1.6 Slopes of Tangents and Instantaneous Rate of Change

Student Text Pages
65 to 73

Suggested Timing
45–60 min

Tools
- grid paper
- graphing calculator

Related Resources
- G–1 Grid Paper
- BLM 1–16 Average Rate of Change Tables
- BLM 1–17 Section 1.6 Practice

Teaching Suggestions

- Allow students to work in pairs or small groups to complete the **Investigate**.
- Students may want to use the sketch created for the **Investigate** using *The Geometer's Sketchpad*® to make the connection between the slopes of secants, the slope of a tangent, and the instantaneous rate of change. Go to *www.mcgrawhill.ca/books/functions12* and follow the links to this GSP file.
- Introduce the arrow notation (see box page 66) to represent "approaches." This notation and the use of the infinity symbol (mentioned in the previous section) are small introductions to the concepts that will be studied in calculus.
- Read and discuss the last paragraph on page 66 before presenting **Example 1**.
- Point out that in the wording of each example, and also in the exercises, the word "after" means exactly at 2 s. This wording refers to the instant in time at which the instantaneous rate of change is to be found.
- For **Example 1**, ensure students know Method 1 and Method 2 to estimate instantaneous rate of change from a given graph.
- Be sure to discuss the points in the paragraph (page 68) following the solution to Method 2 of **Example 1**.
- Point out to students that **Examples 1**, **2**, and **3** represent the same situation, the distance travelled by a parachutist. In **Example 1**, the graph is provided, in **Example 2**, the data is provided, and in **Example 3**, the equation is provided. Emphasize that methods for estimating instantaneous rate of change depend on the given information. An equation offers the opportunity for obtaining the best estimate because we can make the intervals as small as we wish.
- As students consider the **Communicate Your Understanding** questions C1 and C2, draw out that average rate of change takes place over a period or interval, so there is a starting value and an ending value. Instantaneous rate of change takes place at an instant, so there is only a single value. When discussing question C3, draw out that all the different ways to estimate instantaneous rate of change depend on finding the slope, so in fact, the steps are similar to determining average rate of change except that the intervals are smaller. The method used to estimate instantaneous rate of change depends on the given information. If the equation is given, it allows the interval to be as small as we want to make it.
- Remind students that as they complete the exercises, the method used to determine instantaneous rate of change depends on the given information. There are two possible methods for a given graph.
- **Questions 7** to **11** will help students understand how values for average rate of change and instantaneous rate of change can differ, and thus provide useful information in a real-world application.
- **Question 7** allows students to use reflecting and reasoning skills to estimate instantaneous rates of change and then to interpret the average and instantaneous rates of change for this particular situation. They will use selecting tools to represent the curve and its tangent and connecting tools to determine the average rate of change. They will then have to use their communicating skills to make the interpretation necessary.
- **Question 10** uses reflecting and reasoning and proving skills to determine the average rates of change. Connections will have to be made with previously learned mathematical concepts to estimate the instantaneous rates of change. Finally, they will use their communicating skills to interpret the average and instantaneous rates of change found in parts e) and f).

- The point of **question 12** is to develop an algebraic model that can be used to estimate the instantaneous for many different values of h. This approach is more efficient than estimating instantaneous rate of change for individual values. This approach is the basis for the first principles method that will be used in calculus.
- Use **BLM 1–17 Section 1.6 Practice** for remediation or extra practice.

Investigate Answers (pages 65–66)

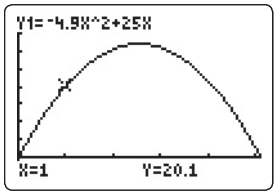
1.

Interval	Δh	Δt	Average rate of change, $\frac{\Delta h}{\Delta t}$
$1 \leq t \leq 2$	10.3	1	10.3
$1 \leq t \leq 1.5$	6.375	0.5	12.75
$1 \leq t \leq 1.1$	1.471	0.1	14.71
$1 \leq t \leq 1.01$	0.1515	0.01	15.15
$1 \leq t \leq 1.001$	0.01520	0.001	15.20

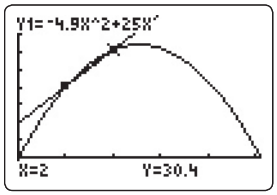
2. Answers may vary. Sample answer: The intervals are getting smaller.

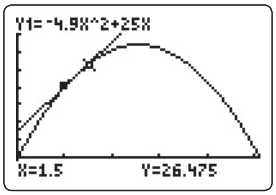
3. Answers may vary. Sample answer: As the interval is getting smaller and closer to 1, the average rate of change is getting closer to the value of the instantaneous rate of change at 1.

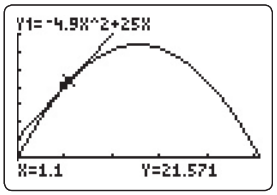
4. a)

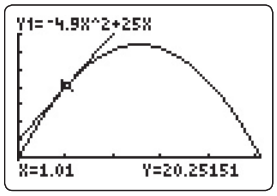


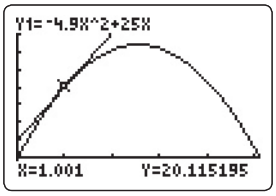
b) The slope of the secant line becomes closer to the slope of the tangent line as the interval gets smaller.



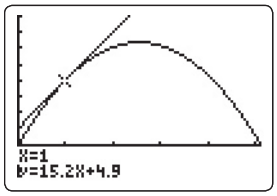








5. a) Answers may vary. Sample answer: Estimate the slope of the tangent line by finding the average rate of change between the given point and another point very close to the given point.



b) instantaneous rate of change

Differentiated Instruction

Use **blastoff** to relate average to instantaneous rate of change.

Use a **journal entry**. Give the topic as "What is the Difference Between Average Rate of Change and Instantaneous Rate of Change?"

Use **Think-Pair-Share** with **think aloud** to have students explain their solution to question 10.

Common Errors

- Students have difficulty distinguishing between real-world situations that represent average rate of change and those that represent instantaneous rate of change.

R_x Point out that average rate of change takes place over an interval, so there are two values that are usually provided in the question. Instantaneous rate of change is estimated at a single value.

- Students forget to include units with their answers.

R_x Remind them that in any exercise involving variables other than x and y, the average rate of change and the instantaneous rate of change require units such as km/h or m/s.

c) Approaching the t-value from either direction will give an estimate of the instantaneous rate of change.

6. a) If P and Q are points on a curve, as Q gets closer to P, the slope of the secant line between the two points gets closer to the slope of the tangent line at P.

b) The slope of a tangent at a point on a curve is the instantaneous rate of change at that point.

Communicate Your Understanding Responses (page 70)

C1. a) instantaneous speed

b) Answers may vary.

C2. a) instantaneous rate of change

b) average rate of change

c) average rate of change

d) instantaneous rate of change

e) average rate of change

f) instantaneous rate of change

C3. Answers may vary. Sample answer: Example 3

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	
Reasoning and Proving	3, 5–8, 10–12
Reflecting	7, 8, 10–12
Selecting Tools and Computational Strategies	4, 7, 11, 12
Connecting	2–12
Representing	3, 7, 11, 12
Communicating	1, 2, 5–12