}$, asymptotes:

$x = 5, y = 0$, slope positive and increasing for $x < 5$, slope positive and decreasing for $x > 5$

b)
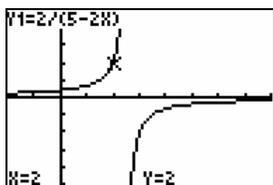
y-intercept $\dfrac{2}{5}$, $\left\{x \in \mathbb{R}, x \ne \dfrac{5}{2}\right\}$,

$\{y \in \mathbb{R}, y \ne 0\}$, asymptotes:

$x = \dfrac{5}{2}, y = 0$, slope positive and

increasing for $x < \dfrac{5}{2}$, slope positive

and decreasing for $x > \dfrac{5}{2}$

7. a) $P(V) = \dfrac{20\,000}{V}$

b)
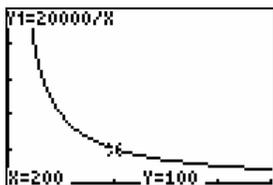
c) $\dfrac{800}{3}$ kPa

d) As the volume increases, the rate of change of the pressure decreases.

**8. a)** $0 < b < 1$, vertical compression by factor of b; $b > 1$, vertical stretch by factor of b

b) i)

ii)

iii)
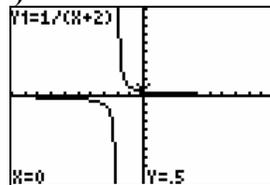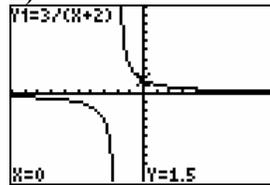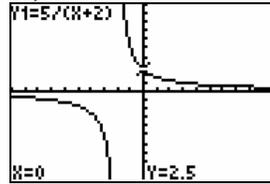
9. $\{x \in \mathbb{R}, x \ne k180°, k \in \mathbb{I}\}$,

$\{y \in \mathbb{R}, y \le -1, y \ge 1\}$, vertical

asymptotes: $x = k180°, k \in \mathbb{I}$
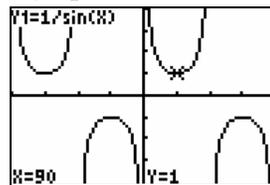
3.2 Reciprocal of a Quadratic Function

1.

As $x\to$	$f(x)\to$
-2^+	$+\infty$
-2^-	$-\infty$
-5^+	$-\infty$
-5^-	$+\infty$
$+\infty$	0
$-\infty$	0

2. a) $x = 1, x = 6$; $\{x \in \mathbb{R}, x \ne 6, x \ne 1\}$

b) none; $\{x \in \mathbb{R}\}$

3. a)

Interval	$x < 2$	$x > 2$
Sign of $f(x)$	+	+
Sign of Slope	+	−
Change in Slope	+	+

$y = \dfrac{1}{(x-2)^2}$

b)

Interval	$x<0$	$0<x<2$	$x=2$	$2<x<4$	$x>4$
Sign of $f(x)$	+	–	–	–	+
Sign of Slope	+	+	0	–	–
Change in Slope	+	–		–	+

$$y = \frac{5}{x(x-4)}$$

4. a) i) $x=2$, $x=-4$, $y=0$

ii) $\{x \in \square, x \neq 2, x \neq -4\}$

iii) no x-intercept, y-intercept $-\dfrac{1}{8}$

iv)
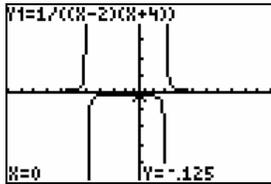
v)

