1.3

Equations and Graphs of Polynomial Functions

Student Text Pages
30 to 41

Suggested Timing
45–90 min

Tools
- graphing calculator
- computer with *The Geometer's Sketchpad®* (optional)
- grid paper

Related Resources
- G–1 Grid Paper
- G–3 Four Quadrant Grids
- T–2 *The Geometer's Sketchpad*-5 pt 4
- BLM 1–10 Function Analysis Tables
- BLM 1–11 Section 1.3 Practice

Teaching Suggestions

- Allow students to work in groups of three or four on **Investigate 1**. The intent of **Investigate 1** is to help students discover
  i) the relationship between the factors of a polynomial function and the x-intercepts of the corresponding graph
  ii) the effect of repeated factors (even number vs odd number) on the graph
- Have each group present their results for one of steps 1, 2, 3, 4, or 5 from **Investigate 1.**
- Refer to **Prerequisite Skills** question 6 for this lesson.
- Before presenting **Example 1**, discuss the information on page 32. Ensure students understand the meaning of the word "order." Point out how test values may be used to verify the sign of the curve on either side of an x-intercept.
- Remind students of the equivalent inequalities for the given interval notations. You may wish to give students the choice of selecting either method for naming intervals.
- The tables in **Example 2** provide an efficient, concise, and thorough way to organize all key features of the graph of each function. Point out how the leading coefficient is obtained in column 2 (it is the product of all the coefficients in the factored form of the polynomial). When sketching the graph of the function ALWAYS begin on the left (i.e. the negative side of the x-axis) and move to the right (i.e. the positive side of the x-axis). At this point, students do not know how to find any minimum or maximum points on the graph, so these will be estimates only. They do however, have enough information to determine the shape of the graph. Use **BLM 1–10 Function Analysis Tables** to support this activity.
- Allow students to work in groups of three or four to complete **Investigate 2.** The intent of **Investigate 2** is to help students explore how point or line symmetry can be determined algebraically.
- If needed, use **T–2 *The Geometer's Sketchpad*® 4** to support **Investigate 1** or **Investigate 2.**
- **Example 3** identifies how the exponents of the terms can be used to determine if a polynomial function is even or odd.
- In the **Communicate Your Understanding** questions C1 and C4, draw out the difference between "odd (or even)-degree" polynomial function and "odd (or even) function" and identify their symmetry. Questions C2 and C3 draw out the connection between factors, zeros, and x-intercepts.
- The information required in **question 2** may be organized in a table such as the one shown.

Graph	x-intercepts	Intervals Where Function is Positive	Intervals Where Function is Negative	Explanation of Order of Zeros
a)				
b)				
c)				
d)				

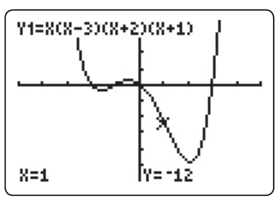
- The intent of **question 7** is to have students work backwards to obtain the factors from the given zeros and then use the given point to determine the leading coefficient.

- To answer all the parts of **question 9** it will be necessary to increase the order of some of the zeros.
- **Question 9** requires students to reflect on the properties of four different functions that all have three of the same zeros to reason out possible equations for these different functions. They will have to select tools to represent a sketch of the graph of each function and use connecting skills to produce the graphs.
- Students may use technology for **question 10,** part d).
- The functions in **question 12** can be factored by letting $m = x^2$. Note that for part a) ii), a common factor must be removed first.
- **Question 13** demonstrates how a vertical translation can change the number of x-intercepts without changing the shape of the graph.
- Technology tip for **question 13:**
  - Students could use *The Geometer's Sketchpad®* to plot the polynomial function $f(x) = (x - 3)(x - 1)(x + 2)^2 + c$ and create a slider for the constant c. By doing this students save time by analysing the changes in the value for c by selecting values instantly with the slider instead of graphing multiple graphs on paper or entering them on the graphing calculator. Students can also create a new parameter c and select values to see how this affects the number of x-intercepts. When creating parameters or sliders for variables refer to the Technology Appendix on page 506.
- **Question 14** uses reflecting and reasoning skills to write the general equations to be represented and then connecting skills along with the selecting of appropriate tools to determine the specific equations required.
- Use **BLM 1–11 Section 1.3 Practice** for remediation or extra practice.

