2.2 The Factor Theorem

Student Text Pages
94 to 103

Suggested Timing
45–60 min

Tools
- computer algebra system
- graphing calculator

Related Resources
- T–4 The Computer Algebra System (CAS) on the TI-89 Calculator
- BLM 2–3 Section 2.2 Practice
- BLM 2–4 Section 2.2 Achievement Check Rubric

Teaching Suggestions

- In the **Investigate**, students discover the factor theorem. This can be completed individually or in pairs.
- Discuss the meaning of "if and only if" in the margin feature on page 95. To support this definition you may wish to provide an example from geometry, such as "A triangle is equilateral if and only if all three angles are equal."
- Discuss with students that up to this point they have learned how to factor quadratics, or polynomials of degree two. The methods in this section will allow them to factor polynomials of degree higher than two.
- Refer to **Prerequisite Skills** questions 2, 4, and 5 for this lesson.
- In **Example 1**, students should understand that the sign of the substituted value b is opposite to the sign of b in $x - b$. In part v), point out how to find the value $\frac{b}{a} = \frac{1}{2}$. In Method 1, students should be careful when evaluating the expression. Remind them of BEDMAS; that is, they must first evaluate exponents, such as $(-2)^3$, then multiply, and then add or subtract. In particular they should be careful when evaluating with negative integers. Fraction evaluations—as in part v)—may need to be explained in detail. The graphing calculator illustrated in Method 2 may be new for many students, and is a very efficient means of finding the y-value of a function at a given x-value. For integral values of x, the corresponding y-values may also be verified using the table values by pressing 2nd [TABLE], though this would not work as easily for rational values.
- Point out that in **Example 1** the factors to be tested were given. This is not the case in **Example 2**, where you are required to find the factors. Trial and error is used to find the first factor. This is where the integral zero theorem and factor theorem are used. Once one factor is determined, use division to draw out the other factors. Division allows you to break up the given polynomial into linear and quadratic factors. Once a quadratic factor is found, previous factoring methods are applied and division is no longer necessary.
- Two options for division (by hand) are long division and synthetic division. The latter is very efficient, in particular when two or more divisions are performed (as in the case for polynomials of degree higher than three).
- Point out that synthetic division also works with the value b if addition is used instead of subtraction. For instance, in **Example 2**, Method 2, you could use 2 instead of -2 in the very first column, and then replace the minus sign with a plus sign. Then, you would *add* instead of subtracting to determine the values in the bottom row.
- The polynomial in **Example 3** is a quartic. Division may be applied twice but in this case it is also possible to factor by grouping, a method that students may not be familiar with.
- Students should understand that the polynomials in **Examples 2** and **3** have a leading coefficient of 1. **Example 4** illustrates how to apply the rational zero theorem when the leading coefficient is not 1. Remind them that a rational number is any number that can be expressed as a fraction.
- For question C2 of **Communicate Your Understanding**, provide an example (such as $6x^2 - x - 2$) that students could use to discuss the methods for factoring trinomials of this form (decomposition, product sum method, fast method, etc.).
- Remind students to use division when completing **questions 6** and **11**.

Differentiated Instruction

Use **concept attainment** to teach the factor theorem.

Use a **Frayer model** to summarize the factor theorem.

Common Errors

- When evaluating $P(b)$, students use the sign of the factor for b. For instance, given $x + 2$, students evaluate $P(2)$ and not $P(-2)$.

R_x Remind them to use the value that makes the factor zero; that is $x = -2$ makes $x + 2 = 0$.

- Students have difficulty evaluating polynomial expressions involving fractions.

R_x Practise more questions such as Prerequisite Skills question 2.

- Students have difficulty factoring quadratics.

R_x Review factoring methods involving quadratics. Practise more questions such as Prerequisite Skills question 4.

Ongoing Assessment

Achievement Check, question 16, on student text page 103.