Interval	$x<-4$	$-4<x<-1$	$x=-1$	$-1<x<2$	$x>2$
Sign of $f(x)$	+	–	–	–	+
Sign of Slope	+	+	0	–	–
Change in Slope	+	–		–	+

vi) $\left\{ y \in \square, y>0, y \leq -\dfrac{1}{9} \right\}$

vii) –0.03

b) i) $x=-4$, $y=0$ **ii)** $\{x \in \square, x \neq -4\}$

iii) no x-intercept, y-intercept $\dfrac{1}{16}$

iv)
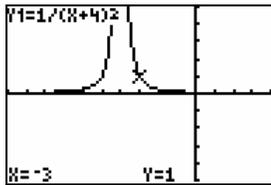
v)

Interval	$x<-4$	$x>-4$
Sign of $f(x)$	+	+
Sign of Slope	+	–
Change in Slope	+	+

vi) $\{y \in \square, y>0\}$ **vii)** –0.03

c) i) $y=0$ **ii)** $\{x \in \square\}$

iii) no x-intercept, y-intercept $\dfrac{1}{4}$

iv)
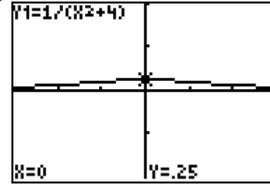
v)

Interval	$x<0$	$x=0$	$x>0$
Sign of $f(x)$	+	+	+
Sign of Slope	+	0	–
Change in Slope	+		+

vi) $\left\{ y \in \square, 0<y \leq \dfrac{1}{4} \right\}$

vii) 0

5. a)
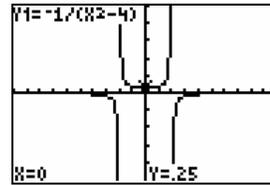
b)
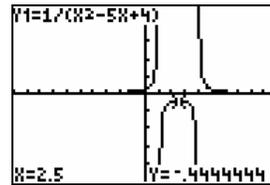
c)
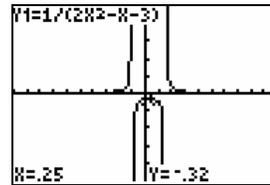
d)
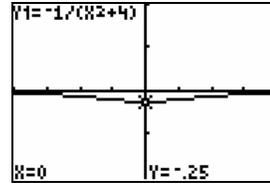
6. a) $\left(0, \dfrac{1}{4} \right)$ **b)** $\left(\dfrac{5}{2}, -\dfrac{4}{9} \right)$

c) $\left(\dfrac{1}{4}, -\dfrac{8}{25} \right)$ **d)** $\left(0, -\dfrac{1}{4} \right)$

7. a) $B = \dfrac{2880}{d^2}$

b)
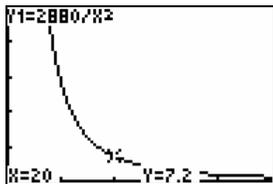
**c)** 20 lux   **d)** $d > \sqrt{28.8}$ m

8. a)
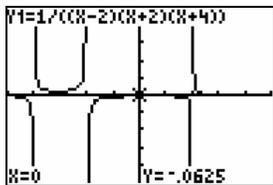
b)
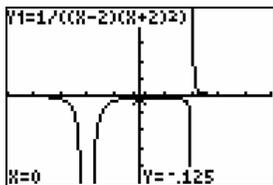
3.3 Rational Functions of the Form
$$f(x) = \frac{ax+b}{cx+d}$$

1. a) i) $x = -3$, $y = 1$

**ii)** $\{x \in \square, x \neq -3\}$, $\{y \in \square, y \neq 1\}$

iii)
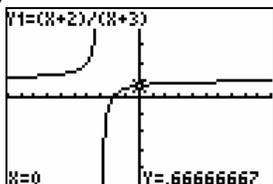
iv)

Interval	$x < -3$	$-3 < x < -2$	$x > -2$
Sign of $f(x)$	+	−	+
Sign of Slope	+	+	+
Change in Slope	+	−	−

b) i) $x = 2$, $y = -4$

ii) $\{x \in \square, x \neq 2\}$, $\{y \in \square, y \neq -4\}$

iii)
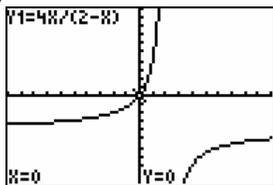
iv)

