

## 6.4 Power Law of Logarithms

BLM 6-5

- Use the power law to evaluate without using a calculator.
  - $\log_3 27^7$
  - $\log_2 4^{-5}$
  - $\log_5 \sqrt[3]{25}$
  - $\log_8 \sqrt{2}$
- Solve for  $n$ , correct to three decimal places.
  - $5 = 2.5^n$
  - $2000 = 500(1.045)^n$
- A child is sitting on a moving swing. The horizontal displacement of the swing away from the vertical is modelled by the function  $a = 2.7(0.9)^n$ , where  $a$  is the displacement, in metres, and  $n$  is the number of swings since the child's father stopped pushing.
  - What is the displacement after the sixth swing?
  - After how many swings is the displacement less than 1 m?
- Use a calculator to evaluate, to one decimal place.
  - $\log_9 12$
  - $\log_{0.25} 52$
- Write as a single logarithm, then evaluate without a calculator.
  - $\frac{\log 16}{\log 4}$
  - $\frac{\log\left(\frac{8}{27}\right)}{\log\left(\frac{2}{3}\right)}$
- Solve, to two decimal places.
  - $\log 4^x = 7$
  - $12 = \log_3 4^m$
- Suppose the temperature of the atmosphere is increasing by 1% each year. The temperature  $T$  can be modelled by the function  $y = 1758.27 + \log_{1.01} T$ , where  $y$  is the year and  $T$  is the average temperature of the atmosphere, in degrees Celsius.
  - Verify that the average temperature was  $12^\circ\text{C}$  in 2008.
  - In what year will the average temperature be  $15^\circ\text{C}$ ?
- A particular radioactive isotope has a half-life of 400 years. The time required for only  $A\%$  of the original amount to remain is modelled by the function  $t = -400 \log_2 \left( \frac{A}{100} \right)$ , where  $t$  is the time, in years.
  - After how many years will only 25% of the original amount of radioactive isotope remain?
  - After 20 years, what percent of the original amount of radioactive isotope remains?
- To understand why  $\log_b 5$  is not defined when  $b = 1$ , do the following.
  - Let  $x = \log_1 5$ , convert to exponential form, and try to solve for  $x$ .
  - Use the power law to express  $\log_1 5$  as a ratio of common logarithms. In both cases, explain why  $b \neq 1$ .
- Use Technology** Graph  $y = \log_a x$  as  $a$  varies from 2 to 10.
  - What kind of transformation appears to be occurring as a result of changing the value of  $a$ ?
  - Write  $y = \log_a x$  in terms of common logarithms. Explain how this way of writing the function relates to your answer to part a).