Chapter 3

Rational Functions

Curriculum Expectations

Polynomial and Rational Functions

Connecting Graphs and Equations of Rational Functions

C2.1 determine, through investigation with and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals) of the graphs of rational functions that are the reciprocals of linear and quadratic functions, and make connections between the algebraic and graphical representations of these rational functions [e.g., make connections between $f(x) = \frac{1}{x^2 - 4}$ and its graph by using graphing technology and by reasoning that there are vertical asymptotes at $x = 2$ and $x = -2$ and a horizontal asymptote at $y = 0$ and that the function maintains the same sign as $f(x) = x^2 - 4$]

Technology Notes

The technology used in this chapter includes graphing calculators, specifically the TI-83 Plus/TI-84 Plus series, a computer algebra system (CAS), specifically the TI-89/89T series, and *The Geometer's Sketchpad*®.

C2.2 determine, through investigation with and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals) of the graphs of rational functions that have linear expressions in the numerator and denominator [e.g., $f(x) = \frac{2x}{x - 3}$, $h(x) = \frac{x - 2}{3x + 4}$], and make connections between the algebraic and graphical representations of these rational functions

C2.3 sketch the graph of a simple rational function using its key features, given the algebraic representation of the function

Solving Polynomial and Rational Equations

C3.5 determine, through investigation using technology (e.g., graphing calculator, computer algebra systems), the connection between the real roots of a rational equation and the x-intercepts of the graph of the corresponding rational function, and describe this connection [e.g., the real root of the equation $\frac{x - 2}{x - 3} = 0$ is 2, which is the x-intercept of the function $f(x) = \frac{x - 2}{x - 3}$; the equation $\frac{1}{x - 3} = 0$ has no real roots, and the function $f(x) = \frac{1}{x - 3}$ does not intersect the x-axis]

C3.6 solve simple rational equations in one variable algebraically, and verify solutions using technology (e.g., using computer algebra systems to determine the roots; using graphing technology to determine the x-intercepts of the graph of the corresponding rational function)

C3.7 solve problems involving applications of polynomial and simple rational functions and equations [e.g., problems involving the factor theorem or remainder theorem, such as determining the values of k for which the function $f(x) = x^3 + 6x^2 + kx - 4$ gives the same remainder when divided by $x - 1$ and $x + 2$]

Solving Inequalities

C4.1 explain, for polynomial and simple rational functions, the difference between the solution to an equation in one variable and the solution to an inequality in one variable, and demonstrate that given solutions satisfy an inequality (e.g., demonstrate numerically and graphically that the solution to $\frac{1}{x + 1} < 5$ is $x < -1$ or $x > -\frac{4}{5}$)

Characteristics of Functions

Understanding Rates of Change

D1.3 sketch a graph that represents a relationship involving rate of change, as described in words, and verify with technology (e.g., motion sensor) when possible

D1.5 recognize examples of instantaneous rates of change arising from real-world situations, and make connections between instantaneous rates of change and average rates of change (e.g., an average rate of change can be used to approximate an instantaneous rate of change)

D1.6 determine, through investigation using various representations of relationships (e.g., tables of values, graphs, equations), approximate instantaneous rates of change arising from real-world applications (e.g., in the natural, physical, and social sciences) by using average rates of change and reducing the interval over which the average rate of change is determined

D1.7 make connections, through investigation, between the slope of a secant on the graph of a function (e.g., quadratic, exponential, sinusoidal) and the average rate of change of the function over an interval, and between the slope of the tangent to a point on the graph of a function and the instantaneous rate of change of the function at that point

D1.8 determine, through investigation using a variety of tools and strategies (e.g., using a table of values to calculate slopes of secants or graphing secants and measuring their slopes with technology), the approximate slope of the tangent to a given point on the graph of a function (e.g., quadratic, exponential, sinusoidal) by using the slopes of secants through the given point (e.g., investigating the slopes of secants that approach the tangent at that point more and more closely), and make connections to average and instantaneous rates of change

D1.9 solve problems involving average and instantaneous rates of change, including problems arising from real-world applications, by using numerical and graphical methods (e.g., by using graphing technology to graph a tangent and measure its slope)

Combining Functions

D2.2 recognize real-world applications of combinations of functions (e.g., the motion of a damped pendulum can be represented by a function that is the product of a trigonometric function and an exponential function; the frequencies of tones associated with the numbers on a telephone involve the addition of two trigonometric functions), and solve related problems graphically

Using Function Models to Solve Problems

D3.1 compare, through investigation using a variety of tools and strategies (e.g., graphing with technology; comparing algebraic representations; comparing finite differences in tables of values) the characteristics (e.g., key features of the graphs, forms of the equations) of various functions (i.e., polynomial, rational, trigonometric, exponential, logarithmic)

D3.2 solve graphically and numerically equations and inequalities whose solutions are not accessible by standard algebraic techniques