Investigate Answers (pages 30–31, 36)

Investigate 1

1. a)



b) **i)** 4 **ii)** + **iii)** 0, 3, −2, −1 **iv)** 0

c)

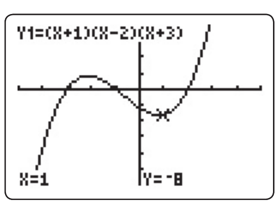
Interval	$x < -2$	$-2 < x < -1$	$-1 < x < 0$	$0 < x < 3$	$x > 3$
Sign of $f(x)$	+	−	+	−	+

d) **i)** Answers may vary. Sample answer: The degree is equal to the sum of all the exponents of the factors. The sign of the leading coefficient is the sign in front of all the xs multiplied and the sign in front of all the factors multiplied together.

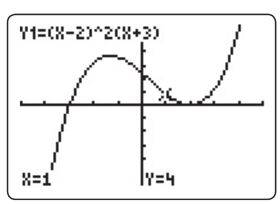
ii) The x-intercepts equal the roots of the equation. The y-intercept is when $x = 0$.

iii) The sign of $f(x)$ changes at each intercept.

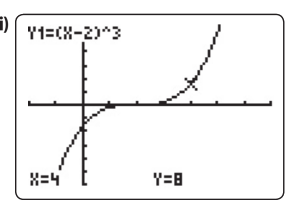
2. a) **i)**



ii)



iii)

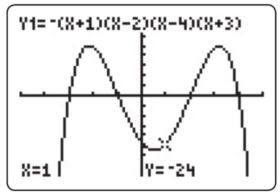


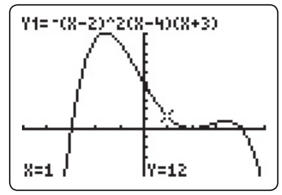
b) **i)** x-intercepts are −1, 2, −3; y-intercept is −6

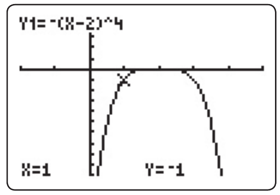
ii) x-intercepts are 2, −3; y-intercept is 12

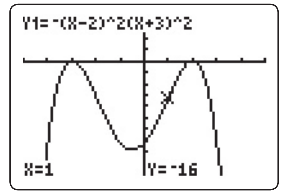
iii) x-intercepts is 2; y-intercept is −8

c) Answers may vary. Sample answer: The number of factors (repeated factors count once) equals the number of x-intercepts; the sign of $f(x)$ changes if the zero is of odd order; the sign of $f(x)$ remains the same if the zero is of even order; the x-intercepts are the roots of the equation.

3. a) **i)** 

ii) 

iii) 

iv) 

b) **i)** x-intercepts are -1, 2, 4, -3; y-intercept is -24

ii) x-intercepts are 2, 4, -3; y-intercept is 48

iii) x-intercepts is 2; y-intercept is -16

iv) x-intercepts are 2 (order 2), -3 (order 2); y-intercept is -36

c) Answers may vary. Sample answer: The number of factors (repeated factors count once) equals the number of x-intercepts; the sign of $f(x)$ changes if the zero is of odd order; the sign of $f(x)$ remains the same if the zero is of even order; the x-intercepts are the roots of the equation.

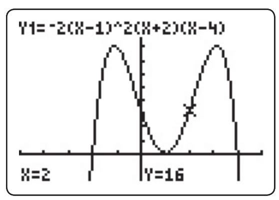
4. a) **i)** touches the x-axis, does not cross it

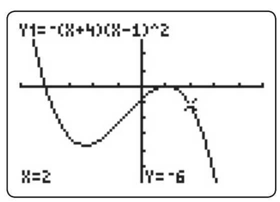
ii) crosses the x-axis

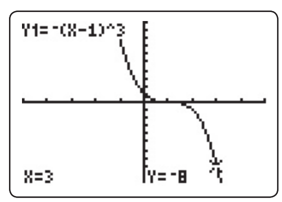
b) **i)** does not change

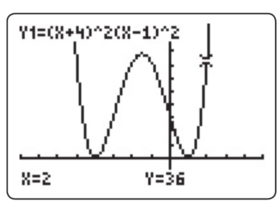
ii) changes

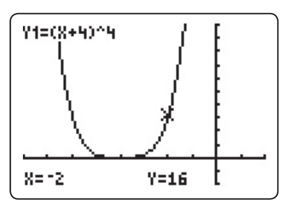
5. a) Answers may vary.

