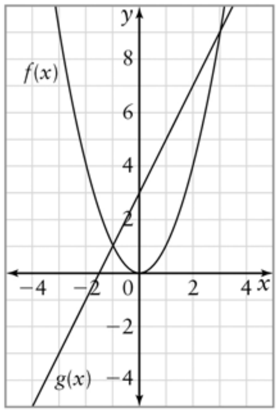
Name: ________________________________ Date: _________________________

8.4 Inequalities of Combined Functions

1. Consider the functions $f(x) = x^2$ and $g(x) = 2x + 3$. The functions are shown on the graph below.

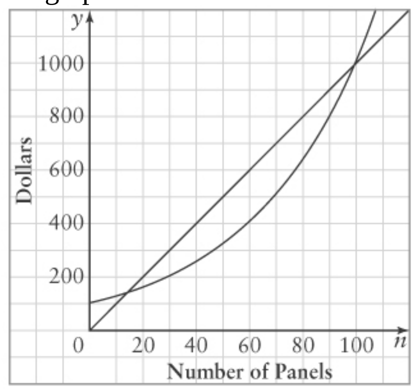


a) For what values of x is $f(x) > g(x)$?
b) Sketch the graph of $y = f(x) - g(x)$.
c) For what interval is the function $f - g$ positive?
d) Explain why your answers to parts a) and c) are the same.

2. Let $f(x) = \frac{1}{4}x^3$ and $g(x) = x$.

a) Graph f and g on the same set of axes.
b) For what values of x is $f(x) < g(x)$?
c) Graph the function $y = \frac{f(x)}{g(x)}$ on the same axes as the original functions.
d) Add the line $y = 1$ to your graph.
e) Calculate the x-value(s) for the point(s) of intersection of $y = \frac{f(x)}{g(x)}$ and $y = 1$.
f) Is the answer for part e) related to the answer for part b)? Explain.

3. A company makes solar panels. The company's revenue function, in dollars, is $R(n) = 10n$, where n is the number of panels produced. The cost function is $C(n) = 100(2)^{\frac{n}{30}}$. R and C are shown on the graph below.



a) Estimate from the graph
i) the break-even points
ii) the number of panels that should be produced for maximum profit
b) Write the equation for the profit function P.
c) Graph P using graphing technology.
d) Use your graph of P to estimate the number of panels that give maximum profit.
e) How would your answers for break-even points and maximum profit change if
i) the number of dollars of revenue per panel is increased slightly?
ii) the cost function is changed to $C(n) = 100(2)^{\frac{n}{35}}$?
f) What does the number that was changed in part e) ii) represent?

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4. In the early 19th century, Thomas Malthus published *An Essay on the Principle of Population*. He argued that, since food supply increases linearly but population growth is exponential, people in the future are doomed to starvation. Consider a food supply model $F(t) = 40t + 1000$ and a population model $P(t) = 500(1.04)^t$, where t represents years from now. Starvation occurs when $F(t) < P(t)$.

a) Calculate the rate of increase of the food supply. What percent of the original supply is this?

b) Calculate the rate of population increase.

c) Use a graphical method to decide whether starvation occurs in this model, and, if so, when?

d) Is there any rate of increase in the food supply that will prevent starvation under this model? Try using graphing technology to animate the rate.

e) Is there any non-zero rate of increase in the population that will prevent starvation? Try animating the rate of growth.