Interval	$x < 0$	$0 < x < 2$	$x > 2$
Sign of $f(x)$	−	+	−
Sign of Slope	+	+	+
Change in Slope	+	+	−

c) i) $x = \dfrac{4}{3}$, $y = -\dfrac{2}{3}$

ii) $\left\{x \in \square, x \neq \dfrac{4}{3}\right\}$, $\left\{y \in \square, y \neq -\dfrac{2}{3}\right\}$

iii)
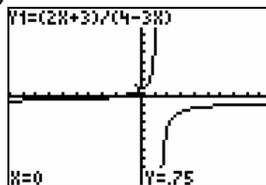
iv)

Interval	$x < -\dfrac{3}{2}$	$-\dfrac{3}{2} < x < \dfrac{4}{3}$	$x > \dfrac{4}{3}$
Sign of $f(x)$	−	+	−
Sign of Slope	+	+	+
Change in Slope	+	+	−

2. a) $x = -\dfrac{d}{c}$, $y = \dfrac{a}{c}$

b) $\left\{x \in \square, x \neq -\dfrac{d}{c}\right\}$, $\left\{y \in \square, y \neq \dfrac{a}{c}\right\}$

3. a) $y = \dfrac{2x+1}{x-2}$ **b)** $y = \dfrac{6x-3}{2x+1}$

4. a) $y = \dfrac{-2x+3}{x-3}$ **b)** $y = \dfrac{3x-9}{x-4}$

5. a)
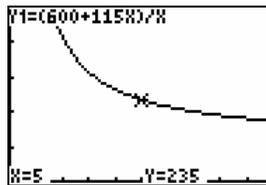
b) as $n \to +\infty$, $C \to 115$

c) approximately after 7 years

6. a) vertically stretched by a factor of 3, translated 2 units right and 4 units up

**b)** $f(x) = \dfrac{4x-5}{x-2}$

c) c and d determine the translation of 2 units to the right; a and c determine the translation of 4 units up.

7. a) If $k > 0$, vertically stretched by a factor of k; if $k < 0$, flipped about $x = p$ and vertically stretched by a factor of $|k|$; if $p > 0$, translated p units to the right; if $p < 0$, translated p units to the left; if $q > 0$, translated q units up; if $q < 0$, translated q units down

b) $f(x) = \dfrac{qx + (k - pq)}{x - p}$

c) $a = q$, $b = k - pq$, $c = 1$, $d = -p$

8. a)
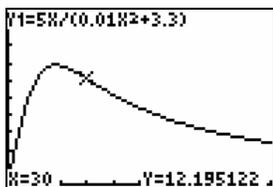
b) The maximum concentration is about 13.8 mg/mL, which occurs at 18.2 min.

c) Increasing the "5" vertically stretches the graph. Decreasing the "5" vertically compresses the graph. Changing the sign of the "5" flips the graph about the x-axis. Increasing the "0.01" vertically compresses the graph and shifts the maximum to the left. Decreasing the "0.01" vertically stretches the graph and shifts the maximum to the right.

