2.1 The Remainder Theorem

Student Text Pages
84 to 93

Suggested Timing
45–65 min

Tools
- computer algebra system

Related Resources
- T–4 The Computer Algebra System (CAS) on the TI-89 Calculator
- BLM 2–2 Section 2.1 Practice

Ongoing Assessment

Use **Assessment Masters A–1** to **A–7** to remind students about the Math Processes expectations and how you may be assessing their integrated use of them.

Teaching Suggestions

- Have students work in pairs on **Investigate 1**. The intent of **Investigate 1** is to help students remember the steps of long division and reinforce the related vocabulary. **Prerequisite Skills** question 1 supports this Investigate.
- When presenting **Example 1**, it is helpful to use different colours of chalk to correspond to each step of the Method 1 solution. Point out the importance of writing the polynomial in descending order of powers.
- Technology tip for **Example 1**, Method 2:
  - Another way to divide a polynomial by a binomial using CAS is to press F2 and select **3: expand**. Enter the division expression, and then press ENTER.
- In **Example 2**, the placeholder is helpful to subtract like terms.
- In **Example 3**, the remainder is 0 and the trinomial quotient can be factored further.
- Have students work in pairs to complete **Investigate 2**. The purpose of this Investigate is to have students discover the remainder theorem by making the connection between $P(b)$ and the remainder when $P(x)$ is divided by $x - \text{b}$. If needed, use **T–4 The Computer Algebra System (CAS) on the TI-89 Calculator** to support Method 2.
- In **Example 4**, stress that even though long division may be used to verify the remainder, the remainder theorem is the preferred method to determine the remainder.
- In **Example 5**, students should understand that long division should not be used to solve for k.
- For question C1 of **Communicate Your Understanding**, students should understand that division with polynomials represents a rational expression where the numerator is the dividend and the denominator is the divisor. Since division by zero is undefined the divisor cannot equal zero hence the restriction. For question C2, students should understand that a placeholder is used when aligning the terms in descending powers. Placeholders help to simplify the subtraction steps in the division. Question C4 points toward the factor theorem, which is dealt with in the next section.
- **Question 5** allows students to solve a problem by reflecting on the strategy needed to determine possible dimensions for the box and by using algebraic reasoning to make conjectures about how to do this. They will also need to select tools and make connections among different strands of mathematics to solve this problem.
- In **questions 12** and **13**, it is not necessary to know the remainder.
- **Question 18** helps students make the connection to the slope formula.
- **Question 18** requires students to reflect and interpret part of the question geometrically and part algebraically. It will be necessary to select tools in order to produce a geometric interpretation and make a representation in the form of a diagram for part d). Reasoning strategies and connecting skills will allow students to ascertain certain information that they will then communicate throughout the question.
- **Questions 20** and **21** lead students to solve a system of linear equations.
- Use **BLM 2–2 Section 2.1 Practice** for remediation or extra practice.

Investigate Answers (pages 84, 87–88)

Investigate 1

1. a) **i)** 753 **ii)** 22 **iii)** 34 **iv)** 5

b) **i)** $x^2 + 5x + 7$ **ii)** $x + 2$ **iii)** $x + 3$ **iv)** 1

2. Answers may vary. Sample answers:

a) The divisor 22 divides into 75, three times. Write 3 above the 5 found under the division sign. Multiply 22 by 3 to get 66. Write 66 below 75. Subtract to get 9. Bring down the 3. 22 divides into 93 four times. Write 4 above the 3. Multiply 22 by 4 to get 88. Write this under 93 and subtract to get the remainder 5.

b) The x in the divisor divides into x^2 of the dividend x times. Write x above the term $5x$. Multiply $x + 2$ with x to get $x^2 + 2x$. Write these terms below $x^2 + 5x$. Subtract to get $3x$. Bring down the 7. The x in the divisor divides into $3x$, three times. Write 3 above the 7. Multiply $x + 2$ with 3 to get $3x + 6$. Write these terms below $3x + 7$ and subtract to get the remainder 1.

c) Similarities: In both long divisions we multiply and subtract to obtain the result. For division of numbers, the digits are lined up by place value. For trinomial division, the divisor terms are lined up with like terms in the dividend.

