1.4 Transformations

Student Text Pages
42 to 51

Suggested Timing
40–70 min

Tools
- graphing calculator
- computer with *The Geometer's Sketchpad*® (optional)
- grid paper

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad*® 4
- BLM 1–12 Section 1.4 Investigate
- BLM 1–13 Section 1.4 Summary
- BLM 1–14 Section 1.4 Practice

Teaching Suggestions

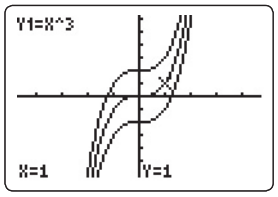
- Allow students to work in groups of three or four to complete the **Investigate**. Encourage students to apply their knowledge of transformations from previous math courses. Use **BLM 1–12 Section 1.4 Investigate** to support the Investigate. If needed, use **T–2 *The Geometer's Sketchpad*® 4**.
- Technology tips for the **Investigate** using *The Geometer's Sketchpad*®:
  - Plot the polynomial function of the form $y = a[k(x - d)]^n + c$ and create integer horizontal sliders for the values of a, k, d, and c or use the ones that come with *The Geometer's Sketchpad*® under **Custom Tools**.
  - Create the integer horizontal sliders first, then choose **New Function** from the **Graph** menu and type the polynomial function $y = a[k(x - d)]^n + c$. Then, select the polynomial function of the form $y = a[k(x - d)]^n + c$ and from the **Graph** menu choose **Plot New Function** so that the polynomial function is graphed on the grid. Drag the integer horizontal sliders in order to analyse the roles of a, k, d, and c in the polynomial function.
  - For the variable n, create a parameter that can be changed for different values of n. Students should refer to the Technology Appendix on page 506 for creating and using parameters with *The Geometer's Sketchpad*®.
- Refer to **Prerequisite Skills** questions 1, 2, 8, 9, and 10 for this lesson.
- Ask students to review the information in the table (pages 44–45) regarding the roles of the parameters a, k, d, and c. Be sure to read the paragraph at the bottom of page 45. Point out that to obtain an accurate sketch of the graph of a transformed function, the order of transformations follows the order of operations. That is, a and k (which correspond to the operations of multiplication and division) must be applied before c and d (which correspond to the operations of addition and subtraction).
- Guide students through the operations required to obtain the points in the table in **Example 1**.
- **Example 3** provides an interesting application of transformations. The equations that form the pattern are found from corresponding graphs.
- Give **BLM 1–13 Section 1.4 Summary** to students to place in their notebook as a reference/memory aid.
- Use the **Communicate Your Understanding** questions C1, C2, and C3 to draw out the distinction between compressions/stretches, vertical/horizontal translations, and reflections. Students should easily identify the parameters that are associated with each transformation. Use question C4 to reinforce the importance of the order of transformations when sketching the graph.
- In **question 5**, students may find it helpful to verify their choices by substituting an x-value, say $x = 2$, into each equation and determine the corresponding y-value, then checking to see if the point is on the graph.
- **Question 5** requires students to reflect on each function and on each graph to determine, by making connections with previously learned material, which graph represented in the question matches with its corresponding function. Communicating skills will be needed to justify their choices.
- Remind students to use the correct and precise terminology when describing transformations in **questions 6** to 8. In particular, the value should be included (i.e., a vertical compression of factor 2), and direction (where it applies) (i.e., a vertical translation of 2 units up).

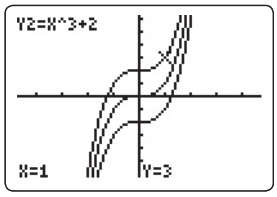
- **Question 7** gives students the opportunity to use reasoning skills to determine how each parameter in part a) transforms the graph of the basic function in part b). Connecting skills will give students the necessary tools to discuss, using their communicating skills, the terms required in part c) and two possible orders in which the transformations can be applied in part d).
- Students should refer to **Example 3** if having difficulty with **questions 10** and **12**.
- The intent of **questions 14** and **15** is to help students make the connection between transforming the entire function and transforming individual terms of the function. You may wish to point out that the given polynomial function is a combination (through addition and subtraction) of individual power functions.
- Use **BLM 1–14 Section 1.4 Practice** for remediation or extra practice.

