

## 7.2 Techniques for Solving Exponential Equations

BLM 7-3

- Solve for  $t$ . Round answers to two decimal places.
  - $5 = (1.12)^t$
  - $7 = 2(1.06)^{2t+1}$
  - $4 = \left(\frac{1}{8}\right)^{t-3}$
  - $6.5 = 1.5\left(\frac{1}{2}\right)^{\frac{t}{15}}$
- The amplitude of a vibration decays according to the equation  $a = 4.5(0.65)^{\frac{t}{60}}$ , where  $a$  is the size of the amplitude, in centimetres, at time  $t$ , in seconds.
  - What will the amplitude be after 30 s, to the nearest tenth of a centimetre?
  - How long (to the nearest second) will it take the amplitude to decay to 30% of its original value?
  - Would it take twice as long as your answer for part b) for the amplitude to decay to 15% of its original value? Explain.
- Solve each equation. Leave answers in exact form.
  - $3^{x-3} = 4^{x-4}$
  - $7^{2x-3} = 2^{-3x}$
- Consider the equation  $3^{2x} - 4(3)^x + 1 = 0$ .
  - Write the equation in the form  $az^2 + bz + c = 0$ , where  $z = 3^x$ .
  - Solve the equation using the quadratic formula.
  - Identify any extraneous roots.
- A 25-mg sample of radioactive francium decays to 20 mg in 13.6 min.
  - Determine the half-life of francium.
  - Graph the amount of francium remaining over a 2-h period.
  - On the same set of axes as part b), sketch the graph of the amount of an element if
    - it has the same half-life as francium, but the original amount was 30 mg
    - it has the same original mass as the francium sample, but its half-life is twice as long
- Solve and check for extraneous roots. Leave answers in exact form.
  - $3^{2x} - 6(3)^x - 27 = 0$
  - $2^x = 2 + 3(2)^{-x}$
  - $5^{2x} + 1 = 3(5)^x$
- The population of a colony of bacteria grows according to the formula  $P(t) = 4(1.40)^{\frac{t}{24}}$ , where  $P$  is the population, in thousands, and  $t$  is the time, in hours.
  - How long does it take the population to reach 10 000, to the nearest hour?
  - Calculate the time it takes for the population to double, to the nearest hour.
- The maximum height that a ball reaches after bounce number  $n$  is given by the equation  $H = 2.0(0.90)^n$ , where  $H$  is the height, in metres.
  - What is the ball's maximum height after the fifth bounce?
  - What is the first bounce after which the maximum height is less than 10 cm?
- Solve  $4^x + 2^{x+2} = 32$ .
- Rewrite the equation  $P(t) = 4(1.40)^{\frac{t}{24}}$  with base 1.40 replaced with 2.
- The general equation for population growth is  $P(t) = P_0\left(1 + \frac{R}{100}\right)^{\frac{t}{t_0}}$ , where  $R$  is the growth rate, in percent, over time period  $t_0$ . Suppose a population grew from 10 000 to 25 000 in six years. If time is measured in years, calculate
  - the yearly growth rate
  - the growth rate per decade (10 years)