3.4 Solving Rational Equations and Inequalities

Student Text Pages
177 to 185

Suggested Timing
75–150 min

Tools
- grid paper
- graphing calculator
- computer algebra system

Related Resources
- G–1 Grid Paper
- G–5 Number Lines
- T–4 The Computer Algebra System (CAS) on the TI-89 Calculator
- BLM 3–5 Section 3.4 Practice
- BLM 3–6 Section 3.4 Achievement Check Rubric

Teaching Suggestions

- Rational equations can be solved using methods from Chapter 2. Caution students to state the restrictions on the variable when dividing both sides by an expression or when reducing rational equations.
- As shown in **Example 2**, for equations that cannot be solved using algebraic methods, the use of technology should be encouraged. Use **T–4 The Computer Algebra System (CAS) on the TI-89 Calculator** to support **Example 2**, Method 2.
- Technology tip for **Example 2**, Method 2:
  - Students using the TI-89 have two options: press F2 and select either **1:solve(** or **4:zeros(**.
- Students should be encouraged to use the method from **Example 4** as soon as they understand the consistencies of the values of rational functions in each branch of the graph. One important key is to allow the column headings to mimic a number line.
- **Example 4** gives the opportunity to reason out different strategies possible to solve the inequality. It will then be necessary to select various tools and make mathematical connections with previously learned material to work through the solution. One of the methods of solving the inequality involves representing the inequality graphically.
- Many students have the ability to visualize the graph of a rational function without the use of a table. With this technique, students need to provide a fully labelled graph of the function and mark the sign of the function in each interval, as evidence for their solution.
- **Question 6** requires the use of algebraic reasoning and making connections with mathematical concepts already learned to write the required rational equation. It will be necessary to represent this described function with an equation and to communicate an explanation of the reasoning process.
- **Questions 7** and **8** can be done with a graphing calculator. Have students enter the rational inequality as two equations. For example, $Y_1 = \frac{x}{x+1}$ and $Y_2 = \frac{2x}{x-2}$.
- **Questions 12** and **13** are short investigations that require some communication of the students' findings.
- **Questions 14**, **15**, and **16** are good applications of rational equations and inequalities.
- **Questions 18** and **20** involve combining two equations and solving them using the skills from this section.
- Use **BLM 3–5 Section 3.4 Practice** for remediation or extra practice.

Differentiated Instruction

Use a **journal entry**. Give the topic as "How to Solve a Rational Inequality."

Common Errors

- Some students will divide both sides of an equation or inequality by a common numerator, thereby eliminating one or more solutions.

R_x Provide the correct solutions and ask students to substitute each value into the original equation. If this occurs with inequalities, use technology to illustrate the regions and show where the discontinuity occurs.

- Some students will neglect to change the inequality when multiplying or dividing by a negative value.

R_x Have students compare the relationships $-4 < 4$ and $(-1)(-4) > (-1)(4)$.

Ongoing Assessment

Achievement Check, question 17, on student text page 185.

Communicate Your Understanding Responses (page 183)

C1. Answers may vary. Sample answer:
Determine restrictions: $x \neq 1, x \neq -5$.
Multiply both sides by: $(x - 1)(x + 5)$
Solve for x.

C2. The expression $\frac{1}{x^2 + 2x + 9}$ is positive for all values of x.

C3. The equality has one solution, $x = -31$. The inequality has a range of solutions, $(-\infty, -31) \cup (-4, 5)$.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	19, 20
Reasoning and Proving	6, 7, 11–20
Reflecting	18–20
Selecting Tools and Computational Strategies	1–5, 7, 8–10, 17–20
Connecting	1, 4–20
Representing	4, 5, 7, 8, 10, 14, 17
Communicating	6, 12, 13

Achievement Check, question 17, student text page 185

This performance task is designed to assess the specific expectations covered in Sections 3.4.

The following Math Process Expectations can be assessed.

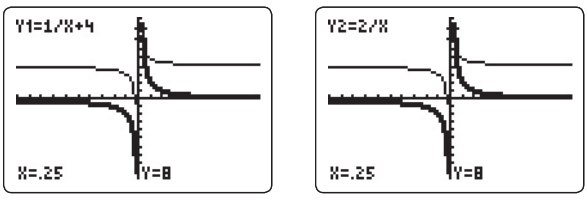
- Reasoning and Proving
- Reflecting
- Representing
- Communicating
- Selecting Tools
- Connecting

Achievement Chart Category	Related Math Processes
Knowledge and Understanding	Selecting tools and computational strategies
Thinking	Problem solving, Reasoning and proving Reflecting
Communication	Communicating, Representing
Application	Selecting tools and computational strategies Connecting

Sample Solution

Provide students with **BLM 3–6 Section 3.4 Achievement Check Rubric** to help them understand what is expected.

a)



The point of intersection is $\left(\frac{1}{4}, 8\right)$.

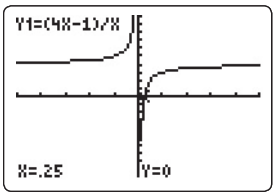
b) $\frac{1}{x} + 4 < \frac{2}{x}$

$\frac{1}{x} + 4 - \frac{2}{x} < 0$

$4 - \frac{1}{x} < 0$

$\frac{4x - 1}{x} < 0$

The vertical asymptote has equation $x = 0$ and the x-intercept is $\frac{1}{4}$.



So, $f(x) < g(x)$ for $0 < x < \frac{1}{4}$.

c) $\frac{1}{x} + 4 = \frac{2}{x}$

$1 + 4x = 2, x \neq 0$

$4x = 1$

$x = \frac{1}{4}$

d) **Case 1:** $x < 0$

$\frac{1}{x} + 4 < \frac{2}{x}$

$1 + 4x > 2$

$4x > 1$

$x > \frac{1}{4}$

No solution.

Case 2: $x > 0$

$\frac{1}{x} + 4 < \frac{2}{x}$

$1 + 4x < 2$

$4x < 1$

$x < \frac{1}{4}$

So, $f(x) < g(x)$ for $0 < x < \frac{1}{4}$.