# Chapter 2

# Polynomial Equations and Inequalities

## Curriculum Expectations

### Polynomial and Rational Functions

**Technology Notes**
The technology used in this chapter is primarily graphing calculators, specifically the TI-83 Plus/TI-84 Plus series, and a computer algebra system (CAS), specifically the TI-89/89T series.

*Connecting Graphs and Equations of Polynomial Functions*

**C1.7** determine an equation of a polynomial function that satisfies a given set of conditions (e.g., degree of the polynomial, intercepts, points on the function), using methods appropriate to the situation (e.g., using the $x$-intercepts of the function; using a trial-and-error process with a graphing calculator or graphing software; using finite differences), and recognize that there may be more than one polynomial function that can satisfy a given set of conditions (e.g., an infinite number of polynomial functions satisfy the condition that they have three given $x$-intercepts)

**C1.8** determine the equation of the family of polynomial functions with a given set of zeros and of the member of the family that passes through another given point [e.g., a family of polynomial functions of degree three with zeros 5, $-3$, and $-2$ is defined by the equation $f(x) = k(x - 5)(x + 3)(x + 2)$, where $k$ is a real number, $k \neq 0$; the member of the family that passes through $(-1, 24)$ is $f(x) = -2(x - 5)(x + 3)(x + 2)$]

*Solving Polynomial and Rational Equations*

**C3.1** make connections, through investigation using technology (e.g., computer algebra systems), between the polynomial function $f(x)$, the divisor $x - a$, the remainder from the division $\frac{f(x)}{x - a}$, and $f(a)$ to verify the remainder theorem and the factor theorem

**C3.2** factor polynomial expressions in one variable, of degree no higher than four, by selecting and applying strategies (i.e., common factoring, difference of squares, trinomial factoring, factoring by grouping, remainder theorem, factor theorem)

**C3.3** determine, through investigation using technology (e.g., graphing calculator, computer algebra systems), the connection between the real roots of a polynomial equation and the $x$-intercepts of the graph of the corresponding polynomial function, and describe this connection [e.g., the real roots of the equation $x^4 - 13x^2 + 36 = 0$ are the $x$-intercepts of the graph of $f(x) = x^4 - 13x^2 + 36$]

**C3.4** solve polynomial equations in one variable, of degree no higher than four (e.g., $2x^3 - 3x^2 + 8x - 12 = 0$), by selecting and applying strategies (i.e., common factoring, difference of squares, trinomial factoring, factoring by grouping, remainder theorem, factor theorem), and verify solutions using technology (e.g., using computer algebra systems to determine the roots; using graphing technology to determine the $x$-intercepts of the graph of the corresponding polynomial function)

**C3.7** solve problems involving applications of polynomial and simple rational functions and equations [e.g., problems involving the factor theorem or remainder theorem, such as determining the values of $k$ for which the function $f(x) = x^3 + 6x^2 + kx - 4$ gives the same remainder when divided by $x - 1$ and $x + 2$]

*Solving Inequalities*

**C4.1** explain, for polynomial and simple rational functions, the difference between the solution to an equation in one variable and the solution to an inequality in one variable, and demonstrate that given solutions satisfy an inequality (e.g., demonstrate numerically and graphically that the solution to $\frac{1}{x + 1} < 5$ is $x < -1$ or $x > -\frac{4}{5}$)

**C4.2** determine solutions to polynomial inequalities in one variable [e.g., solve $f(x) \geq 0$, where $f(x) = x^3 - x^2 + 3x - 9$] and to simple rational inequalities in one variable by graphing the corresponding functions, using graphing technology, and identifying intervals for which $x$ satisfies the inequalities

**C4.3** solve linear inequalities and factorable polynomial inequalities in one variable (e.g., $x^3 + x^2 > 0$) in a variety of ways (e.g., by determining intervals using $x$-intercepts and evaluating the corresponding function for a single $x$-value within each interval; by factoring the polynomial and identifying the conditions for which the product satisfies the inequality), and represent the solutions on a number line or algebraically (e.g., for the inequality $x^4 - 5x^2 + 4 < 0$, the solution represented algebraically is $-2 < x < -1$ or $1 < x < 2$)

