2.6 Solving Factorable Polynomial Inequalities Algebraically

Student Text Pages
132 to 139

Suggested Timing
45–60 min

Related Resources
- G–5 Number Lines
- BLM 2–8 Section 2.6 Practice

Teaching Suggestions

- Whereas in Section 2.5 technology was used to solve non-factorable polynomial inequalities, this section develops algebraic methods to solve factorable polynomial inequalities.
- In the solution for **Example 1**, part b), be sure to explain why the inequality must be reversed. Remind students to use an open (or hollow) circle when showing a number line solution for an inequality with $>$ or $<$ and a solid circle for solutions to inequalities with $\leq$ or $\geq$.
- In **Example 2**, part a), Method 1, remind students to look for values of x that make the product positive, since > 0 means positive. There are many values that will make this happen and so they must consider all possibilities. In Method 2, the number line intervals are based on the roots of the corresponding equation.
- In **Example 2**, part b), the polynomial must be factored first to obtain a product before it can be solved. Method 2 achieves the same result, but by using intervals and test values. The chart is an efficient way to summarize the results. Remind students that the results will be the same for any test value in the interval.
- In the solution of **Example 3**, the quadratic $w^2 + 19w + 176$ cannot be factored since the discriminant is not a perfect square; in fact, $b^2 - 4ac = -343$ is negative, so there are no other real roots.
- **Example 3** requires students to reflect on the wording of the question and reason out how to put the words into an inequality to solve the problem, finding the minimum dimensions. Once the inequality is produced, they will use selecting tools and connecting of concepts previously learned to work out the identity, using two cases in order to solve the problem.
- As students consider the **Communicate Your Understanding** questions, draw out that solving a polynomial inequality involves factoring it to break it down to a product of linear inequalities, and then determining the solution by considering solutions to the linear equalities. When discussing question C3, point out that the higher the degree of the polynomial the more cases that must be considered. The interval method is more efficient to organize the information.
- Students may choose their preferred method for **questions 4** and **7**.
- **Question 8** uses reflecting, reasoning and proving, and problem solving skills to determine the minimum dimensions of the box. The wording has to be considered, tools have to be selected, and connections have to be made with previously learned mathematical concepts to solve the problem.
- The point of **questions 11** and **12** is for students to apply the most efficient method based on the degree of the polynomial inequality.
- Use **BLM 2–8 Section 2.6 Practice** for remediation or extra practice.

Communicate Your Understanding Responses (page 138)

C1. Examples may vary. The inequality must be reversed to make the statement true. For instance, suppose we choose a value, $x = 3$, and substitute it in $-2x < 16$. The result is $-6 < 16$, which is a true statement. If we use $x > -8$, which is the simplified form of $-2x < 16$, with the inequality reversed, the result is $3 > -8$, which is also true. However, if the inequality is not reversed, the simplified form would be $x < -8$ and then $3 < -8$, which is not a true statement.

C2. A polynomial inequality is solved by using the factors, each of which is linear. The results of these solutions must then be combined to give the final solution.

C3. Both methods are efficient for polynomials of degree one or two, however for polynomials of degree three or more, there are more cases to be considered and so the interval method is more efficient.

Differentiated Instruction

Use **think aloud** during the teaching of this section.

Common Errors

- Students have difficulty determining the cases.

R_x Point out that the cases depend on the number of factors. Remind students to think of the factors in the inequality in terms of the product of two, three, or four numbers, such as *ab* or *abc* or *abcd* as they consider cases.

- Students have difficulty determining the intervals.

R_x Students should first find the zeros of the function, indicate these in numerical order on the number line, and then use them to determine the intervals from left to right.

Ongoing Assessment

Achievement Check, question 10, on student text page 139.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	8, 11, 12, 14
Reasoning and Proving	8, 9, 11–14
Reflecting	8, 9, 11, 14
Selecting Tools and Computational Strategies	2, 4, 7–9, 11–14
Connecting	2–14
Representing	1–3, 5
Communicating	10–12

Achievement Check, question 10, student text page 139

This performance task is designed to assess the specific expectations covered in Section 2.6.

The following Math Process Expectations can be assessed.

- Reasoning and Proving
- Reflecting
- Representing
- Communicating
- Selecting Tools
- Connecting

Sample Solution

a) Factor $x^3 - 5x^2 + 2x + 8$ using the factor theorem.

$x^3 - 5x^2 + 2x + 8 = (x - 4)(x - 2)(x + 1)$

Solve $(x + 1)(x - 2)(x - 4) < 0$.

i)

Factor \ Interval	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$2 < x < 4$	$x = 4$	$x > 4$
$(x + 1)$	–	0	+	+	+	+	+
$(x - 2)$	–	–	–	0	+	+	+
$(x - 4)$	–	–	–	–	–	0	+
$(x + 1)(x - 2)(x - 4)$	–	0	+	0	–	0	+

The solution is $x < -1$ or $2 < x < 4$.

ii) **Case 1**

$(x + 1) < 0$	$(x - 2) < 0$	$(x - 4) < 0$
$x < -1$	$x < 2$	$x < 4$

So, $x < -1$ is a solution.

Case 2

$(x + 1) > 0$	$(x - 2) > 0$	$(x - 4) < 0$
$x > -1$	$x > 2$	$x < 4$

So, $2 < x < 4$ is a solution.

Case 3

$(x + 1) > 0$	$(x - 2) < 0$	$(x - 4) > 0$
$x > -1$	$x < 2$	$x > 4$

Case 3 has no solution.

Case 4

$(x + 1) < 0$	$(x - 2) > 0$	$(x - 4) > 0$
$x < -1$	$x > 2$	$x > 4$

Case 4 has no solution.

Combining the results, the solution is $x < -1$ or $2 < x < 4$.

b) The methods are the same in that the polynomial must be factored and the zeros of the linear factors are used to determine the solution. The methods are different in that a single number line and test values are used in the interval method, whereas the case method requires listing all the possible solutions within each and then considering if their common values meet the criteria of the inequality.

Level 3 Notes

Look for the following:

- Factored form of equation, chart intervals, and factors are mostly accuracy
- Understanding of chart method is evident and final solution is mostly accurate
- Most of the cases are identified and solved fairly accurately
- Results from cases are interpreted fairly accurately
- Justification for how the two methods are the same and how they are different is mostly valid

What Distinguishes Level 2

- Factored form of equation, chart intervals, and factors are somewhat accurate
- Understanding of chart method is evident and final solution is somewhat accurate
- Most of the cases are identified and solved with some accuracy
- Results from cases are interpreted with some errors
- Justification for how the two methods are the same and how they are different is somewhat valid

What Distinguishes Level 4

- Factored form of equation, chart intervals, and factors have only minor errors
- Understanding of chart method is evident and final solution has only minor errors
- All of the cases are identified and solved with a high degree of accuracy
- Results from cases are interpreted with only minor errors
- Justification for how the two methods are the same and how they are different is accurate