3.1 Reciprocal of a Linear Function

Student Text Pages
148 to 155

Suggested Timing
75 min

Tools
- grid paper
- graphing calculator

OR
- computer
- *The Geometer's Sketchpad®*

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad®* 4
- BLM 3–2 Section 3.1 Practice

Ongoing Assessment

Use **Assessment Masters A–1** to **A–7** to remind students about the Math Processes expectations and how you may be assessing their integrated use of them.

Teaching Suggestions

- Although students will have studied the reciprocal function in grade 11, this section is developed through the concepts of vertical and horizontal asymptotes, rather than through transformations. In the **Investigate**, students consider restrictions on the domain, along with investigating the end behaviour of the functions. If needed, use **T–2** ***The Geometer's Sketchpad*® 4** to support this activity.
- Students should investigate the behaviour of a function as x approaches the restriction on the domain from both sides, and as x approaches $\pm\infty$, by using tables of values. This way, they get a sense of how the function behaves.
- Caution: Do not introduce the concept of a limit. Once the students understand the concept of behaviour near an asymptote, encourage them to shorten their steps, provided they give reasons for their decisions. Encourage the use of descriptors, such as "as x approaches 3 from the left, the function approaches $-\infty$," or "as x approaches $+\infty$, $f(x)$ approaches 0." Leave the formal part of limits to the calculus course.
- **Example 3** uses reflecting and reasoning and proving skills to determine where the slope is increasing or decreasing. Making connections with work done in Chapter 1 with rates of change will be helpful. In addition, the rational function should be represented by a graph to assess the slope visually, and communicating skills will provide the description required.
- When investigating rates of change, students should continue to use tables of values to illustrate how the slope calculations become more accurate as the secant approaches a tangent. The spreadsheet characteristics of a graphing calculator can help quicken the calculations.
- **Questions 8** and **9** require students to work backward, given the characteristics of a reciprocal function. Suggest that they set up equations to solve.
- **Question 10** allows students to reason out how to write the function required and to use connecting skills to write the function in part a). Representing skills will be necessary to sketch a graph of this function. A description of the rates of change of the time as the average speed increases will then be communicated.
- **Both questions 10** and **11** involve inverse proportionality. It is recommended that at least one of these questions be assigned.
- **Question 12** requires students to apply their investigation skills.
- **Question 14** involves the extension of the skills learned in this section to other types of reciprocal functions.
- Use **BLM 3–2 Section 3.1 Practice** for remediation or extra practice.

Investigate Answers (pages 148–149)

1. a) $\{x \in \mathbb{R}, x \neq 0\}$

b)

x	y
-2	$-\frac{1}{2}$
-1	-1
0	undefined
1	1
2	$\frac{1}{2}$

Window variables: $x \in [-5, 5]$, $y \in [-3, 3]$

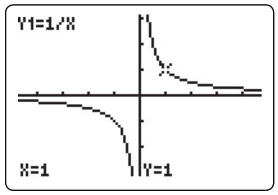
Differentiated Instruction

Build a **word wall/information wall** for the terminology in this chapter/section.

Common Errors

- Students may assign the incorrect sign to the vertical asymptote.

R_x Remind them of how transformations of *x*-coordinates are always the reverse of what seems to be the case. For example, the vertex of $y = (x + 3)^2 + 4$ is $(-3, 4)$.



c) **i)** The function approaches $-\infty$. **ii)** The function approaches $+\infty$.

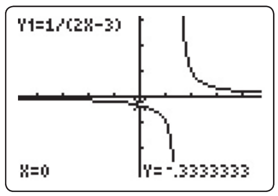
d) **i)** The function approaches 0. **ii)** The function approaches 0.

e) at $x = 0.1$, m $= -100$; at $x = 1$, $m = -1$; at $x = 10$, $m = -0.01$; $x = -0.1$, $m = -100$; $x = -1$, $m = -1$; $x = -10$, $m = -0.01$

f) Answers may vary. Sample answer: As x approaches 0 from the left or the right, the slope of the curve approaches undefined. There is a vertical asymptote at $x = 0$. As x approaches $-\infty$ or $+\infty$, the slope of the curve approaches 0. The horizontal asymptote has equation $y = 0$.

