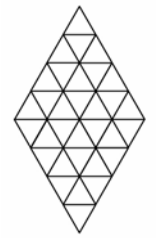
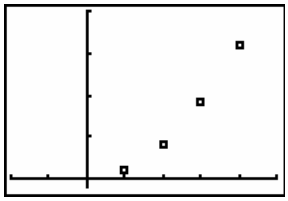
Prerequisite Skills

1. a)



b) quadratic

c)

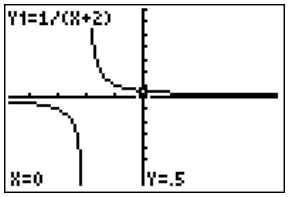


d) $T = 2n^2$

2. a) even values $2k, k \in \mathbb{I}$

b) odd values $2k-1, k \in \mathbb{I}$

3.



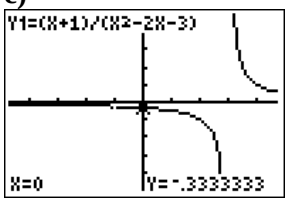
$\{x \in \mathbb{R}, x \neq -2\}$, $\{y \in \mathbb{R}, y \neq 0\}$

4. a) $y = \dfrac{1}{x-3}$, $x \neq 3, x \neq -1, y \neq 0$

b) $\{x \in \mathbb{R}, x \neq 3, x \neq -1\}$,

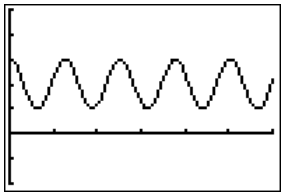
$\left\{y \in \mathbb{R}, y \neq 0, y \neq -\dfrac{1}{4}\right\}$

c)



d) asymptote $x = 3$, $y = 0$, hole at $\left(-1, -\dfrac{1}{4}\right)$

5.



6. a) $f^{-1}(x) = \pm\sqrt{x-5} - 2$

b) $f^{-1}(x) = \dfrac{1}{x-4} + 2$

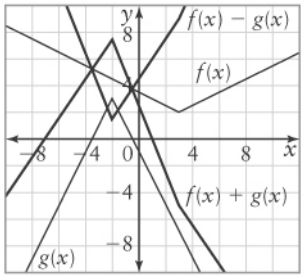
7. The inverse of 6 b) is a function, because the original function is 1 to 1.

8.1 Sums and Differences of Functions

1. a) $h(x) = 5x - 2$ **b)** 8

c) $k(x) = x - 8$ **d)** –6

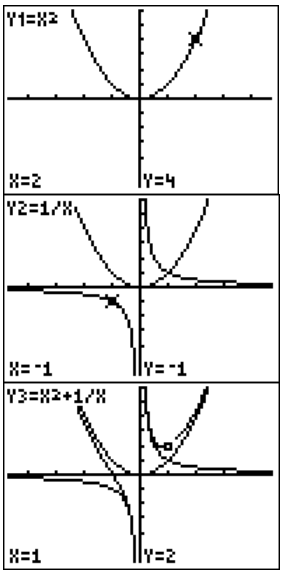
2. a)



b) $\{x \in \mathbb{R}\}$, $\{x \in \mathbb{R}, x \leq 7.5\}$

c) $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq 1.5\}$

3. a)



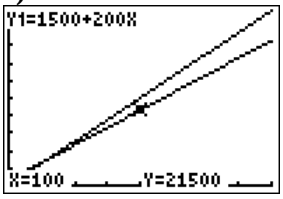
b) $\{x \in \mathbb{R}, x \neq 0\}$, $\{y \in \mathbb{R}\}$

c) $f(x) = x^2$, as $x \to +\infty$, $f(x) \to x^2$

4. a) $C(n) = 1500 + 200n$

b) $R(n) = 250n$

c)