- **Question 8** gives students the opportunity to make connections with material they have learned previously, to select the necessary tools to do this, and to reason through strategies that will help them to solve the problem of finding the possible dimensions of the block.
- In **questions 9** and **10**, students need to remember what it means to be a factor.
- **Questions 12** and **13** focus on patterns for factoring sums and differences of cubes.
- **Question 12** allows students to use reasoning and reflecting skills to solve the problem of pattern prediction. They will have to select tools to do this and make connections with concepts learned in their past. Once they feel that they can predict a pattern, they will use their communicating skills to describe the prediction.
- For **question 14**, students need only test the values found using the integral root theorem.
- **Question 15** illustrates the value of synthetic division for repeated division.
- The method in **question 17** changes the quartic expression to a quadratic expression. Remind students to give the final answer in terms of x and not x^2.
- **Question 18** involves solving a system of linear equations.
- The point of **question 19** is to have students work backwards from the zeros to the factors. Members of a family of polynomial functions share the same factors. The given point is needed to determine a particular family member.
- **Question 21** develops a pattern to factor difference of powers.
- Use **BLM 2–3 Section 2.2 Practice** for remediation or extra practice.

Investigate Answers (page 94)

1. a) 0

b) $x^3 + 2x^2 - x - 2 = (x - 1)(x^2 + 3x + 2)$

c) $(x - 1)(x + 1)(x + 2)$

d) If $P(b) = 0$, then $x - b$ is a factor of $P(x)$.

2. a) B, C, E

b) If $(x - b)$ is a factor of $P(x)$, then $P(b) = 0$.

3. a) If $P(b) = 0$, then $(x - \mathrm{b})$ is a factor of $P(x)$.

b) $(x - 2)(x - 1)(x + 1)$

Communicate Your Understanding Responses (page 101)

C1. a) B, D, F

b) $(x - 2)(x + 1)(2x + 3)$

C2. When factoring trinomials, using either the product and sum method or the method of decomposition, we determine two numbers whose product is ac. Both methods take into account the leading coefficient, as does the rational zero theorem.

C3. Find a value $x = b$ such that $P(b) = 0$. Then, use division to determine the other factors.

C4. None of the test values—±1, ±2, or ±4—results in 0 when evaluated, so the polynomial does not have integral factors.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	8, 12–15, 19–21
Reasoning and Proving	8–10, 12–16, 17–21
Reflecting	12, 13, 19–21
Selecting Tools and Computational Strategies	1–8, 11–16, 18–21
Connecting	3–6, 8–21
Representing	16
Communicating	12, 13, 16, 20, 21

Achievement Check, question 16, student text page 103

This performance task is designed to assess the specific expectations covered in Section 2.2.
The following Math Process Expectations can be assessed.

- Connecting
- Selecting Tools
- Communicating
- Reflecting

Achievement Chart Category	Related Math Processes
Knowledge and Understanding	Selecting tools and computational strategies
Thinking	Problem solving Reasoning and proving Reflecting
Communication	Communicating, Representing
Application	Selecting tools and computational strategies Connecting

Sample Solution

Provide students with **BLM 2–4 Section 2.2 Achievement Check Rubric** to help them understand what is expected.

a) $P(x) = x^3 - 6x^2 + 9x$

$$\begin{aligned} P(1) &= 1^3 - 6(1)^2 + 9(1) \\ &= 1 - 6 + 9 \\ &= 4 \end{aligned}$$

Since $P(1) \neq 0$, $x - 1$ is not a factor of $P(x)$.

b) Possible values of $\dfrac{b}{a}$ are ± 1, ± 3, and ± 9.

c)

$$\begin{array}{r}
x^2 - 3x \phantom{{}+9x} \\
x - 3 \overline{\smash{\big)}\, x^3 - 6x^2 + 9x} \\
\underline{x^3 - 3x^2} \phantom{{}+9x} \\
-3x^2 + 9x \\
\underline{-3x^2 + 9x} \\
0
\end{array}$$

d)

$$\begin{aligned} P(x) &= 4x^3 + 12x^2 - 16x \\ &= 4x(x^2 + 3x - 4x) \\ &= 4x(x + 4)(x - 1) \end{aligned}$$