2. a) $\left\{x \in \mathbb{R}, x \neq \frac{3}{2}\right\}$ The denominator is zero when $x = \frac{3}{2}$.

b) Window variables: $x \in [-5, 5]$, $y \in [-3, 3]$



c) **i)** The function approaches $-\infty$. **ii)** The function approaches $+\infty$.

d) **i)** The function approaches 0. **ii)** The function approaches 0.

e) at $x = 1.6$, $m = -50$; $x = 2$, $m = -2$; $x = 10$, $m \doteq -0.007$; $x = 1.4$, $m = -50$; $x = 0$, $m \doteq -0.222$; $x = -10$, $m \doteq -0.004$

f) Answers may vary. Sample answer: As x approaches $\frac{3}{2}$ from the left or the right, the slope of the curve approaches undefined. There is a vertical asymptote at $x = \frac{3}{2}$. As x approaches $-\infty$ or $+\infty$, the slope of the curve approaches 0. The horizontal asymptote has equation $y = 0$.

3. Domain: $\left\{x \in \mathbb{R}, x \neq \frac{c}{k}, k \neq 0\right\}$

Range: $\{y \in \mathbb{R}, y \neq 0\}$

Asymptotes: $x = \frac{c}{k}$, $k \neq 0$; $y = 0$

Communicate Your Understanding Responses (page 153)

C1. The domain is $\{x \in \mathbb{R}, x \neq 3\}$. There is an asymptote at $x = 3$.

C2. a) The function value decreases and approaches zero.

b) The function value increases and approaches undefined.

C3. No. Answers may vary.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	
Reasoning and Proving	8, 10, 11, 12, 14
Reflecting	
Selecting Tools and Computational Strategies	2, 4, 10–13
Connecting	1, 3, 5–13, 14, 15
Representing	1–4, 6, 7, 10–15
Communicating	10, 11, 14, 15

Extension

Student Text
Page 156

Suggested Timing
20–30 min

Tools
- graphing calculator

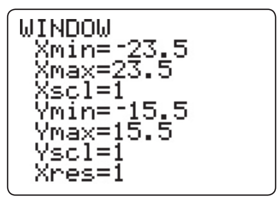
Asymptotes and the TI-83 Plus or TI-84 Plus Graphing Calculator

Teaching Suggestions

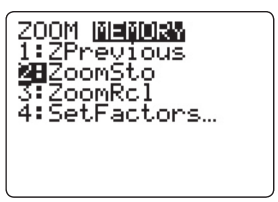
- For classes that are using the TI-83 Plus or TI-84 Plus graphing calculator, this is a valuable activity. Not only does it help overcome the limitations of the graphing calculators, but it helps to consolidate the students' understanding of vertical asymptotes.
- The ZDecimal settings provide a friendly square window. It is friendly because it has the window variables $x \in [-4.7, 4.7]$, $y \in [-3.1, 3.1]$. It is square because the horizontal and vertical scales in the screen are equal in size. A square window is often the preferred window to use because it displays no visual distortion.
- To ensure that a window is square, the size of a horizontal step (Δx) must equal the size of a vertical step (Δy). For example:
  - For window variables $x \in [-4.7, 4.7]$, $y \in [-3.1, 3.1]$, Δx is 0.1.
  - For window variables $x \in [-9.4, 9.4]$, $y \in [-6.2, 6.2]$, Δx is 0.2.
- To get a friendly window of 0.5 pixel increments, for example, on the TI-83 Plus:

$$x: \frac{0.5 \times 94}{2} = 23.5$$

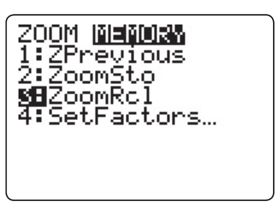
$$y: \frac{0.5 \times 62}{2} = 15.5$$



- To save this friendly window and recall it quickly, follow these steps.
  - First enter the values into the **WINDOW** screen.
  - Press ZOOM, cursor over to the **MEMORY** menu, and select **2:ZoomSto**.



  - The next time you need this window, press ZOOM, cursor over to the **MEMORY** menu, and select **3:ZoomRcl**.



- Note: Such asymptote issues do not occur on CAS calculators, such as the TI-89, TI-92, or TI-Nspire™ handheld.