# Families of Polynomial Equations

## Teaching Suggestions

- Have students work in pairs or small groups to complete the **Investigate**.
- In addition to using graphing calculators, students may find using *The Geometer's Sketchpad®* useful since colours may be assigned to polynomial functions with the same zeros.
- Refer to **Prerequisite Skills** question 8 for this lesson.
- In **Examples 1** and **2**, point out that the equation for the family of functions must include the constant $k$ to represent the leading coefficient. Encourage students to use fractions and not decimals where appropriate. Apply Chapter 1 skills to graph the polynomials, as shown in **Example 2** part d).
- **Example 3** involves zeros with radicals (or irrational numbers). Do not change these to decimals; leave them as exact values. The solution to part a) may require further scaffolding if students have difficulty simplifying using a difference of squares.
- As students consider the **Communicate Your Understanding** questions, draw out the distinction between the equation for a family of functions and the equation for a member of the family. The similarities and differences in their graphs should be emphasized in terms of the effect of the leading coefficient. It is important for students to understand that infinite functions share the same $x$-intercepts.
- The factors corresponding to the rational zeros in **questions 10** and **11** should be expressed in the form $ax - b$. All equations should be expanded and simplified.
- The purpose of **question 12** is to have students recall (from Chapter 1) the meaning of "order 2."
- **Question 20** requires students to reason out the actual creation of the box and the wording of the question that will help them establish the functions required. They will need to use connecting skills to produce the functions in parts a) and b) and to compare them, and then they will use representing skills to sketch graphs of all three functions.
- Use **BLM 2–6 Section 2.4 Practice** for remediation or extra practice.

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### Investigate Answers (page 114)

**1. b)** Similarities: All graphs are parabolas with $x$-intercepts 1 and $-2$. In set A, the graphs open upward. In set B, the graphs open downward.

Differences: The parabolas have different $y$-intercepts, different vertices, and different stretch factors. Set A i) and set B iv) are reflections of each other in the $x$-axis. This is also true for set A ii) and set B v) and set A iii) and set B vi).

**2.** All have the same $x$-intercepts.

**3. a)** Same factors, but different leading coefficient.

**b)** They are cubic functions and will have the same $x$-intercepts. Their shape and direction will be different due to the sign and value of the leading coefficient.

**4. a)** Window variables: $x \in [-5, 5]$, $y \in [-30, 30]$, Yscl = 3
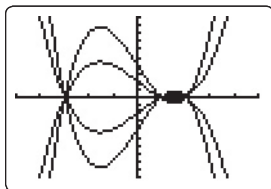


**b)** Answers may vary.

**5.** Families have the same $x$-intercepts and share similar characteristics.

**Communicate Your Understanding Responses (page 118)**

**C1.** Infinite number.

**C2.** The zeros.

**C3.** A point on the graph, other than an *x*-intercept.

**C4.** Similar: Same *x*-intercepts and same general shape.

Different: *y*-intercepts and shape is different by a stretch factor.

# Mathematical Process Expectations

| Process Expectation | Selected Questions |
|---|---|
| Problem Solving | 18–22 |
| Reasoning and Proving | 1–23 |
| Reflecting | 18, 21–23 |
| Selecting Tools and Computational Strategies | 1–23 |
| Connecting | 5–23 |
| Representing | 4, 7–10, 18, 19, 20–22 |
| Communicating | 2, 3, 4, 17, 18 |

**Achievement Check, question 21, student text page 122**

This performance task is designed to assess the specific expectations covered in Section 2.4.

The following Math Process Expectations can be assessed.

- Reasoning and Proving
- Reflecting
- Representing
- Communicating
- Problem Solving
- Selecting Tools
- Connecting

**Sample Solution**

**a)** An equation for the family of functions used to create the design is $y = k(x + 3)(x + 1)(x - 5)$ or $y = k(x^3 - x^2 - 17x - 15)$.

**b)** Using the *y*-intercepts $(0, 7.5)$, $(0, 15)$, $(0, 22.5)$, $(0, 30)$, $(0, 37.5)$, $(0, 45)$, $(0, -7.5)$, $(0, -15)$, $(0, -22.5)$, $(0, -30)$, $(0, -37.5)$, and $(0, -45)$, the respective *k*-values are $-0.5$, $-1$, $-1.5$, $-2$, $-2.5$, $-3$, $0.5$, $1$, $1.5$, $2$, $2.5$, and $3$.

**c)** Answers may vary.

## Level 3 Notes

Look for the following:

### Part a)

- Most *x*-intercepts of the graph are correct
- Most factors in the equation for the family of functions correspond with the *x*-intercepts
- Equation of the family of functions, in factored or expanded form, is mostly accurate

### Part b)

- *y*-intercepts, or other points, of the graphs are mostly accurate
- Corresponding *k*-values are mostly accurate
- Equations for the graphs in the design are mostly accurate

### Part c)
- Design with some degree of complexity
- *x*-intercepts and associated factors for each graph in the design are mostly accurate
- Equations of the graphs in the design are mostly accurate

## What Distinguishes Level 2

### Part a)
- Some x-intercepts of the graph are correct
- Some factors in the equation for the family of functions correspond with the *x*-intercepts
- Equation for the family of functions, in factored or expanded form, has some errors

### Part b)
- *y*-intercepts, or other points, of the graphs are somewhat accurate
- Corresponding *k*-values are somewhat accurate
- Equations for the graphs in the design have some errors

### Part c)
- Design is simple
- *x*-intercepts and associated factors for each graph in the design are somewhat accurate
- Equations of the graphs in the design have some errors

## What Distinguishes Level 4

### Part a)
- *x*-intercepts of the graph are correct
- Factors in the equation for the family of functions correspond with the *x*-intercepts
- Equation of the family of functions is provided, in factored and expanded form, with very minor errors

### Part b)
- *y*-intercepts, or other points, of the graphs have only minor errors
- Corresponding *k*-values have only minor errors
- Equations for the graphs in the design have only minor errors

### Part c)
- Design has a high degree of complexity
- *x*-intercepts and associated factors for each graph in the design have only minor errors
- Equations of the graphs in the design have only minor errors