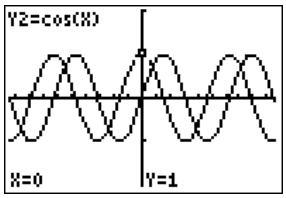


d) 30 **e)** $P(n) = 50n - 1500$

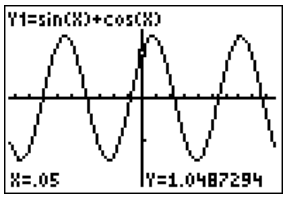
f) $\{n \in \mathbb{I}, 0 \le n \le 200\}$,
Range: $\{-1500, -1450, -1400, \cdots, 8500\}$

g) \$175 per unit sold

5. a)

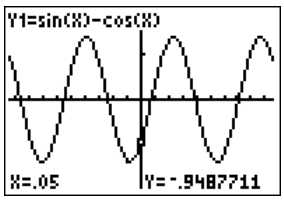


b)



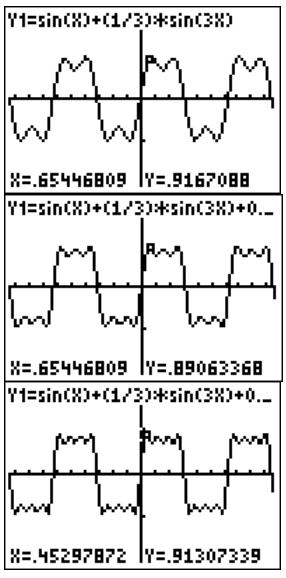
c) $y = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

d)

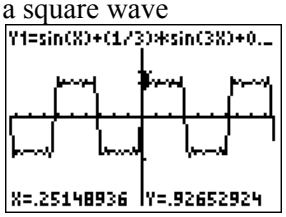


e) $y = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$

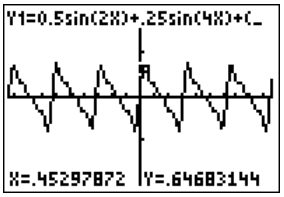
6. a)



b) a square wave



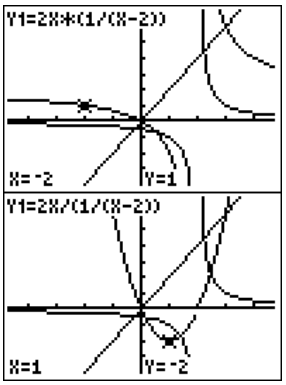
c) a saw-tooth wave



8.2 Products and Quotients of Functions

1. a) for $f(x)$: $\{x \in \mathbb{I}\}$, $\{y \in \mathbb{I}\}$, for $g(x)$:
$\{x \in \mathbb{I}, x \ne 2\}, \{y \in \mathbb{I}, y \ne 0\}$

b)



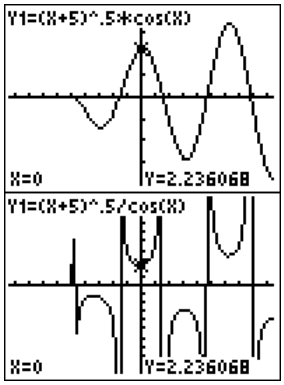
c) for *fg:* $\{x \in \mathbb{I}, x \ne 2\}, \{y \in \mathbb{I}, y \ne 2\}$;
for $\frac{f}{g}$: $\{x \in \mathbb{I}, x \ne 2\}$,
$\{y \in \mathbb{I}, y \ne 0, y \ge -2\}$

d) for fg, vertical asymptote $x = 2$,
horizontal asymptote $y = 2$;
for $\frac{f}{g}$, hole at (2, 0)

2. a) for $f(x)$: $\{x \in \mathbb{I}, x \ge -5\}$,
$\{y \in \mathbb{I}, y \ge 0\}$; for $g(x)$: $\{x \in \mathbb{I}\}$,
$\{y \in \mathbb{I}, -1 \le y \le 1\}$

b) $f(x)g(x) = \left(\sqrt{x+5}\right)\cos x$,
$\frac{f(x)}{g(x)} = \frac{\sqrt{x+5}}{\cos x}$

c)



d) for *fg:* $\{x \in R, x \geq -5\}$, $\{y \in R\}$; for $\frac{f}{g}$:

