1.2 Characteristics of Polynomial Functions

Student Text Pages
15 to 29

Suggested Timing
45–90 min

Tools
- graphing calculator
- computer with *The Geometer's Sketchpad®* (optional)
- grid paper

Related Resources
- G–1 Grid Paper
- G–3 Four Quadrant Grids
- T–2 *The Geometer's Sketchpad®* 4
- BLM 1–4 Investigate 1 Part A
- BLM 1–5 Investigate 1 Part B
- BLM 1–6 Investigate 2 Finite Differences Table
- BLM 1–7 Section 1.2 Summary
- BLM 1–8 Section 1.2 Practice
- BLM 1–9 Section 1.2 Achievement Check Rubric

Teaching Suggestions

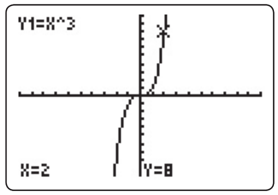
- Allow students to complete **Investigate 1** in groups of three or four. Have students record their results on **BLM 1–4 Investigate 1 Part A** and **BLM 1–5 Investigate 1 Part B**. If needed, use **T–2 *The Geometer's Sketchpad®* 4** to support this activity.
- If using pencil and paper for **Investigate 2,** have students complete the finite difference table on **BLM 1–6 Investigate 2 Finite Differences Table**. Method 2 of **Investigate 2** provides the steps for using a graphing calculator to determine finite differences. This may be the first time that students determine finite differences using a graphing calculator.
- Refer to **Prerequisite Skills** question 5 for this lesson.
- In **Example 1,** have students describe the key features by analysing the equations first and then match the equations with the graphs.
- In preparation for **Examples 2** and **3,** discuss finite differences and factorial notation as referred to on page 20.
- **Example 2** demonstrates how finite differences can be used to determine the degree, the sign, and the value of the leading coefficient.
- In **Example 3,** finite differences are found using technology. They are used to identify the degree of the polynomial function that models the data and a regression is performed to determine the equation that models the data.
- Give **BLM 1–7 Section 1.2 Summary** to students to use as a reference/memory aid.
- As you discuss the **Communicate Your Understanding** questions, draw out the relationships that exist between linear/odd-degree polynomial functions and quadratic/even-degree polynomial functions and, the key features of their graphs.
- For **question 1** refer to the Connections and discuss the wording "least possible degree."
- **Questions 8, 11, 12,** and **13** demonstrate that a restricted domain is needed when considering real-world applications of polynomial functions.
- **Question 8** allows students to reflect on finite differences for the function given and to reason through an explanation for the value of the constant. Communication will be needed to give a description of the end behaviour for this function and to discuss the meaning of its x-intercepts. Connecting skills, using knowledge previously learned, will enable the students to discuss restrictions on the domain and to determine profit asked for in part f).
- **Question 11** requires the use of reasoning skills to determine the value of the constant finite differences for the function given. Students will have to select tools and use connecting skills to factor the function and to represent the function graphically. Communicating skills will be needed to discuss the relationship between the factored form of the equation and the graph.
- **Question 15** requires students to work backwards and create polynomial functions that satisfy given criteria.
- **Question 17** has students investigate how repeated factors in polynomial functions are related to the x-intercepts of the graph.
- Use **BLM 1–8 Section 1.2 Practice** for remediation or extra practice.

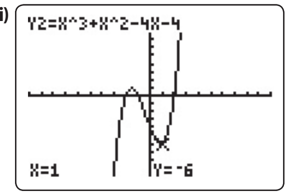
Investigate Answers (pages 15–18)

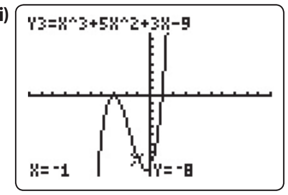
Investigate 1

Part A

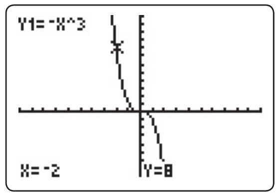
1. a) Group A

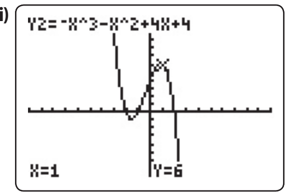
i) 