3.4 Solve Rational Equations and Inequalities

1. a) $x = 1.1$ **b)** $x = 6$ or $x = -1$

c) $x = 3$ or $x = -\dfrac{1}{3}$ **d)** $x = 7$

2. a) $x \doteq 3.52$ or $x \doteq 0.28$
b) $x \doteq -0.61$

3. a) $x < -\dfrac{19}{2}$ or $-4 < x < \dfrac{3}{2}$

b) $-\dfrac{1}{3} < x \le \dfrac{6}{17}$ or $x > 3$

c) $x < -4$ or $\dfrac{1}{2} < x < 3$ or $x > 5$

d) $-4 < x \le -3$ or $-1 < x \le \dfrac{1}{2}$

4. a) $x = -\dfrac{20}{7}$

b) $x = 1$ or $x = \dfrac{3}{2}$ or 3

c) $x = \dfrac{1}{5}$ **d)** $x = \pm\sqrt{0.4}$

5. a) $x < -3$ or $0 < x < 1$
b) $x < -2$ or $x > 0$

6. more than 7 years old

7. a) 20 000 km **b)** 2×10^{10} joules

8.
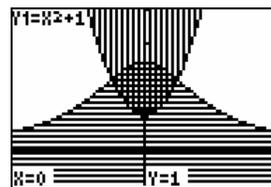
9. $A = 3$, $B = 5$

3.5 Making Connections With Rational Functions and Equations

1. a) 160 min
b) Yes. The upper limit is 40 g/L, because there is a horizontal asymptote.
c) $0 \le t \le 224$

2. a) $3 million **b)** $2.5 million
c) approximately 2001 and 2013
d) 2003
**e)** Do not hire more people after 2003 because sales decline after that.

3. a) approximately 977 N
b) approximately 974 N
c) 671 km

4. a) $R(n) = \dfrac{2n^2}{(n+3)^2}$

b)
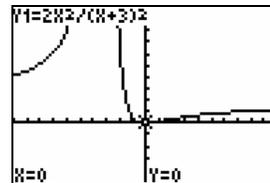
c) This is true only for positive integers, since as $n \to +\infty$, $R \to 2^-$.
d) $-1 \le n \le 3$
e) $-6 < n < -3$ or $-3 < n < -2$

5. a) $b > 0$
b)
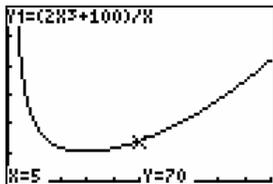
c) approximately 2.9 cm
d) $y = 2x^2$

Chapter 3 Review

1. a) $x = 2.5, y = 0$ **b)** $x = -5, y = 0$

2. $y = -\dfrac{3}{x+2}$

3. a)

b)

4. a) i) $\left\{ x \in \square, x \neq \dfrac{7}{4} \right\}, \{ y \in \square, y \neq 0 \}$
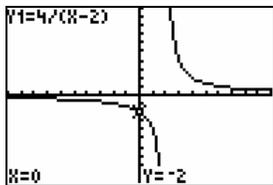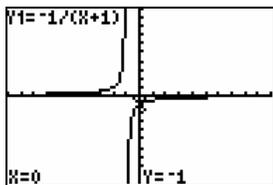
ii) no x-intercept, y-intercept $\dfrac{5}{7}$

b) i) $\{ x \in \square, x \neq 5 \}, \{ y \in \square, y \neq 0 \}$

ii) no x-intercept, y-intercept $-\dfrac{1}{5}$

5. a) i) $x = -2, x = 1, y = 0$

ii) no x-intercept, y-intercept $-\dfrac{1}{2}$

iii)

**iv)** $\{ x \in \square, x \neq -2, x \neq 1 \},$
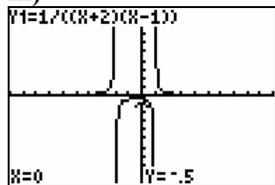
$\left\{ y \in \square, y > 0, y \leq -\dfrac{4}{9} \right\}$

v) $x < -2$ or $-2 < x < -\dfrac{1}{2}$

vi) $x < -2$ or $x > 1$
**b) i)** $x = -2, x = 2, y = 0$
ii) no x-intercept, y-intercept 2
iii)

iv) $\{ x \in \square, x \neq -2, x \neq 2 \},$
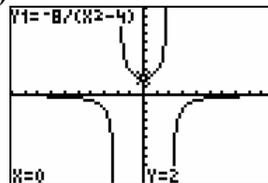
$\{ y \in \square, y < 0, y \geq 2 \}$