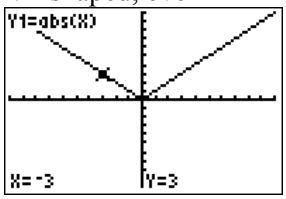
$\left\{x \in \mathbb{R}, x \geq 5, x \neq (2k-1)\frac{\pi}{2}, k \in \mathbb{I}\right\}$,

$\{y \in \mathbb{R}, y \geq 0 \text{ or } y \leq -1.30663\}$

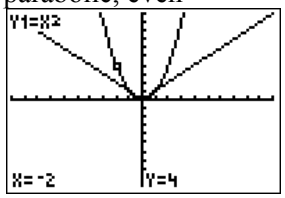
e) vertical asymptotes at

$x = (2k-1)\frac{\pi}{2}, k \in \mathbb{I}$

3. a) "V"-shaped, even

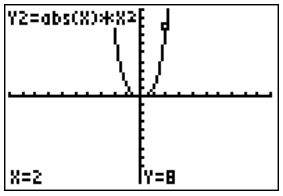


b) parabolic, even

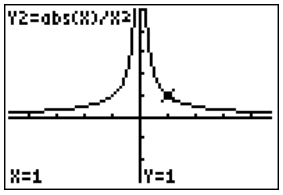


c) both even

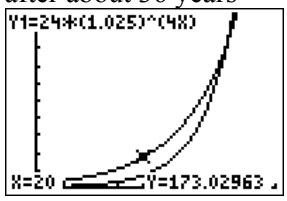
d) $y = f(x)g(x)$



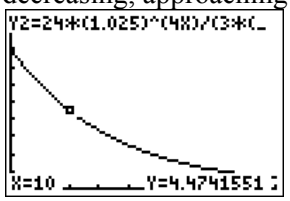
$y = \frac{f(x)}{g(x)}$



4. a) both increase, but *P* increases faster after about 36 years



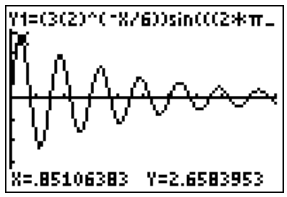
b) decreasing, approaching 0



c) \$8000/person, \$5983/person

d) yes, after approximately 10 years

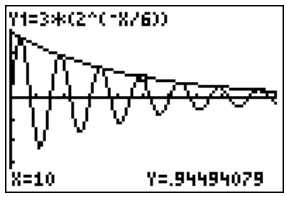
5. b)



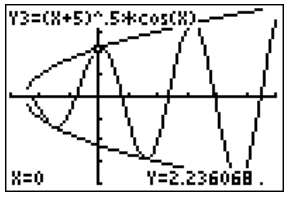
c) makes the sine curve's amplitude get smaller

6. d) for functions, odd × odd = even, odd × even = odd, and even × even = even. This is exactly the opposite of multiplication of numbers.

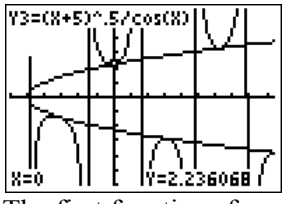
7. a)



b)



c)



d) The first functions form "boundaries" for the combination functions.

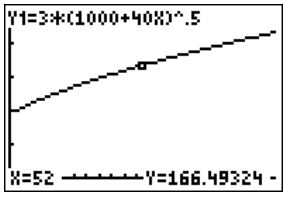
8.3 Composite Functions

1. a) i) $y = \frac{4}{x} + 1$ **ii)** $y = x$

b) i) 7 **ii)** $\frac{2}{7}$

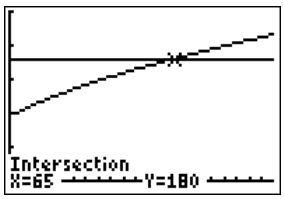
2. a) $P(t) = 3\sqrt{1000 + 40t}$

b)



c) 65 weeks

d)



