2.3 Polynomial Equations

Student Text Pages
104 to 112

Suggested Timing
40–55 min

Tools
- graphing calculator

Related Resources
- BLM 2–5 Section 2.3 Practice

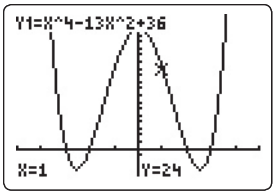
Teaching Suggestions

- Allow students to work individually or in pairs on the **Investigate**.
- Refer to **Prerequisite Skills** questions 6 and 7 for this lesson.
- The polynomials in **Example 1** may be factored without using division. Emphasize the importance of using the most efficient strategy when factoring a polynomial. Reinforce that the word *solve* means to find all x-values that satisfy the equation. The first part of the solution involves factoring. Once the factors are found, the roots are determined by setting each factor to zero.
- The factor theorem and division are needed to factor the polynomial in **Example 2**. Discuss whether you want students to show all the test values, as in the solution to part a). Perhaps it is more efficient for them to verify only the test value that they use to find the first factor. Remind them that depending on the test value selected, it is possible to have a different first factor. For instance, in **Example 2**, if $x = -3$ was tested first, you would divide by $x + 3$.
- Before presenting **Example 3**, point out that a polynomial equation of degree n will always have n roots, however, not all the roots may be real numbers. The x-intercepts of the graph correspond to the real roots, also demonstrated in **Example 3**. Students should only use the quadratic formula when the corresponding trinomial cannot be factored, that is, when the discriminant $b^2 - 4ac$ is not a perfect square. Once again, it should not be necessary for students to list all the test values indicated in **Example 3**.
- Students should understand that many polynomial equations do not have any roots that are integers, and so factoring cannot be used. **Example 4** demonstrates how technology is used to find the roots of these polynomials. Have students refer to the Technology Appendix on page 506 for instructions on how to use the **Zero** operation. Discuss why the solution may be found by graphing the two functions (as suggested in the margin feature) and then using the **Intersect** operation.
- **Example 4** demonstrates that the test values may not provide the solutions to the given equation.
- In **Communicate Your Understanding** questions C1 and C2, draw out the difference between a root and an x-intercept. A root could also be a complex number, but an x-intercept is only a real number.
- The x-intercepts of the graphs in **question 2** are all integer values. Students should pay attention to the given scale for each axis.
- Remind students to use the indicated examples as a reference as they complete questions.
- **Question 5** gives students an opportunity to reflect on each given statement, to reason out possible alternative wording if a statement is false, and to use their connecting and communicating skills to develop and reword any false statements to make them true.
- In **question 8**, all terms should be moved to one side of the equal sign so that the equation is equal to 0. It is beneficial to have a positive leading coefficient.
- **Question 10** requires students to apply their reasoning and proving skills to think through the problem given and to solve it. They will have to select tools and make connections with problem solving strategies they have learned previously to determine the dimensions of the tank.

- A hint for **question 17** part a), is to let $m = x - 1$. For part b), to let $m = x^2 - 4x$.
- A hint for **question 18**, part a), is to use the definition of a root. To solve part b), students should first substitute the value found in part a).
- **Question 19** uses thinking skills of reflecting, reasoning and proving, and problem solving to determine possible dimensions of the open-top boxes. The students will have to show how they can select tools and connect mathematical skills that they have previously learned to solve this problem.
- **Question 20** introduces students to complex numbers. Have students research more information regarding complex numbers and possibly extend the lesson to include complex roots.
- In **question 21**, students are to consider two boxes: a smaller one with dimensions that are consecutive integers and then a larger one with the modified dimensions.
- A hint for **question 22** is to first solve the given polynomial equation.
- Use **BLM 2–5 Section 2.3 Practice** for remediation or extra practice.

Investigate Answers (page 104)

1. a) Window variables: $x \in [-5, 5]$, $y \in [-8, 40]$, Yscl = 2



b) The x-intercepts are -3, -2, 2, and 3.

c) $f(x) = (x - 3)(x - 2)(x + 2)(x + 3)$
The zeros are 3, 2, -2, and -3.

d) The x-intercepts of the graph are the same as the zeros of the function.

2. a) The roots are 3, 2, -2, and -3.

b) The x-intercepts of the graph and the roots of the equation are equal.

c) They are equal.

Communicate Your Understanding Responses (page 109)

C1. A root of an equation is a value for x that produces a result of zero. A root may be real or non-real. A zero is a value of x that makes a function equal to 0. A zero may be real or non-real. An x-intercept is where the graph intersects the x-axis, and is only a real number.

C2. Method 1: Substitute 2, -1, 3, and -2 for x in the left side of the polynomial equation and evaluate to check if the result is 0.
Method 2: Use technology to verify that 2, -1, 3, and -2 are the x-intercepts of the corresponding graph.

C3. 2

C4. Possible methods are: using the factor theorem and long division or synthetic division; using technology to find the x-intercepts (if they are integer values); combining methods (such as factor by grouping, common factor, difference of squares, factor trinomials); and substituting a different letter, such as letting $m = x^2$.

C5. The maximum number of real roots of the corresponding equation is n. The number of x-intercepts is equal to the number of distinct (different) real roots. If a real root is repeated, then the number of x-intercepts is less than the number of real roots.

Common Errors

- Students write the wrong factor for a corresponding root.

R_x Remind students that, for a root $x = 2$, the corresponding factor has the opposite sign, $x - 2$.

- Students mix up factors and roots.

R_x Have students write that $x = b$ is the root and $x - b$ is the factor when applying the factor theorem.

- Students test values to solve the equation.

R_x Students should understand that the test values may or may not comprise all the possible solutions.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	10, 19, 21, 22
Reasoning and Proving	5, 10–13, 15–22
Reflecting	5, 12, 13, 19, 22
Selecting Tools and Computational Strategies	3, 4, 6–11, 14–17, 19–22
Connecting	2, 4–9, 10–22
Representing	16
Communicating	12, 13, 16, 18, 20