Prerequisite Skills

1. a) c^{12} **b)** $12x^3y^4$ **c)** $10mn^3$

2. a) w^5 **b)** $\frac{p^2}{2q}$ **c)** $-3y^2$

3. a) x^{15} **b)** m^8n^{12} **c)** $27p^6q^3$

4. a) $\frac{n}{m^2}$ **b)** x^5z^2

5. a) $x = 3$ or $x = -1$

b) $m = \frac{4}{3}$ or $m = -\frac{3}{2}$

c) $p = -\frac{3}{2}$

d) $r = 4$ or $r = \frac{1}{2}$

e) $y = -\frac{2}{3}$ or $y = -\frac{5}{2}$

6. a) $x = \frac{1 \pm \sqrt{33}}{4}$ **b)** $v = 1 \pm \sqrt{6}$

c) $d = \frac{1 \pm \sqrt{17}}{8}$ **d)** $m = \frac{-1 \pm \sqrt{13}}{3}$

e) $y = \frac{-6 \pm 5\sqrt{2}}{2}$

7. **a)** $2\sqrt{3}$ **b)** $9\sqrt{3}$

c) $2 + \sqrt{2}$ **d)** $-3 + 2\sqrt{2}$

8. a) 8 **b)** –3

c) $\frac{2}{3}$ **d)** $-\frac{3}{2}$

9. a) 2.322 **b)** 0.528

c) 3.322 **d)** 2.710

10. a) 1.40 **b)** 7.85

c) 1.30 **d)** 0.54

7.1 Equivalent Forms of Exponential Equations

1. a) 4^9 **b)** 2^{-10} **c)** $4^{\frac{3\log 5}{\log 4}}$

2. a) $3^{\frac{4}{3}}$ **b)** 3^1

3. a) $x = -10$ **b)** $t = 28$ **c)** $x = \frac{17}{9}$

4. a) $x = -\frac{1}{3}$ **b)** $x = -\frac{1}{3}$

5. $x = \frac{17}{7}$

6. a) i) $x = \log 5$ **ii)** $x = \log 3$

iii) $x = \log 7$

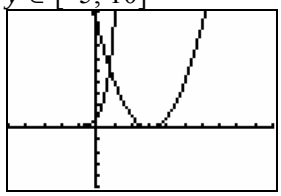
b) $x = \log b$

7. a) $x = \frac{11}{15}$ **b)** $x = \frac{11}{15}$

c) $x = \frac{11}{15}$

d) Answers may vary.

8. a) Window variables $x \in [-5, 10]$, $y \in [-5, 10]$



b) approximately $x \leq 0.75$

c) $x \leq 0.74$

9. Answers may vary. Sample answers:

a) 4, 4, 5.5

b) 4, 4000, 1 452 600

c) exponential

7.2 Techniques for Solving Exponential Equations

1. a) 14.20 **b)** 10.25

c) 2.33 **d)** –31.73

2. a) 3.6 cm **b)** 168 s

c) No, 264 s

3. a) $x = \frac{3\log 3 - 4\log 4}{\log 3 - \log 4}$

b) $x = \frac{3\log 7}{2\log 7 + 3\log 2}$

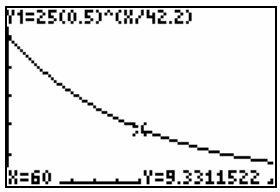
4. a) $z^2 - 4z + 1 = 0$

b) $x = \frac{\log\left(2 \pm \sqrt{3}\right)}{\log 3}$

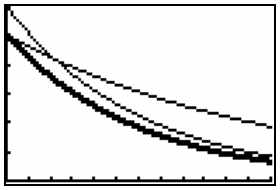
c) no extraneous roots

5. a) approximately 42.2 min

b)



c) i) $y = 30\left(\frac{1}{2}\right)^{\frac{t}{42.2}}$ **ii)** $y = 25\left(\frac{1}{2}\right)^{\frac{t}{84.4}}$



6. a) $x = 2$ **b)** $x = \frac{\log 3}{\log 2}$

c) $x = \frac{\log\left(\frac{3 \pm \sqrt{5}}{2}\right)}{\log 5}$

7. a) 65 h **b)** 49 h

8. a) 1.18 m **b)** 29

9. $x = 2$

10. $P(t) = 4(2)^{\frac{t \log 1.4}{24 \log 2}}$

11. a) 16.5% **b)** 360.5 %

7.3 Product and Quotient Laws of Logarithms

1. a) $\log 11$, 1.041 **b)** $\log_5 60$, 2.544

2. a) $\log_4 \frac{2xy}{z-3}$, $x > 0, y > 0, z > 3$

b) $\log \frac{ac^3}{b^2}$, $a > 0, b > 0, c > 0$

3. a) 2 **b)** 2 **c)** 1

4. a) $2\log x + \log y - 3\log z$

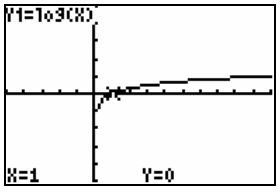
b) $\log_3 5 + \frac{1}{3}\log_3 m - 4\log_3 n$

5. a) $4\log m, m > 0$ **b)** $\log p, p > 0$

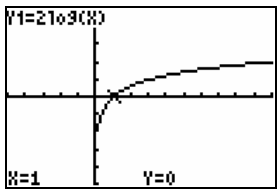
c) $\log \frac{x^2 - 5x - 6}{x-3}, x > 6$

d) $\log(3x-2), x > \frac{3}{2}$

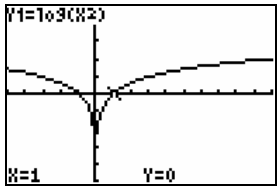
6. a)



b)



c)



d) same for $x > 0$; power law

7. a) approximately 22 years

b) The first job offer pays better until year 22.

c) Yes. If Jorge retires after 30 years, the second job offer has the better pension plan.

