

8.3 Composite Functions

BLM 8-4

1. Let $f(x) = 2x + 1$ and $g(x) = \frac{2}{x}$.
- Determine a simplified algebraic model for each.
 - $y = f(g(x))$
 - $y = g(g(x))$
 - Evaluate.
 - $y = f(f(1))$
 - $y = g(f(3))$
2. The number of items, $n(t)$, made per week by a company is given by $n(t) = 2000 + 40t$, where t is time, in weeks. The profit of the company, in thousands of dollars, depends on the number of items produced, according to the formula $P(n) = 3\sqrt{n} - 1000$.
- Determine the equation for weekly profit of the company.
 - Use graphing technology to graph the weekly profit for the next 2 years.
 - Use algebraic techniques to determine when the weekly profit first reaches \$180 000.
 - Use a graphical method to check your answer to part c).
3. Let $f(x) = \frac{2}{x+1}$ and $g(x) = x + 3$.
- Write the equations for $y = f^{-1}(x)$ and $y = g^{-1}(x)$.
 - Determine a simplified algebraic model for each.
 - $y = (f^{-1} \circ g^{-1})(x)$
 - $y = (g^{-1} \circ f^{-1})(x)$
 - $y = (f \circ g)^{-1}(x)$
 - $y = (g \circ f)^{-1}(x)$
 - Which of the equations in part b) are the same?
4. Let $f(x) = x^2$ and $g(x) = \cos x$.
- Write the equation for $y = f(g(x))$.
 - Use graphing technology to graph $y = f(g(x))$.
 - Is $y = f(g(x))$ periodic?
 - Write the equation for $y = g(f(x))$.
 - Use graphing technology to graph $y = g(f(x))$.
 - Is $y = g(f(x))$ periodic?
5. Define f^n by the rule that $f^n(x) = \underbrace{f(f(f \dots f(x)))}_{n \text{ times}}$. For example, $f^3(x) = f(f(f(x)))$.
- If $f(x) = \sqrt{x}$, calculate $f^4(65\,536)$.
 - For $f(x) = \frac{1}{x}$, calculate each value.
 - $f^2(5)$
 - $f^4(5)$
 - $f^{200}(5)$
 - $f^{1001}(5)$
 - Determine what happens to the value of $f^n(x)$ as $n \rightarrow +\infty$ for each function.
 - $f(x) = \sqrt{x}$
 - $f(x) = x^2, 0 < x < 1$
 - $f(x) = x^2, x = 1$
 - $f(x) = x^2, x > 1$