Differences: When dividing the trinomial, we only use the x in the divisor to determine how many times it divides into x^2, then the other terms as the subtraction occurs.

3. a) Multiply the quotient by the divisor and add the remainder to get the dividend.

b) $753 = (22)(34) + 5$; $x^2 + 5x + 7 = (x + 2)(x + 3) + 1$

Investigate 2

METHOD 1

1. a) **i)** 1 **ii)** -1 **iii)** 2 **iv)** -2

b) **i)** 1 **ii)** -1 **iii)** 2 **iv)** -2

2. a) the remainders **i)** 5 **ii)** -1 **iii)** 32 **iv)** 8

b) **i)** 5 **ii)** -1 **iii)** 32 **iv)** 8

c) They are the same.

d) Substitute the value b, from the binomial $x - b$, into the polynomial. The value $P(b)$ is the remainder when a polynomial function $P(x)$ is divided by $(x - b)$.

3. a) **i)** 7 **ii)** 15 **iii)** 55 **iv)** 19

b) Long division gives the same remainders.

4. The remainder is equal to $P(b)$. The name *remainder theorem* is appropriate because it identifies that the resulting value is the remainder when a long division is performed.

METHOD 2

2. 5

3. 5

4. a) Yes, the remainder of $P(x) \div (x - 1)$ is $P(1)$.

b) -1, 32, 8

c) -1, 32, 8

d) The answers are the same.

e) Substitute the value b, from the binomial $x - b$, into the polynomial. The value $P(b)$ is the remainder when a polynomial function $P(x)$ is divided by $(x - b)$.

5. a) **i)** 7 **ii)** 15 **iii)** 55 **iv)** 19

b) The CAS gives the same remainders.

6. The remainder is equal to $P(b)$. The name remainder theorem is appropriate because it identifies that the resulting value is the remainder when a long division is performed.

Communicate Your Understanding Responses (page 90)

C1. Division by zero is not defined. The value that makes the divisor zero is the restriction. If the divisor is $x - b$, then $x - b \neq 0$, so $x \neq b$.

C2. A placeholder might be necessary to replace a missing x^n-term, where n is less than the degree of the polynomial. The placeholder is helpful when subtracting like terms in the process of long division.

C3. The remainder is incorrect; it should be $R = \dfrac{15}{x - 2}$.

C4. The divisor is $(x + 3)$. The remainder is 0. The divisor is a factor of $P(x)$.

Differentiated Instruction

Use a **journal entry**. Give the topic as "Convince Me That You Understand the Remainder Theorem."

Common Errors

- Students use long division to determine a remainder or solve for an unknown coefficient.

R_x Remind students that the remainder theorem is an efficient and simple means of determining a remainder and solving for unknown values, as shown in Examples 4 and 5.

- Students have difficulty with the steps of long division with polynomials.

R_x Have students practise long division with numbers, and then transfer those steps to polynomial division.

- Students write inaccurate division statements.

R_x Have students practise writing division statements to represent long divisions with numbers. Stress the importance of knowing the vocabulary (*quotient*, *divisor*, *dividend*, and *remainder*) and understanding the relationship between these. It helps to verify that the division statement results in the given polynomial.

C5.

	Dividend	Divisor	Quotient	Remainder
a)	$6x^2 + 5x - 7$	$3x + 1$	$2x + 1$	-8
b)	$12x^3 + 2x^2 + 11x + 14$	$3x + 2$	$4x^2 - 2x + 5$	4
c)	$5x^3 - 7x^2 - x + 6$	$x - 1$	$5x^2 - 2x - 3$	3

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	5, 6
Reasoning and Proving	4–6, 10–22
Reflecting	5, 6, 16, 18
Selecting Tools and Computational Strategies	1–6, 10–22
Connecting	5–22
Representing	18
Communicating	16–19