Chapter 3 Web Task Level 3 Sample Solution

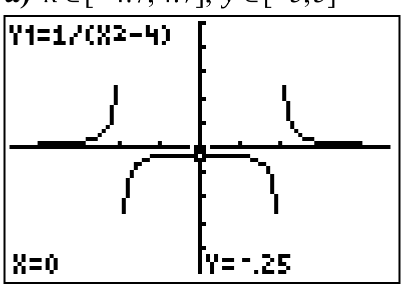
All of the functions listed, except the first, pass through (0, 0). The first has y-intercept 1/4.
They all have asymptotes at $x = 2$ and $x = -2$.
For all of these functions, the domain is $\{x \in \mathbb{R},\ x \neq 2, -2\}$, the range is all real numbers.

I analysed the behaviour of each function on either side of the asymptotes.

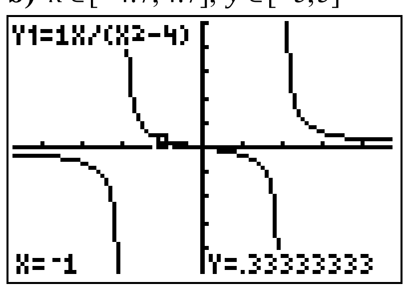
Function	***x* large and negative**	***x* → −2⁻**	***x* → −2⁺**	***x* → 2⁻**	***x* → 2⁺**	***x* large and positive**
a) $y = \frac{1}{x^2 - 4}$	$y \to 0^+$	$y \to \infty$	$y \to -\infty$	$y \to -\infty$	$y \to \infty$	$y \to 0^+$
b) $y = \frac{x}{x^2 - 4}$	$y \to 0^-$	$y \to -\infty$	$y \to \infty$	$y \to -\infty$	$y \to \infty$	$y \to 0^+$
c) $y = \frac{x^2}{x^2 - 4}$	$y \to 0^+$	$y \to \infty$	$y \to -\infty$	$y \to -\infty$	$y \to \infty$	$y \to 0^+$
d) $y = \frac{x^3}{x^2 - 4}$	approaches $y = x$ from below	$y \to -\infty$	$y \to \infty$	$y \to -\infty$	$y \to \infty$	approaches $y = x$ from above
e) $y = \frac{x^4}{x^2 - 4}$	$y \to \infty$	$y \to \infty$	$y \to -\infty$	$y \to -\infty$	$y \to \infty$	$y \to \infty$
f) $y = \frac{x^5}{x^2 - 4}$	$y \to -\infty$	$y \to -\infty$	$y \to \infty$	$y \to -\infty$	$y \to \infty$	$y \to \infty$

Graphs:

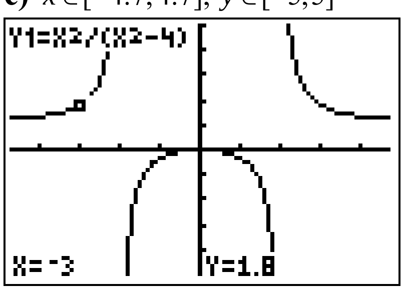
a) $x \in [-4.7, 4.7],\ y \in [-5, 5]$



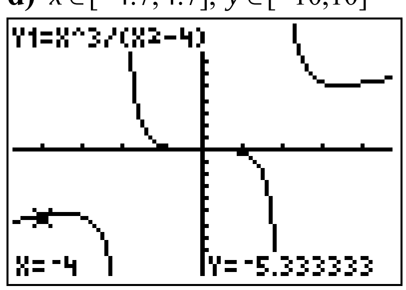
b) $x \in [-4.7, 4.7],\ y \in [-5, 5]$



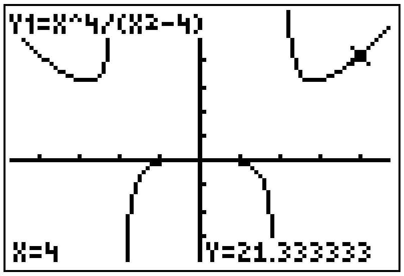
c) $x \in [-4.7, 4.7],\ y \in [-5, 5]$



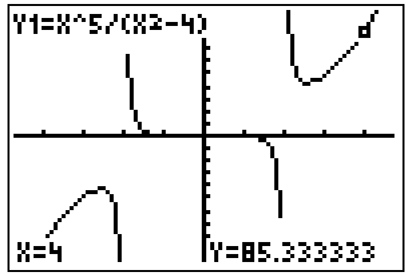
d) $x \in [-4.7, 4.7],\ y \in [-10, 10]$



e) $x \in [-4.7, 4.7]$, $y \in [-20, 30]$, Yscl = 5



f) $x \in [-4.7, 4.7]$, $y \in [-100, 100]$, Yscl = 10



For rational functions of the form $y = \frac{x^n}{x^2 - a^2}$:

The *y*-intercept will be 0.
There will be asymptotes at $x = a$ and $x = -a$.
The domain is $\{x \in \mathbb{R},\ x \neq a, -a\}$, the range is all real numbers.
If *n* is an even number, the function has one local maximum at (0, 0). The function has line symmetry about the y-axis. For $x < -a$, the function will be in the second quadrant going from $+\infty$ to the left of $-a$, will approach the value $y = a^{n-2}$, then go back to $+\infty$ as *x* approaches $-\infty$. The reflection of this will occur in the first quadrant for $x > a$.

If *n* is an odd number, there is no maximum or minimum. There is a point of inflections at the origin. The graph has rotational symmetry about the origin.
For $x > a$, the graph will be in the first quadrant, going from $+\infty$ to the right of *a*, will approach the value $y = a^{n-2}$, then go back to $+\infty$ as *x* approaches $+\infty$. For $x < a$, a similar shape will occur in the third quadrant.

Test: $y = \frac{x^8}{x^2 - 9}$. I predict this will have asymptotes at 3 and −3, the parts either side of the asymptotes will be in the first and second quadrant, while the part between will go from $-\infty$ in third quadrant to $-\infty$ in fourth quadrant.
Check: