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8.2 Products and Quotients of Functions

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1. Let $f(x) = 2x$ and $g(x) = \frac{1}{x-2}$.

a) Give the domain and range of each function.

b) Develop a graphical model for $y = f(x)g(x)$ and $y = \frac{f(x)}{g(x)}$.

c) Give the domain and range of each combined function.

d) Identify any asymptotes and holes in the combined functions

2. Let $f(x) = \sqrt{x+5}$ and $g(x) = \cos x$.

a) State the domain and range of each function.

b) Develop an algebraic model for $y = f(x)g(x)$ and $y = \frac{f(x)}{g(x)}$.

c) Use graphing technology to graph $y = f(x)g(x)$ and $y = \frac{f(x)}{g(x)}$

d) Give the domain and range of each combined function. (Estimate the range.)

e) Identify any asymptotes or holes in the combined functions.

3. Let $f(x) = |x|$ and $g(x) = x^2$

a) Graph $y = f(x)$ and describe its shape. Is *f*(*x*) even, odd, or neither?

b) Graph $y = g(x)$ on the same set of axes as $y = f(x)$ and describe its shape. Is *g*(*x*) even, odd, or neither?

c) Sketch graphs of $y = f(x)g(x)$ and $y = \frac{f(x)}{g(x)}$. Are the combined functions even, odd or neither?

d) Use graphing technology to confirm your answers in part c).

4. Use Technology The gross domestic product (GDP) of a country is modelled by $G(t) = 24(1.025)^{4t}$, where *G* is the GDP, in billions of dollars, and *t* is time, in years from now. The population of the country is growing according to $P(t) = 3(1.04)^{4t}$, where *P* is population, in millions, and *t* is time, in years, from now.

a) Graph *G* and *P* on the same set of axes, and describe their trends over the next 45 years.

b) Graph $y = \frac{G(t)}{P(t)}$ on a new set of axes, and describe its trend.

c) $y = \frac{G(t)}{P(t)}$ gives the GDP per person and provides a measure of the efficiency of workers. Calculate the GDP per person at the present time, and 5 years from now. Use your answer to part b) to describe what happens to worker efficiency as time goes on.

d) If the GDP per person falls below $4500, a recession is predicted. Will a recession occur in this country? If so, when?

5. Use Technology The horizontal distance of a child's swing from its resting position is modelled by $y = A(t)p(t)$, where $A(t) = 3(2)^{-\frac{t}{6}}$ gives the amplitude of the vibration, in metres, $p(t) = \sin\left(\frac{2\pi}{3}t\right)$ governs the back-and-forth part of the swing's motion, and *t* is time, in seconds.

a) From what you know by watching a real swing, sketch a possible position time graph for the motion of a real swing over an extended period of time.

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b) Graph $y = A(t)p(t)$, over a 20-s time period. Comment on any differences between this graph and your answer to part a).

c) What seems to be the effect of multiplying a sine function by the amplitude function?

6. The algebraic tests used to decide whether a function is even or odd are as follows.

• A function *f* is even provided $f(-x) = f(x)$.

• A function *f* is odd provided $f(-x) = -f(x)$.

a) Suppose *f* and *g* are both odd. Prove that $y = f(x)g(x)$ is even.

b) Suppose *f* is even and *g* is odd. Prove that $y = f(x)g(x)$ is odd.

c) Suppose *f* and *g* are both even. Prove that $y = f(x)g(x)$ is even.

d) Is the product of functions in any way analogous to the multiplication of numbers when it comes to evenness and oddness? Explain.

7. a) Graph $y = 3(2)^{-\frac{x}{6}}$ and $y = 3(2)^{-\frac{x}{6}} \sin\left(\frac{2\pi}{3}x\right)$ on the same set of axes.

b) Graph $y = \pm\sqrt{x+5}$ and $y = \left(\sqrt{x+5}\right)\cos x$ on the same set of axes.

c) Graph $y = \pm\sqrt{x+5}$ and $y = \frac{\sqrt{x+5}}{\cos x}$ on the same set of axes.

d) Describe how the graphs for parts a), b) and c) are similar.