3. a) $f^{-1}(x) = \frac{2}{x} - 1$, $g^{-1}(x) = x - 3$

b) i) $\left(f^{-1} \circ g^{-1}\right)(x) = \frac{2}{x-3} - 1$

ii) $\left(g^{-1} \circ f^{-1}\right)(x) = \frac{2}{x} - 4$

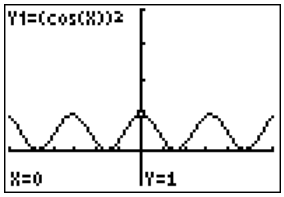
iii) $\left(f \circ g\right)^{-1}(x) = \frac{2}{x} - 4$

iv) $\left(g \circ f\right)^{-1}(x) = \frac{2}{x-3} - 1$

c) $\left(g^{-1} \circ f^{-1}\right)(x) = \left(f \circ g\right)^{-1}(x)$,
$\left(f^{-1} \circ g^{-1}\right)(x) = \left(g \circ f\right)^{-1}(x)$

4. a) $y = \cos^2 x$

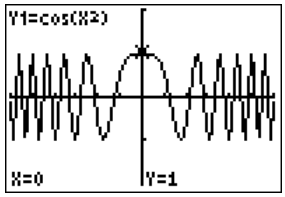
b)



c) yes, period π

d) $y = \cos\left(x^2\right)$

e)



f) no

5. a) 2

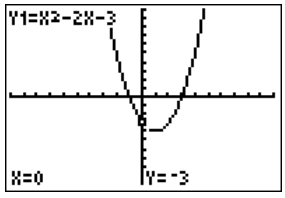
b) i) 5 **ii)** 5 **iii)** 5 **iv)** 0.2

c) i) 0 **ii)** 0 **iii)** 1 **iv)** ∞

8.4 Inequalities of Combined Functions

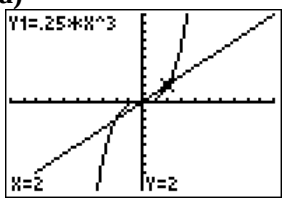
1. a) $x < -1$ or $x > 3$

b)



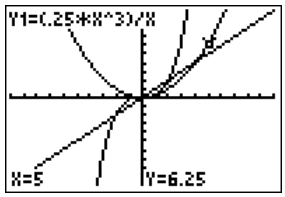
c) $x < -1$ or $x > 3$

2. a)



b) $(-\infty, -2)$ or $(0, 2)$

c)

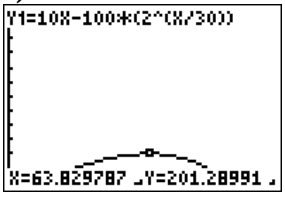


e) –2, 2 **f)** yes, when $y = 1$, $f(x) = g(x)$, which is where $f(x)$ changes from being less than $g(x)$ to greater than $g(x)$.

3. a) i) 14 and 99 panels **ii)** 63 panels

b) $P(n) = 10n - 100(2)^{\frac{n}{30}}$

c)

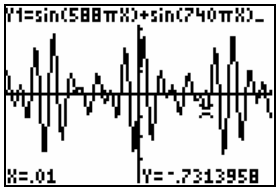


d) 63
e) i) the first break-even point would be less, the second break-even point would be greater, maximum profit would increase
ii) same effect as in i)
f) the number of panels produced in order to double the original cost

4. a) 40 units per year, 4%
b) 4%
c) occurs in 43 years
d) no **e)** no

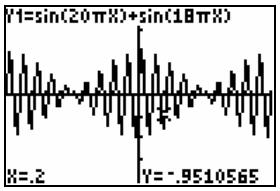
8.5 Making Connections: Modelling With Combined Functions

1. a)



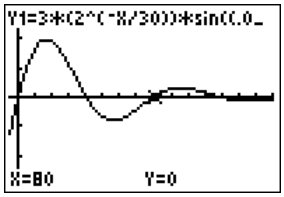
b) very similar

2. a)

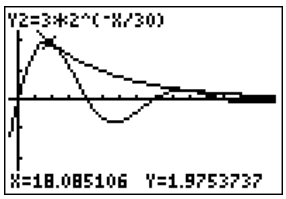


b) amplitude regularly increases and decreases
c) 1 s
d) pattern of loud-soft-loud-soft…

3. a)



b)