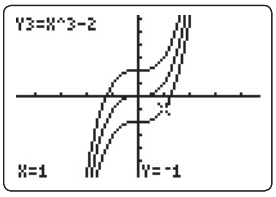
Investigate Answers (pages 42–43)

PART A

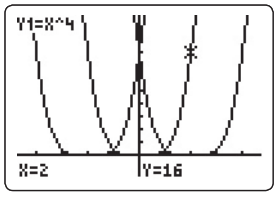
1. a) Group A

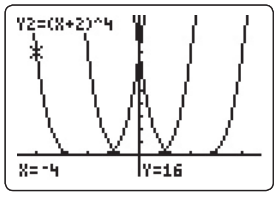
i) 

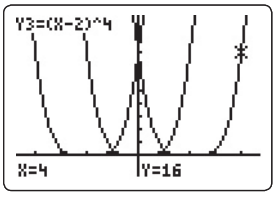
ii) 

iii) 

Group B

i) 

ii) 

iii) 

b) Answers may vary. Sample answer: c corresponds to a vertical translation, up or down.

c) Answers may vary. Sample answer: d corresponds to a horizontal translation, left or right.

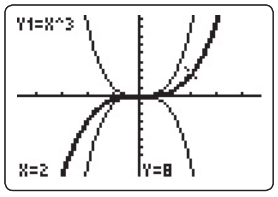
2.

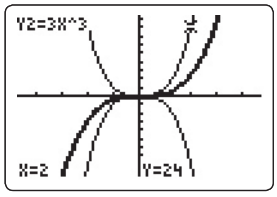
Value of c in $y = a[k(x - d)]^n + c$	Effect on the Graph of $y = x^n$
$c > 0$	Vertical translation c units up
$c < 0$	Vertical translation c units down

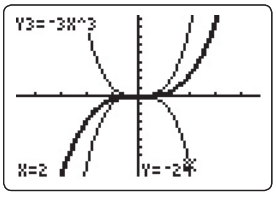
Value of d in $y = a[k(x - d)]^n + c$	Effect on the Graph of $y = x^n$
$d > 0$	Horizontal translation d units right
$d < 0$	Horizontal translation d units left

PART B

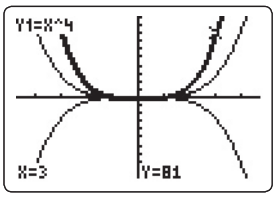
1. a) Group A

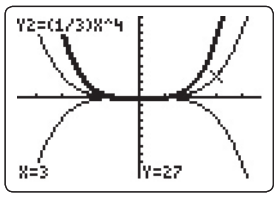
i) 

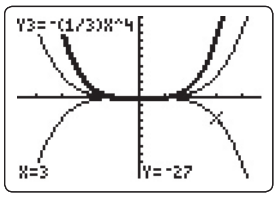
ii) 

iii) 

Group B

i) 

ii) 

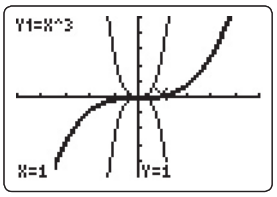
iii) 

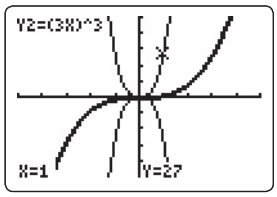
b) $a > 1$: a vertical stretch by a factor of a; $a < -1$: a vertical stretch by a factor of $|a|$ and a reflection in the x-axis

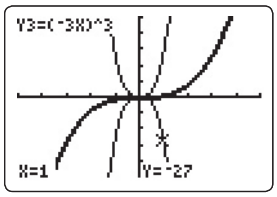
c) $0 < a < 1$: vertical compression by a factor of a; $-1 < a < 0$: vertical compression by a factor of $|a|$ and a reflection in the x-axis

d) a corresponds to a vertical stretch or compression, if $a < 0$ a reflection in the x-axis

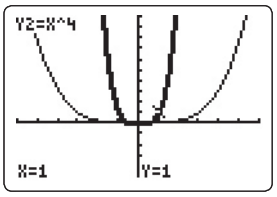
2. a) **Group A**

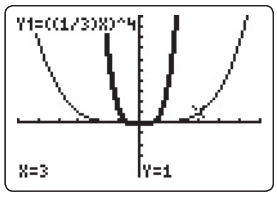
i) 

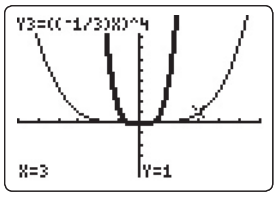
ii) 

iii) 

Group B

i) 

ii) 

iii) 

b) $k > 1$ horizontal compression by a factor of $\frac{1}{k}$; $k < -1$ horizontal compression by a factor of $\left|\frac{1}{k}\right|$ and a reflection in the y-axis

c) $0 < k < 1$ horizontal stretch by a factor of $\frac{1}{k}$; $-1 < k < 0$ horizontal stretch by a factor of $\left|\frac{1}{k}\right|$ and a reflection in the y-axis

d) k corresponds to a horizontal stretch or compression, if $k < 0$ a reflection in the y-axis

3.

