Name: ______________________________ Date: ________________________

7.5 Making Connections: Mathematical Modelling With Exponential and Logarithmic Equations

BLM 7–8

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1. Refer to the two exponential models developed in Example 1 of the text for the population of Decimal Point since 1920.

$P = 1006(1.016)^t$ $\quad$ $P = 1000 \times 2^{\frac{t}{43.5}}$

Decimal Point will require an upgrade to its water infrastructure once the population reaches 8000. The town must start to plan for the cost 15 years before the work must be done. When should the town start its financial plan?

2. Refer to question 1. The population of Decimal Point could be modelled by using an exponential equation based on its "tripling time" (the time it takes the population to triple).

The equation would be $P = 1000 \times 3^{\frac{t}{T}}$, where T is the tripling time.

a) Calculate the value of T:

i) Use one of the existing equations developed in Example 1 (substitute 3000 for P and solve for t).

ii) Use Technology Create a dynamic model with a single slider for T and adjust the value of T until the curve of best fit is obtained.

b) Compare the values of T obtained in part a). If there is a difference, explain why.

3. Decimal Point needs \$80 000 to begin building a recreation centre. Two investment option for a surplus of \$50 000 were explored in Example 2 of the text.

Investment Option	Years Before Work Can Begin
Lakeland Savings Bond	7.96
Northern Equity Mutual Fund	7.95

Suppose that a financial company offered Decimal Point another investment option— an interest rate of 5.5% compounded quarterly, with the condition that should Decimal Point leave the money invested for longer than 7 years, Decimal Point would be paid a bonus of 6% of the original investment when the reserve fund is withdrawn. Should this option be considered? Justify your reasoning.

4. Use Technology A tub of warm water is left outside on a mild winter day. Its temperature is recorded every 4 min, as shown in the table.

Time (min)	Temperature (°C)
0	40
4	23
8	13
12	8
16	4
20	3

a) Create a scatter plot of temperature versus time. Does the curve appear linear, quadratic, or exponential? Justify your answer.

b) Perform linear, quadratic, and exponential regression on the data. Record the equations, correct to two decimal places.

c) Can any of the models be discounted immediately? Explain.

d) Use both the quadratic model and the exponential model to calculate the temperature of the tub of water after 25 min. Explain how this calculation helps you decide which model is better.

e) Use the model you consider to be the best to calculate the outdoor temperature. (Hint: after a very long time, the tub of water will be the same temperature as the outdoors.)

5. The actual equation that models heating or cooling of material is $T=(T_0-T_A)(2)^{-\frac{t}{D}}+T_A$, where T is the temperature at time t, T_0 is the initial temperature of the material, T_A is the "ambient" temperature (the temperature of the surroundings), and D is the time it takes for the difference between the temperature of the material and the ambient temperature to be halved.

An object placed in an oven has temperature modelled by $T=-327(2)^{-\frac{t}{8}}+350$.

a) How long does it take the object's temperature to reach 325°C?

b) What was the original temperature of the object?

6. **Use Technology** Refer to question 5. The table shows the temperature of material as a function of time.

Time (min)	Temperature (°C)
0	6
5	12
12	18
17	21
27	25
31	26
36	27
43	28
55	29

a) Make a scatter plot of the data.

b) Estimate the ambient temperature from your plot. You now know the values of T_0 and T_A in $T=(T_0-T_A)(2)^{-\frac{t}{D}}+T_A$.

c) Use the estimate for the ambient temperature to transform the data so that it has an asymptote of 0 instead of the ambient temperature. Create a new scatter plot of this transformed data.

d) Perform an exponential regression on the transformed data. This equation will be of the form $y=k(b)^x$. Use this equation to calculate the value of D in $T=(T_0-T_A)(2)^{-\frac{t}{D}}+T_A$.

(Hint: remember that $2^{-\frac{t}{D}}=\left(2^{-\frac{1}{D}}\right)^t$, so the base in your regression will equal $2^{-\frac{1}{D}}$.)

e) Write an equation that models the given temperature data.