

## 8.3 Composite Functions

BLM 8-4

1. Let  $f(x) = 2x + 1$  and  $g(x) = \frac{2}{x}$ .
  - a) Determine a simplified algebraic model for each.
    - i)  $y = f(g(x))$
    - ii)  $y = g(g(x))$
  - b) Evaluate.
    - i)  $y = f(f(1))$
    - ii)  $y = g(f(3))$
  
2. The number of items,  $n(t)$ , made per week by a company is given by  $n(t) = 2000 + 40t$ , where  $t$  is time, in weeks. The profit of the company, in thousands of dollars, depends on the number of items produced, according to the formula  $P(n) = 3\sqrt{n - 1000}$ .
  - a) Determine the equation for weekly profit of the company.
  - b) Use graphing technology to graph the weekly profit for the next 2 years.
  - c) Use algebraic techniques to determine when the weekly profit first reaches \$180 000.
  - d) Use a graphical method to check your answer to part c).
  
3. Let  $f(x) = \frac{2}{x+1}$  and  $g(x) = x + 3$ .
  - a) Write the equations for  $y = f^{-1}(x)$  and  $y = g^{-1}(x)$ .
  - b) Determine a simplified algebraic model for each.
    - i)  $y = (f^{-1} \circ g^{-1})(x)$
    - ii)  $y = (g^{-1} \circ f^{-1})(x)$
    - iii)  $y = (f \circ g)^{-1}(x)$
    - iv)  $y = (g \circ f)^{-1}(x)$
  - c) Which of the equations in part b) are the same?
  
4. Let  $f(x) = x^2$  and  $g(x) = \cos x$ .
  - a) Write the equation for  $y = f(g(x))$ .
  - b) Use graphing technology to graph  $y = f(g(x))$ .
  - c) Is  $y = f(g(x))$  periodic?
  - d) Write the equation for  $y = g(f(x))$ .
  - e) Use graphing technology to graph  $y = g(f(x))$ .
  - f) Is  $y = g(f(x))$  periodic?
  
5. Define  $f^n$  by the rule that
 
$$f^n(x) = \underbrace{f(f(f \dots f(x)))}_{n \text{ times}}$$
 For example,  $f^3(x) = f(f(f(x)))$ .
  - a) If  $f(x) = \sqrt{x}$ , calculate  $f^4(65\,536)$ .
  - b) For  $f(x) = \frac{1}{x}$ , calculate each value.
    - i)  $f^2(5)$
    - ii)  $f^4(5)$
    - iii)  $f^{200}(5)$
    - iv)  $f^{1001}(5)$
  - c) Determine what happens to the value of  $f^n(x)$  as  $n \rightarrow +\infty$  for each function.
    - i)  $f(x) = \sqrt{x}$
    - ii)  $f(x) = x^2, 0 < x < 1$
    - iii)  $f(x) = x^2, x = 1$
    - iv)  $f(x) = x^2, x > 1$