8. a) $y = \frac{(x-3)^2}{x}$, $\{x \in \mathbb{R}, x > 3\}$

b) $y = \frac{x}{1000 - x}$, $\{x \in \mathbb{R}, x > 0, x \neq 1000\}$

c) $y = \frac{x^2}{(x+5)^2}$, $\{x \in \mathbb{R}, x > 0\}$

7.4 Techniques for Solving Logarithmic Equations

1. a) $x = 95$ **b)** $p = 8 + \sqrt{10}$

c) $m = 4$

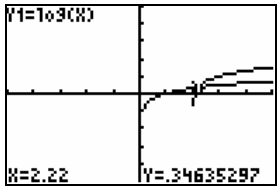
2. a) $x = 12$ **b)** $v = 99$

3. a) $x = 6$ **b)** $x = 10$ **c)** $m = 100$

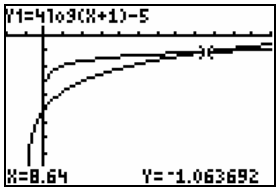
4. a) $x = 5$ **b)** $x = \frac{5 + \sqrt{389}}{2}$

5. a) $x = 4$ **b)** $x = \frac{1 + \sqrt{425}}{2}$

6. a) $x \approx 2.22$



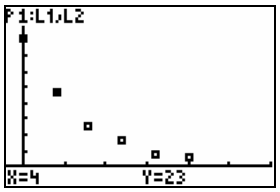
b) $x \approx 0.001$ and $x \approx 8.64$



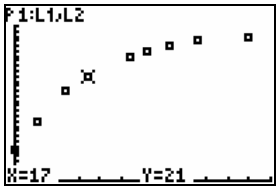
7. a) 6 **b)** \$2800 **c)** 6.5%
8. $x = 32$
9. a) approximately 3.32
b) same

7.5 Making Connections: Mathematical Modelling With Exponential and Logarithmic Equations

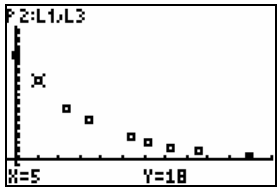
1. 2036
2. a) 68.9 years
b) Answers may vary. Sample answer: There are differences due to judgment of best fit.
3. Yes; reaches \$80 000 in 7.90 years.
4. a)



Answers may vary. Sample answer: exponential
b) $T = -1.76t + 32.81$,
$T = 0.12t^2 - 4.09t + 39$,
$T = 38.87(0.88)^t$
c) Linear; data does not follow a straight line.
d) Quadratic: 11.75°C; exponential: 1.59°C; cannot be quadratic, since it predicts water temperature will increase.
e) 0°C
5. a) approximately 30 min
b) 23°C
6. a)



b) 30°C
c)

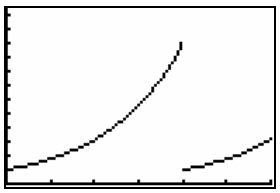


d) $y \approx 24.00 \times 0.94^x$; $D \approx 11$
e) $T = -24(2)^{-\frac{x}{11}} + 30$

Chapter 7 Review

1. a) 8^3 **b)** 8^{-2} **c)** $8^{\frac{\frac{1}{3}\log 32}{\log 8}}$
2. a) $8^{1.441}$ **b)** $8^{0.723}$
3. a) $x = \frac{10}{9}$ **b)** $x = \frac{6}{5}$
4. $x = \frac{5\log 4}{2\log 4 - \log 7}$
5. –0.72
6. a) approximately 82.24 h
b) 213 h
7. a) $x = 0$ or $x = 1$ **b)** $x = \frac{\log 3}{\log 2}$
8. a) 1.58 months **b)** 6.8 months
9. a) 1 **b)** 2
10. $\log_3 5(x-2)^2$
11. $\log a + 4\log b - \frac{1}{4}\log c$
12. a) $\log\frac{x+2}{3}, x > 6$
b) $\log(x^2 + 3x + 9), x > 3$
13. a) $x = \frac{2}{3}$ **b)** $x = \frac{1}{2}$
14. 1.19
15. a) approximately 15 min
b) approximately 423°C
16. a) $P = 200(1.122)^t$
b) 1999
c) approximately 6 years
d) $P = 200(1.122)^{t-20}$

e)



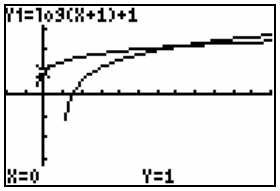
Chapter 7 Test

1. a) $\log_6 36$; 2 **b)** $\log_3 9$; 2

2. a) $x = 2$ **b)** $x = -15$

c) $x = \frac{41}{2}$ **d)** $x = 2$

3. a)



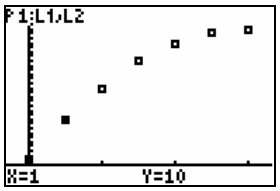
b) 14.4 **c)** $7 + \sqrt{55}$

4. a) 4.33 years

b) approximately 6 years

5. approximately 89 min

6. a)



b) Yes; vertical reflection and translation, approaching horizontal asymptote.

c) Yes; vertex (6, 30).

d) i) the height of an object thrown into the air over time (height reaches a maximum)

ii) the warming of an object over time (up to 30°C)