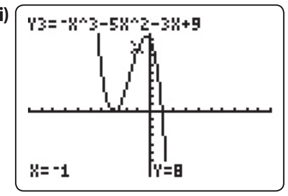
ii) 

iii) 

Group B

i) 

ii) 

iii) 

b) Group A

$y = x^3$

i) quadrant 3 to 1
ii) no minimum, no maximum
iii) no local maximum, no local minimum
iv) 1

$y = x^3 + x^2 - 4x - 4$

i) quadrant 3 to 1
ii) no minimum, no maximum
iii) one local maximum and one local minimum
iv) 3

$y = x^3 + 5x^2 + 3x - 9$

i) quadrant 3 to 1
ii) no minimum, no maximum
iii) one local maximum and one local minimum
iv) 2

Group B

$y = -x^3$

i) quadrant 2 to 4
ii) no minimum, no maximum
iii) no local maximum or local minimum
iv) 1

$y = -x^3 - x^2 + 4x + 4$

i) quadrant 2 to 4
ii) no minimum, no maximum
iii) one local maximum and one local minimum
iv) 3

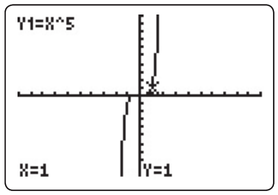
$y = -x^3 - 5x^2 - 3x + 9$

i) quadrant 2 to 4
ii) no minimum, no maximum
iii) one local maximum and one local minimum
iv) 2

c) Answers may vary. Sample answer: Similarities: same domain, same range; no maximum or minimum points; at most $(n - 1)$ local minimum and local maximum points; at least one x-intercept and at most n x-intercepts, where n is the degree of the function

Differences: group A all go from quadrant 3 to 1; group B all go from quadrant 2 to 4

d) i) Group A ii) Group B

2. a) i)


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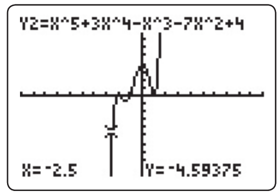

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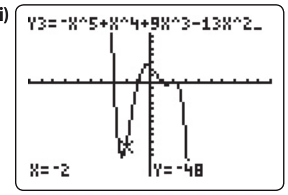


b) Answers may vary. Sample answer: Similarities: all have same domain and range, all odd degree, no maximum, no minimum; Differences: graphs extend from quadrant 3 to 1 (i and ii) or quadrant 2 to 4 (iii), *x*-intercepts, *y*-intercepts, local maximum(s), local minimum(s)

3. a) Answers may vary. Sample answer: Similarities: if the leading coefficient is positive the graphs tend from quadrant 3 to 1; if the leading coefficient is negative the graphs tend from quadrant 2 to 4; same domain and range; no maximum and no minimum

Differences: number of *x*-intercepts may be different, number of local maxima and local minima

b) 1, 3

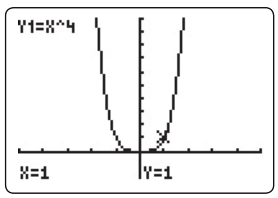
c) A polynomial function of degree n, where n is odd, has no minimum, no maximum, and at most $n - 1$ local minimum and local maximum points.

d) positive: quadrant 3 to 1; negative: quadrant 2 to 4

e) Yes.

Part B

1. a) Group A

i)


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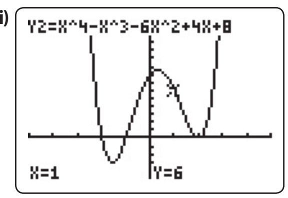

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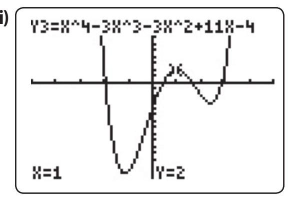
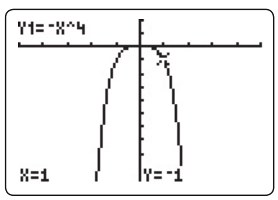


Group B

i)


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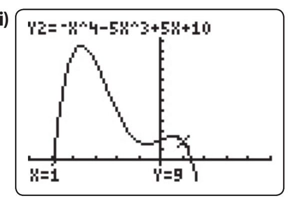