Value of a in $y = a[k(x - d)]^n + c$	Effect on the Graph of $y = x^n$
$a > 1$	Vertical stretch by a factor of a
$0 < a < 1$	Vertical compression by a factor of a
$-1 < a < 0$	Vertical compression by a factor of $\|a\|$ and a reflection in the x-axis
$a < -1$	Vertical stretch by a factor of $\|a\|$ and a reflection in the x-axis

Value of k in $y = a[k(x - d)]^n + c$	Effect on the Graph of $y = x^n$
$k > 1$	Horizontal compression by a factor of $\frac{1}{k}$
$0 < k < 1$	Horizontal stretch by a factor of $\frac{1}{k}$
$-1 < k < 0$	Horizontal stretch by a factor of $\left\|\frac{1}{k}\right\|$ and a reflection in the y-axis
$k < -1$	Horizontal compression by a factor of $\left\|\frac{1}{k}\right\|$ and a reflection in the y-axis

Differentiated Instruction

Use **jigsaw** to learn the effect of transformations on polynomial functions.

Common Errors

- Students incorrectly describe the transformations associated with the parameter k.

R_x Emphasize that k represents a horizontal stretch (when the value of k is a proper fraction).

Otherwise, k represents a horizontal compression. The factor value is always the reciprocal $\frac{1}{k}$.

- Students have difficulty sketching the transformed graph.

R_x Have students select points on the original function and then make a table, similar to the one shown in **Example 1**, to determine the transformed points, which can then be plotted. Remind them to first apply the "a" and "k" transformations before "c" and "d."

Ongoing Assessment

Achievement Check, question 13, on student text page 52.

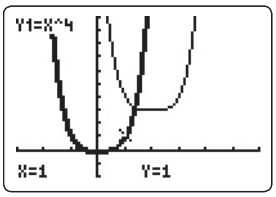
Communicate Your Understanding Responses (page 49)

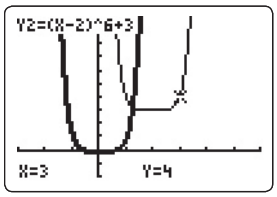
C1. a) a and k

b) i) $0 < a < 1$, $-1 < a < 0$, $0 < k < 1$, $-1 < k < 0$

ii) $a > 1$, $a < -1$, $k > 1$, $k < -1$

C2. c and d; answers may vary. Sample answers:





C3. a and k; if $a < 0$ the graph is reflected in the x-axis, if $k < 0$ the graph is reflected in the y-axis.

C4. a) stretch/compression, reflection, translation (vertical and horizontal)

b) A vertical and horizontal translation could be applied in any order; a vertical and horizontal stretch/compression could be applied in any order.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	11
Reasoning and Proving	4, 7, 11, 14, 16
Reflecting	5, 11, 14
Selecting Tools and Computational Strategies	1, 7, 9, 14–16
Connecting	1, 4, 5, 7, 8, 10–16
Representing	1, 5, 6, 8–16
Communicating	1, 3–8, 13, 15, 16

Achievement Check, question 13, student text page 52

This performance task is designed to assess the specific expectations covered in Section 1.4.

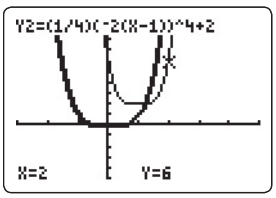
The following Math Process Expectations can be assessed.

- Connecting
- Representing
- Communicating
- Reflecting

Sample Solution

a) $a = \frac{1}{4}$, vertical compression by a factor of $\frac{1}{4}$; $k = -2$, horizontal compression by a factor of $\frac{1}{2}$ and reflection in the y-axis; $d = 1$, translation 1 unit right; $c = 2$, translation 2 units up

b)



Level 3 Notes

Look for the following:

- Most of the parameters are correctly identified and described with accuracy
- Graphs of the original function and transformed function contains only minor errors
- Graphs are well-labelled and some key points are identified

What Distinguishes Level 2

- A few of the parameters are correctly identified and described with some accuracy
- Graphs of the original function and transformed function contains some errors
- Graphs are labelled but no key points are identified

What Distinguishes Level 4

- All of the parameters are correctly identified and described accurately
- Graphs of the original function and transformed function are drawn without error
- Graphs are well-labelled and all key points are identified, with some justification or description provided