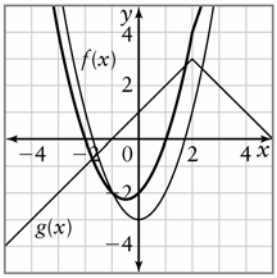


c) it forms a boundary for the wave

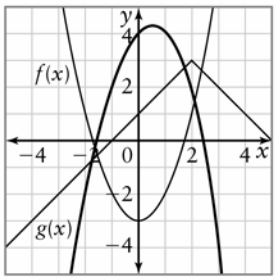
4. a) $y = 5\sin(2\pi x) + x^2$
b) $y = 5\sin(2\pi x) + 2x$
c) $y = 5\sin(2\pi x) + 2^x$

Chapter 8 Review

1. a)



b)

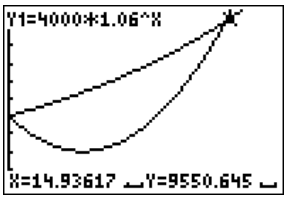


2. a) $y = -x^2 + x + 3$ **b)** $y = x$

3. a) Yes, since both original functions are odd, the product function will be even.
b) Yes, since one original function is odd and the other even, the product will be odd.
d) Yes, when you multiply numbers there is no restriction.
e) No, when you divide numbers, you cannot divide by 0. Any domain elements for $v(x) = \sin x$ that yield $y = 0$ must be excluded from the domain of $y = \dfrac{u(x)}{v(x)}$.

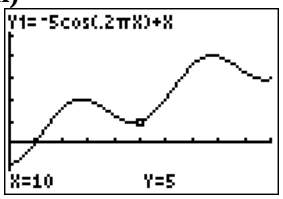
4. a) $y = 2x^2 - 3$
b) $y = 4x^2 + 14x + 12$
c) $y = x$

5. a)

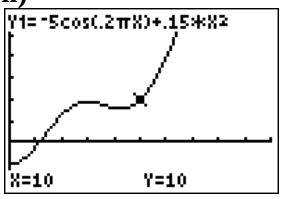


b) money fund **c)** 14.6 years

6. a) ii)



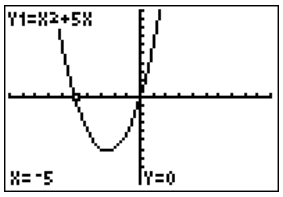
b) ii)



Chapter 8 Test

1. a) i) –6 **ii)** −5

b)



c) i) $(-\infty,-4)$ or $(1,+\infty)$

ii) $(-4,-2)$ or $(1,+\infty)$

d) –2

2. a) i) 12 **ii)** $\frac{19}{5}$

b) i) $f(x)+g(x)=x^2+5x$

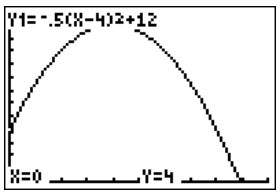
ii) $f(x)g(x)=x^3+6x^2+6x-4$

iii) $(f\circ g)(x)=x^2+4x$

3. a) $A(C)=\frac{C^2}{4\pi}$ **b)** $A(b(V))=6V^{\frac{2}{3}}$

4. a) 19 600

b)



Initially a poor team, winning only 4 of the 14 games in the first season, then becomes a good team after 4 years, then becomes poor again.

c) $0 \le t < 9$ starts in 2009, number of wins cannot be less than 0

d) No. $N(w(4))=48\ 400$

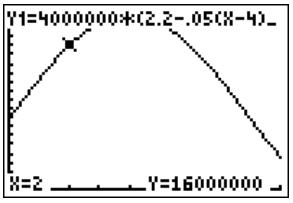
e) It will make team better right away, so would move vertex to the left.

f) It will make team better in 4 years, so would move vertex up.

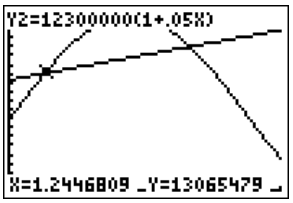
5. a)

$$R(t)=4\ 000\ 000\left(1+\frac{-\frac{1}{2}(t-4)^2+12}{10}\right)^2$$

b)



c)



d) 2010 to 2015