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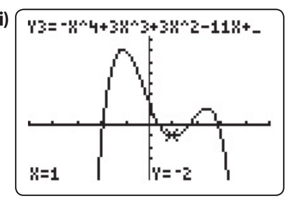


b) Group A

$y = x^4$

i) quadrant 2 to 1

ii) no maximum, one minimum

iii) one local minimum

iv) 1

$y = x^4 - x^3 - 6x^2 + 4x + 8$

i) quadrant 2 to 1

ii) no maximum, one minimum

iii) one local maximum, two local minima

iv) 3

$y = x^4 - 3x^3 - 3x^2 + 11x - 4$

i) quadrant 2 to 1

ii) no maximum, one minimum

iii) one local maximum, two local minima

iv) 4

Group B

$y = -x^4$

i) quadrant 3 to 4
ii) one maximum, no minimum
iii) one local maximum
iv) 1

$y = -x^4 - 5x^3 + 5x + 10$

i) quadrant 3 to 4
ii) one maximum, no minimum
iii) two local maxima, one local minimum
iv) 2

$y = -x^4 + 3x^3 + 3x^2 - 11x + 4$

i) quadrant 3 to 4
ii) one maximum, no minimum
iii) two local maxima, one local minimum
iv) 4

c) Answers may vary. Sample answer:

Similarities: all graphs have at most $(n - 1)$ local minimum and local maximum points; all graphs have same domain; all graphs have at least one x-intercept and at most n x-intercepts, where n is the degree of the function

Differences: group A graphs extend from quadrant 2 to 1, have one minimum point and no maximum; group B graphs extend from quadrant 3 to 4, have one maximum point and no minimum; graphs have different range

d) i) Group A ii) Group B

2. a) Answers may vary. Sample answer:

Similarities: if the leading coefficient is positive the graphs tend from quadrant 2 to 1, and have a minimum point; if the leading coefficient is negative the graphs tend from quadrant 3 to 4, and have a maximum point; same domain

Differences: number of x-intercepts may be different; number of local maxima and local minima; range

b) 0, 4

c) An even-degree polynomial will have at least one minimum point (if the leading coefficient is positive) or at least one maximum point (if the leading coefficient is negative); a polynomial function of degree n may have at most $(n - 1)$ local minimum and local maximum points.

d) positive: quadrant 2 to 1; negative: quadrant 3 to 4

e) Yes.

Investigate 2

1.

		Differences		
x	y	First	Second	Third
−3	−27			
−2	−8	19		
−1	−1	7	−12	
0	0	1	−6	6
1	1	1	0	6
2	8	7	6	6
3	27	19	12	6
4	64	37	18	6

2\. a)

		Differences		
x	y	First	Second	Third
−3	54			
−2	16	−38		
−1	2	−14	24	
0	0	−2	12	−12
1	−2	−2	0	−12
2	−16	−14	−12	−12
3	−54	−38	−24	−12
4	−128	−74	−36	−12

b)

		Differences			
x	y	First	Second	Third	Fourth
−3	81				
−2	16	−65			
−1	1	−15	50		
0	0	−1	14	−36	
1	1	1	2	−12	24
2	16	65	14	12	24
3	81	65	50	36	24
4	256	175	110	60	24

c)

		Differences			
x	y	First	Second	Third	Fourth
−3	−162				
−2	−32	130			
−1	−2	30	−100		
0	0	2	−28	72	
1	−2	−2	−4	24	−48
2	−32	−30	−28	−24	−48
3	−162	−130	−100	−72	−48
4	−512	−350	−220	−120	−48

3\. a) same value

b) same value

c) For a polynomial function of degree n (n positive integer), the nth differences are equal to $a[n(n-1)(n-2)\ldots 2 \times 1]$, where a is the leading coefficient.

d) i) If the degree is n, then the nth finite differences are constant.

ii) The sign of the nth differences equals the sign of the leading coefficient of the function.

iii) The constant value for the nth finite differences is equal to the value of the leading coefficient multiplied by $n!$.

e) i)

