Slopes of Secants and Average Rate of Change

Teaching Suggestions

- Allow students to work in pairs or small groups to complete the **Investigate**. The purpose of the **Investigate** is to introduce students to average rate of change and to help students make the connection between slope and average rate of change. In the next section, instantaneous rate of change is introduced. These concepts are found throughout this course and are key concepts for students who plan on taking the Calculus and Vectors course.
- Refer to **Prerequisite Skills** questions 3 and 4 for this lesson.
- Discuss with students the concept in the box on page 55 before presenting the Examples. It is important for students to understand that average rate of change is simply calculated by determining the slope between two points.
- In **Example 1**, the connection is made between average rate of change and lines with positive slope, zero slope, and negative slope.
- **Example 2** demonstrates how to determine average rate of change given a table of values, while **Example 3** shows how to do this with an equation.
- As students consider the **Communicate Your Understanding** questions, draw out that average rate of change is directly connected to the slope of a secant line so their understanding of the direction and steepness of the line provides the information needed to describe the average rate of change in a particular situation.
- **Question 5** provides real data for students to work with. Students should realize that as technology has become more prominent, the percentages decrease.
- Technology tip for **question 6**:
  - Students could use *The Geometer's Sketchpad*® to plot the function $P(t) = -0.2t^3 + 2t^2 + 8t + 2$ and restrict the domain to $t \in [0, 13]$. To restrict the domain for the graph, select the graph then right-click and choose **Properties…**. Select the tab **Plot** and change the domain.
  - Students may not see the entire graph for its domain. To crop the graph, select the **Unit Point** on the x-axis and drag it to the left so that the values for the x- and y-axis increase in increments therefore shrinking the graph so that it may be viewed.
- Students may benefit from drawing a diagram for **question 8**. Ask them to think of themselves as the cyclist in this situation.
- **Question 7** requires students to select tools to assist them in representing a graph of each function described and in selecting a way to determine the average rates of change. Making connections with mathematical strands from their past learning experiences will help them sketch the graphs and determine the average rates of change. Reasoning skills will be needed, along with their connecting skills, to interpret the answers found in parts c) and d).
- **Question 11** connects finite differences and average rate of change.
- **Question 11** gives students the opportunity to reflect and reason through the relationship between the finite differences and the average rates of change. They will have to select appropriate tools and use connecting skills to determine the average rates of change and to apply the appropriate representation of a table of finite differences for the data. An explanation of whether a relationship exists will then be communicated.
- **Question 12** connects average rate of change to instantaneous rate of change.
- Use **BLM 1–15 Section 1.5 Practice** for remediation or extra practice.

COMMON ERRORS

- Students have difficulty distinguishing between constant and non-constant average rate of change.

R_x Situations that can be represented by straight lines reflect constant average rate of change, but situations that can be represented by any non-linear curve reflect non-constant average rate of change.

- Students incorrectly calculate slope and thus average rate of change.

R_x Point out that if the dependent and independent variables are mixed-up, the average rate of change will be wrong. Remind students to think about which variable is the dependent (i.e., the y-variable) and which is the independent variable (i.e., the x-variable), in the slope formula.

Investigate Answers (pages 54–55)

1. a) independent variable t, dependent variable r

b) i) 2 **ii)** 2 **iii)** 2

c) average rate of change (speed) metres per second

d)
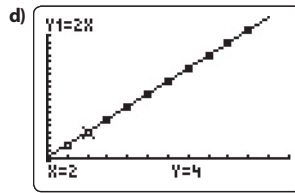
The graph represents a linear function.

e) 2

f) The average rate of change between two points corresponds to the slope of the secant between the points.

2. a) independent variable r, dependent variable A

b) approximate values

i) 62.83 **ii)** 12.57 **iii)** 56.55 **iv)** 6.29 **v)** 94.25

c) average rate of change in square metres per second

d)
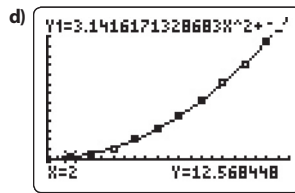
e) same as the values for b

f) The average rate of change between two points corresponds to the slope of the secant between the points.

Communicate Your Understanding Responses (page 61)

C1. a) line with positive slope

b) line with negative slope

c) horizontal line with zero slope

C2. -4

C3. a) Positive rate of change means the function is increasing; negative rate of change means the function is decreasing.

b) If as x increases, $f(x)$ increases, then the rate of change is positive. If as x increases, $f(x)$ decreases, then the rate of change is negative.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	
Reasoning and Proving	1, 2, 5–12
Reflecting	2, 11, 12
Selecting Tools and Computational Strategies	6–11
Connecting	4–12
Representing	6–11
Communicating	1, 2, 5–12