v) $0 < x < 2$ or $x > 2$ **vi)** $-2 < x < 2$
c) i) $y = 0$

ii) no x-intercept, y-intercept 1
iii)

iv) $\{ x \in \square \}, \{ y \in \square \mid 0 < y \leq 1 \}$
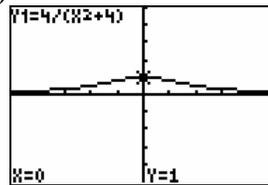
v) $x < 0$ **vi)** $\square$

6. $f(x) = -\dfrac{1}{(x+2)(x-6)}$

7. a) $x = -2, y = 1, \{ x \in \square, x \neq -2 \},$

$\{ y \in \square, y \neq 1 \}, x$-intercept 3,

y-intercept $-\dfrac{3}{2}$; for $x < -2, f(x)$ is

positive and increasing and the slope
is positive and increasing; for
$-2 < x < 3, f(x)$ is negative and
increasing and the slope is positive
and decreasing; for $x > 3, f(x)$ is
positive and increasing and the slope
is positive and decreasing
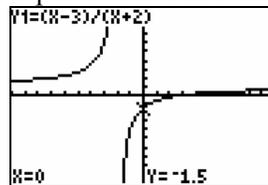
b) $x = -\dfrac{1}{2}$, $y = -\dfrac{3}{2}$, $\left\{ x \in \square, x \neq -\dfrac{1}{2} \right\}$,

$\left\{ y \in \square, y \neq -\dfrac{3}{2} \right\}$, x-intercept $\dfrac{4}{3}$,

y-intercept 4; for $x < -\dfrac{1}{2}$, $f(x)$ is

negative and decreasing and the slope
is negative and decreasing;

$-\dfrac{1}{2} < x < \dfrac{4}{3}$, $f(x)$ is positive and

decreasing and the slope is negative

and increasing; for $x > \dfrac{4}{3}$, $f(x)$ is

negative and decreasing and the slope
is negative and increasing
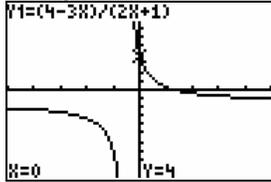
8. $y = \dfrac{3x + 3}{x + 2}$

9. $y = \dfrac{x - 2}{x + 3}$

10. a) $x = -\dfrac{3}{2}$ or $x = 2$

b) $x < -5$ or $-1 \leq x < 3$

11. a) $x \square -1.81$ or $x \square 0.14$ or $x \square 7.66$

b) $x < -2.95$ or $-1 < x < 2$

12. a) 12 000 **b)** 2004

c) Yes; as $t \to +\infty, P \to 40^-$.

Chapter 3 Test
1. a)

b)

c)
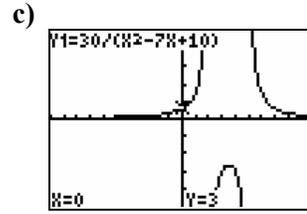
2. a) $y = \dfrac{6}{x^2 + 1}$ **b)** $y = \dfrac{3x - 2}{x + 2}$

c) $y = \dfrac{9}{(x + 4)(x - 2)}$

3. a) $\{ x \in \square, x \neq -4, x \neq 2 \}$,

$\{ y \in \square, y > 0, y \leq -1 \}$

b) $x < -4$ or $-4 < x < -1$

c) $-4 < x < 2$

4. $x = 0$ or $x = 7$

5. $-4 < x < -3$ or $x > 2$

6. a)
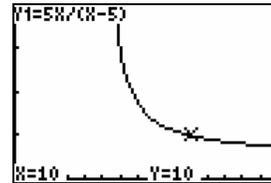
b) 11.25 cm **c)** 5 cm

d) greater than 21.7 cm