		Differences		
x	*y*	First	Second	Third
−3	−116			
−2	−39	77		
−1	−6	33	−44	
0	1	7	−26	18
1	0	−1	−8	18
2	9	9	10	18
3	46	37	28	18
4	129	83	46	18

ii)

		Differences			
x	*y*	First	Second	Third	Fourth
−3	−199				
−2	−47	152			
−1	−7	40	−112		
0	−1	6	−34	78	
1	1	2	−4	30	−48
2	−19	−20	−22	−18	−48
3	−127	−108	−88	−66	−48
4	−437	−310	−202	−114	−48

Communicate Your Understanding Responses (page 25)

C1. a) Answers may vary. Sample answer: If the leading coefficient is positive, the graph extends from quadrant 3 to 1; if the leading coefficient is negative, the graph extends from quadrant 2 to 4; no maximum, no minimum; minimum one x-intercept; same domain and range.

b) Answers may vary. Sample answer: If the leading coefficient is positive, the graph extends from quadrant 2 to 1, and has at least one minimum point; if the leading coefficient is negative, the graph extends from quadrant 3 to 4, and has at least one maximum point; minimum no x-intercepts, maximum n x-intercepts (n = degree); same domain.

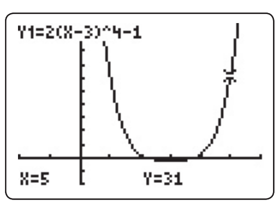
C2. a) Degree n is even: minimum 0 x-intercepts, maximum n x-intercepts; degree n is odd: minimum one x-intercept, maximum n x-intercepts.

b) Degree n is even and leading coefficient is positive: at least one minimum point; degree n is even and leading coefficient is negative: at least 1 maximum point; degree n is odd: no minimum point and no maximum point.

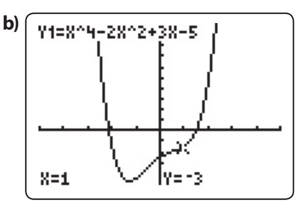
c) The number of local maxima and local minima is equal to or less than the degree of the function n minus 1.

C3. Answers may vary. Sample answers:

a)



b)



C4. Answers may vary. Sample answer: Even degree polynomials start and end in the same direction; if y approaches ∞ there is a minimum value for y; if y approaches $-\infty$ there is a maximum value for y.

Differentiated Instruction

Use an **anticipation guide** to get students thinking about the concepts in this section.

Common Errors

- Students assume that the degree of a polynomial function is the exponent value of the first term of the function.

R_x Point out that it may be the case that the terms are not written in descending powers and so the degree of a polynomial function is the highest exponent value when all terms in the function are considered.

Ongoing Assessment

Achievement Check, question 14, on student text page 29.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	
Reasoning and Proving	1, 4, 5, 7, 8, 11, 12, 15, 16, 18
Reflecting	8, 16
Selecting Tools and Computational Strategies	3, 4, 7–11, 13, 15, 17, 18
Connecting	3, 4, 8, 10–13, 17, 18
Representing	9–11, 14–17
Communicating	1–3, 5, 6, 8, 10–14, 17, 18

Achievement Check, question 14, student text page 29

This performance task is designed to assess the specific expectations covered in Section 1.2.
The following Math Process Expectations can be assessed.

- Connecting
- Representing
- Communicating
- Reasoning and Proving

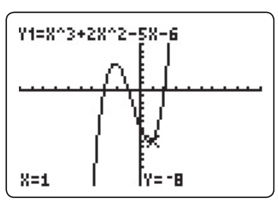
Achievement Chart Category	Related Math Processes
Knowledge and Understanding	Selecting tools and computational strategies
Thinking	Problem solving Reasoning and proving Reflecting
Communication	Communicating, Representing
Application	Selecting tools and computational strategies Connecting

Sample Solution

Provide students with **BLM 1–9 Section 1.2 Achievement Check Rubric** to help them understand what is expected.

a) The function $f(x) = x^3 + 2x^2 - 5x - 6$ is an odd-degree polynomial with a positive leading coefficient so the graph extends from quadrant 3 to quadrant 1.

b)



c) The third differences will be constant, positive, and have a $1[3 \times 2 \times 1]$, or 6.