Chapter 3 Planning Chart

Section Suggested Timing	Student Text Page(s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 3 Opener • 10 min	145			
Prerequisite Skills • 30 min	146–147	• G–3 Four Quadrant Grids • G–5 Number Lines • BLM 3–1 Prerequisite Skills		• grid paper
3.1 Reciprocal of a Linear Function • 75 min	148–155	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad*® 4 • BLM 3–2 Section 3.1 Practice		• grid paper • graphing calculator OR • computer • *The Geometer's Sketchpad*®
Extension: Asymptotes and the TI-83 Plus or TI-84 Plus Graphing Calculator • 20–30 min	156			• graphing calculator
3.2 Reciprocal of a Quadratic Function • 150 min	157–167	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad*® 4 • BLM 3–3 Section 3.2 Practice		• grid paper • graphing calculator OR • computer • *The Geometer's Sketchpad*®
3.3 Rational Functions of the Form $f(x) = \frac{ax + b}{cx + d}$ • 75 min	168–176	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad*® 4 • BLM 3–4 Section 3.3 Practice		• grid paper • graphing calculator OR • computer • *The Geometer's Sketchpad*®
3.4 Solving Rational Equations and Inequalities • 75–150 min	177–185	• G–1 Grid Paper • G–5 Number Lines • T–2 The Computer Algebra System (CAS) on the TI-89 Calculator • BLM 3–5 Section 3.4 Practice	• BLM 3–6 Section 3.4 Achievement Check Rubric	• grid paper • graphing calculator • computer algebra system
3.5 Making Connections With Rational Functions and Equations • 75–150 min	186–191	• G–1 Grid Paper • BLM 3–7 Section 3.5 Practice		• grid paper • graphing calculator
Chapter 3 Review • 75 min	192–193	• G–1 Grid Paper • G–5 Number Lines • BLM 3–8 Chapter 3 Review		• grid paper • graphing calculator
Chapter 3 Problem Wrap-Up • 40–60 min	193	• G–1 Grid Paper	• BLM 3–9 Chapter 3 Problem Wrap-Up Rubric	• grid paper • graphing calculator
Chapter 3 Practice Test • 75 min	194–195	• G–1 Grid Paper • G–5 Number Lines	• BLM 3–10 Chapter 3 Test	• grid paper
Chapters 1 to 3 Review • 60–75 min	196–197	• G–1 Grid Paper • G–5 Number Lines		• grid paper • graphing calculator
Chapter 3 Task: ZENN and Now • 60–75 min	198	• BLM 3–12 BLM Answers	• BLM 3–11 Task: ZENN and Now Rubric	

Chapter 3 Blackline Masters Checklist

	BLM	Title	Purpose
Prerequisite Skills			
	G–3	Four Quadrant Grids	Student Support
	G–5	Number Lines	Student Support
	BLM 3–1	Prerequisite Skills	Practice
3.1 Reciprocal of a Linear Function			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	BLM 3–2	Section 3.1 Practice	Practice
Extension: Asymptotes and the TI-83 Plus or TI-84 Plus Graphing Calculator			
3.2 Reciprocal of a Quadratic Function			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	BLM 3–3	Section 3.2 Practice	Practice
3.3 Rational Functions of the Form $f(x) = \frac{ax + b}{cx + d}$			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	BLM 3–4	Section 3.3 Practice	Practice
3.4 Solving Rational Equations and Inequalities			
	G–1	Grid Paper	Student Support
	G–5	Number Lines	Student Support
	T–4	The Computer Algebra System (CAS) on the TI-89 Calculator	Technology
	BLM 3–5	Section 3.4 Practice	Practice
	BLM 3–6	Section 3.4 Achievement Check Rubric	Assessment
3.5 Making Connections With Rational Functions and Equations			
	G–1	Grid Paper	Student Support
	BLM 3–7	Section 3.5 Practice	Practice
Chapter 3 Review			
	G–1	Grid Paper	Student Support
	G–5	Number Lines	Student Support
	BLM 3–8	Chapter 3 Review	Practice
Chapter 3 Problem Wrap-Up			
	G–1	Grid Paper	Student Support
	BLM 3–9	Chapter 3 Problem Wrap-Up Rubric	Assessment
Chapter 3 Practice Test			
	G–1	Grid Paper	Student Support
	G–5	Number Lines	Student Support
	BLM 3–10	Chapter 3 Test	Summative Assessment
Chapters 1 to 3 Review			
	G–1	Grid Paper	Student Support
	G–5	Number Lines	Student Support
Chapter 3 Task: ZENN and Now			
	BLM 3–11	Task: ZENN and Now Rubric	Assessment
	BLM 3–12	BLM Answers	Answers

Prerequisite Skills

Student Text Pages
146 to 147

Suggested Timing
30 min

Tools
- grid paper

Related Resources
- G–3 Four Quadrant Grids
- G–5 Number Lines
- BLM 3–1 Prerequisite Skills

Assessment
You may wish to use **BLM 3–1 Prerequisite Skills** as a diagnostic assessment. Refer students to the Skills Appendix for examples and further practice of topics.

Chapter Problem

- The Chapter Problem is introduced on page 147. Have students discuss their understanding of the topic. The Chapter Problem is revisited in Sections 3.2 (question 13), 3.3 (question 10), and 3.4 (question 15). These questions are designed to help students move toward the Chapter 3 Problem Wrap-Up on page 193. Alternatively, you may wish to assign the Chapter Problem questions and Chapter Problem Wrap-Up when students have completed the chapter, as part of a summative assessment.