## Chapter 2 Planning Chart

| Section<br>Suggested Timing | Student Text<br>Page(s) | Teacher's Resource<br>Blackline Masters | Assessment | Tools |
|---|---|---|---|---|
| **Chapter 2 Opener**<br>• 10 min | 81 | | | |
| **Prerequisite Skills**<br>• 45–60 min | 82–83 | • BLM 2–1 Prerequisite Skills | | |
| **2.1 The Remainder Theorem**<br>• 45–65 min | 84–93 | • T–4 The Computer Algebra System (CAS) on the TI-89 Calculator<br>• BLM 2–2 Section 2.1 Practice | | • computer algebra system |
| **2.2 The Factor Theorem**<br>• 45–60 min | 94–103 | • T–4 The Computer Algebra System (CAS) on the TI-89 Calculator<br>• BLM 2–3 Section 2.2 Practice | • BLM 2–4 Section 2.2 Achievement Check Rubric | • computer algebra system<br>• graphing calculator |
| **2.3 Polynomial Equations**<br>• 40–55 min | 104–112 | • BLM 2–5 Section 2.3 Practice | | • graphing calculator |
| **2.4 Families of Polynomial Functions**<br>• 40–60 min | 113–122 | • G–3 Four Quadrant Grids<br>• BLM 2–6 Section 2.4 Practice | | • grid paper<br>• graphing calculator |
| **2.5 Solve Inequalities Using Technology**<br>• 40–60 min | 123–131 | • G–1 Grid Paper<br>• T–4 The Computer Algebra System (CAS) on the TI-89 Calculator<br>• BLM 2–7 Section 2.5 Practice | | • grid paper<br>• graphing calculator<br>• computer algebra system |
| **2.6 Solve Factorable Polynomial Inequalities Algebraically**<br>• 45–60 min | 132–139 | • G–5 Number Lines<br>• BLM 2–8 Section 2.6 Practice | | |
| **Chapter 2 Review**<br>• 45–50 min | 140–141 | • G–5 Number Lines<br>• T–4 The Computer Algebra System (CAS) on the TI-89 Calculator<br>• BLM 2–9 Chapter 2 Review | | • graphing calculator<br>• computer algebra system |
| **Chapter 2 Problem Wrap-Up**<br>• 30–50 min | 141 | • G–1 Grid Paper | • BLM 2–10 Chapter 2 Problem Wrap-Up Rubric | • grid paper<br>• graphing calculator<br>• computer with *The Geometer's Sketchpad®* |
| **Chapter 2 Practice Test**<br>• 30–45 min | 142–143 | • G–1 Grid Paper<br>• T–4 The Computer Algebra System (CAS) on the TI-89 Calculator | • BLM 2–11 Chapter 2 Test | • grid paper<br>• graphing calculator<br>• computer algebra system |
| **Chapter 2 Task: Can You Tell Just by Looking?**<br>• 60–75 min | 144 | • T–2 *The Geometer's Sketchpad®* 4<br>• BLM 2–13 BLM Answers | • BLM 2–12 Task: Can You Tell Just by Looking? Rubric | • graphing calculator |

# Chapter 2 Blackline Masters Checklist

| | BLM | Title | Purpose |
|---|---|---|---|
| **Prerequisite Skills** | | | |
| | BLM 2–1 | Prerequisite Skills | Practice |
| **2.1 The Remainder Theorem** | | | |
| | T–4 | The Computer Algebra System (CAS) on the TI-89 Calculator | Technology |
| | BLM 2–2 | Section 2.1 Practice | Practice |
| **2.2 The Factor Theorem** | | | |
| | T–4 | The Computer Algebra System (CAS) on the TI-89 Calculator | Technology |
| | BLM 2–3 | Section 2.2 Practice | Practice |
| | BLM 2–4 | Section 2.2 Achievement Check Rubric | Assessment |
| **2.3 Polynomial Equations** | | | |
| | BLM 2–5 | Section 2.3 Practice | Practice |
| **2.4 Families of Polynomials Functions** | | | |
| | G–3 | Four Quadrant Grids | Student Support |
| | BLM 2–6 | Section 2.4 Practice | Practice |
| **2.5 Solve Inequalities Using Technology** | | | |
| | G–1 | Grid Paper | Student Support |
| | T–4 | The Computer Algebra System (CAS) on the TI-89 Calculator | Technology |
| | BLM 2–7 | Section 2.5 Practice | Practice |
| **2.6 Solve Factorable Polynomial Inequalities Algebraically** | | | |
| | G–5 | Number Lines | Student Support |
| | BLM 2–8 | Section 2.6 Practice | Practice |
| **Chapter 2 Review** | | | |
| | G–5 | Number Lines | Student Support |
| | T–4 | The Computer Algebra System (CAS) on the TI-89 Calculator | Technology |
| | BLM 2–9 | Chapter 2 Review | Practice |
| **Chapter 2 Problem Wrap-Up** | | | |
| | G–1 | Grid Paper | Student Support |
| | BLM 2–10 | Chapter 2 Problem Wrap-Up Rubric | Assessment |
| **Chapter 2 Practice Test** | | | |
| | G–1 | Grid Paper | Student Support |
| | T–4 | The Computer Algebra System (CAS) on the TI-89 Calculator | Technology |
| | BLM 2–11 | Chapter 2 Test | Summative Assessment |
| **Chapter 2 Task: Can You Tell Just by Looking?** | | | |
| | BLM 2–12 | Task: Can You Tell Just by Looking? Rubric | Assessment |
| | BLM 2–13 | BLM Answers | Answers |

# Prerequisite Skills

**Student Text Pages**
82 to 83

**Suggested Timing**
45–60 min

**Related Resources**
- BLM 2–1 Prerequisite Skills

**Differentiated Instruction**
Use an **anticipation guide** to get students thinking about the concepts in this chapter.

**Assessment**
You may wish to use **BLM 2–1 Prerequisite Skills** as a diagnostic assessment. Refer students to the Skills Appendix for examples and further practice of topics.

## Chapter Problem

The Chapter Problem is introduced on page 93. Have students discuss their understanding of the problem, and how mathematics is relevant to running a business. The Chapter Problem is revisited in Sections 2.1 (question 17), 2.2 (question 16), 2.3 (question 16), 2.4 (question 18), and 2.5 (question 13). These questions are designed to help students move toward the Chapter 2 Problem Wrap-Up on page 141. The Chapter Problem questions may be assigned in each section where they appear. Alternatively, you may wish to assign them all with the Chapter Problem Wrap-Up when students have completed the chapter, as part of a summative assessment.