b) 

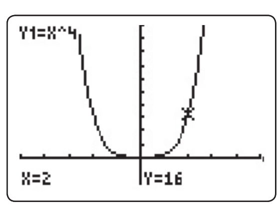
c) **i)** 

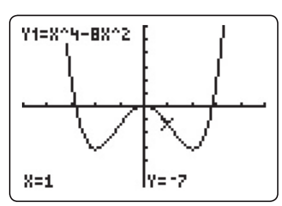
ii) 

iii) 

iv) 

Investigate 2 Part A

1. a) **i)** 

ii) 

Differentiated Instruction

Use a **Frayer model** for cubic functions and another for quartic functions to summarize important characteristics.

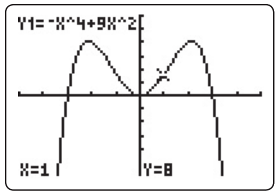
Common Errors

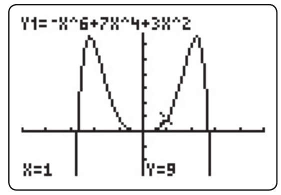
- Students have difficulty sketching the graphs using the factored form of the equation.

R_x Remind students to consider each of the following:
  - **i)** the degree of the function
  - **ii)** the sign of the leading coefficient
  - **iii)** the end behaviour
  - **iv)** the x-intercepts and their order

- Students have difficulty determining the intervals where a function is positive or negative.

R_x Encourage students to first determine the x-intercepts of the graph of the function. These will be used to create intervals along the x-axis from $-\infty$ to ∞. The graph will change signs at each x-intercept that corresponds to a zero of odd order but will not change signs for zeros of even order. Some students may benefit from sketching the function to help them determine the required intervals.

iii) 

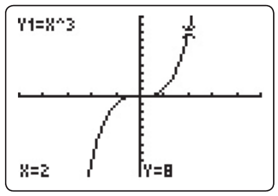
iv) 

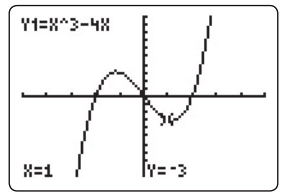
b) line

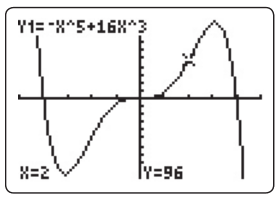
c) Since the exponents of each term of the equation are even, then the polynomial function is an even function.

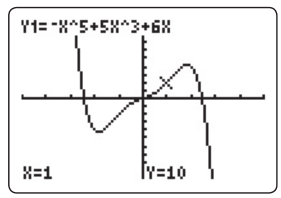
2. a), b) $f(-x) = f(x)$

Part B

1. a) i) 

ii) 

iii) 

iv) 

b) point

c) Since the exponents of each term of the equation are odd, then the function is odd.

2. a), b) $f(-x) = -f(x)$

3. If the function is odd, then it has point symmetry about the origin. If the function is even, then it has line symmetry about the y-axis.

Communicate Your Understanding Responses (page 38)

C1. No. For example, $f(x) = x^2 + x + 3$ is an even-degree polynomial but the function is not even, and $f(x) = x^3 + x^2 + 2$ is an odd-degree polynomial but the function is not odd.

C2. Answers may vary. Sample answer: It is easier to determine the x-intercepts.

C3. a) If the order is even, the sign of the function does not change. If the order is odd, the sign of the function changes.

b) Answers may vary. Sample answer:
Order 1: graph crosses the x-axis and the function changes sign
Order 2: graph touches the x-axis and the function does not change sign
Order 3: graph is very close to the x-axis before it crosses and the function changes sign

C4. Answers may vary.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	
Reasoning and Proving	7, 9, 10, 14, 15
Reflecting	9, 10, 14, 15
Selecting Tools and Computational Strategies	16–15
Connecting	2–15
Representing	3, 7, 9, 10, 12–14
Communicating	2, 4, 5, 10, 15