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# Chapter 4

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## Trigonometry

### Curriculum Expectations

#### Trigonometric Functions

##### *Understanding and Applying Radian Measure*

**B1.1** recognize the radian as an alternative unit to the degree for angle measurement, define the radian measure of an angle as the length of the arc that subtends this angle at the centre of a unit circle, and develop and apply the relationship between radian and degree measure

**B1.2** represent radian measure in terms of  $\pi$  (e.g.,  $\frac{\pi}{3}$  radians,  $2\pi$  radians) and as a rational number (e.g., 1.05 radians, 6.28 radians)

**B1.3** determine, with technology, the primary trigonometric ratios (i.e., sine, cosine, tangent) and the reciprocal trigonometric ratios (i.e., cosecant, secant, cotangent) of angles expressed in radian measure

**B1.4** determine, without technology, the exact values of the primary trigonometric ratios and the reciprocal trigonometric ratios for the special angles  $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ , and their multiples less than or equal to  $2\pi$

##### *Solving Trigonometric Equations*

**B3.1** recognize equivalent trigonometric expressions [e.g., by using the angles in a right triangle to recognize that  $\sin x$  and  $\cos\left(\frac{\pi}{2} - x\right)$  are equivalent; by using transformations to recognize that  $\cos\left(\frac{\pi}{2} + x\right)$  and  $-\sin x$  are equivalent], and verify equivalence using graphing technology

**B3.2** explore the algebraic development of the compound angle formulas (e.g., verify the formulas in numerical examples, using technology; follow a demonstration of the algebraic development [student reproduction of the development of the general case is not required]), and use the formulas to determine exact values of trigonometric ratios [e.g., determining the exact value of  $\sin\left(\frac{\pi}{12}\right)$  by first rewriting it in terms of special angles as  $\sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$ ]

**B3.3** recognize that trigonometric identities are equations that are true for every value in the domain (i.e., a counter-example can be used to show that an equation is not an identity), prove trigonometric identities through the application of reasoning skills, using a variety of relationships (e.g.,  $\tan x = \frac{\sin x}{\cos x}$ ;  $\sin^2 x + \cos^2 x = 1$ ; the reciprocal identities; the compound angle formulas), and verify identities using technology.

#### Technology Notes

- The expectations for this course mandate heavy use of technology. Strong support for technology use has been included in the lesson design.
- To avoid the necessity for multiple learning curves, the primary technologies in use are
  - graphing calculators, specifically the TI-83 Plus/ TI-84 Plus series
  - computer algebra system (CAS), specifically for the TI-89/TI-89T series
  - *The Geometer's Sketchpad*®
- *The Geometer's Sketchpad*® is licensed in Ontario for use by students at home. Consider providing each student with a copy to install on a home computer. This greatly expands the kinds of homework you can assign, and gets around some of the access problems you may run into using school computers. Ensure that students without home computers have an alternative. One possibility is to pair up with a classmate who is willing to share a computer. Another is availability of the software on public use computers in the school, such as library computers.
- Activities involving technology include specific instructions. In addition, you can refer to the Technology Appendix for help or review.
- A scientific calculator provides strong support for students, and is intended to be a standard tool available to the student at all times.

## Chapter 4 Planning Chart

Section Suggested Timing	Student Text Page(s)	Teacher's Resource Blackline Masters	Assessment	Tools
<b>Chapter 4 Opener</b> • 10 min	199			
<b>Prerequisite Skills</b> • 40–50 min	200–201	<ul style="list-style-type: none"> <li>• G–1 Grid Paper</li> <li>• BLM 4–1 Prerequisite Skills</li> </ul>		<ul style="list-style-type: none"> <li>• grid paper</li> <li>• scientific calculator</li> </ul>
<b>4.1 Radian Measure</b> • 60–75 min	202–210	<ul style="list-style-type: none"> <li>• G–4 Protractor</li> <li>• T–2 <i>The Geometer's Sketchpad</i>® 4</li> <li>• T–4 The Computer Algebra System (CAS) on the TI-89 Calculator</li> <li>• BLM 4–2 Section 4.1 Practice</li> </ul>	<ul style="list-style-type: none"> <li>• BLM 4–3 Section 4.1 Achievement Check Rubric</li> </ul>	<ul style="list-style-type: none"> <li>• a large wheel, such as a bicycle wheel</li> <li>• a length of heavy string</li> <li>• masking tape</li> <li>• protractor</li> <li>• scissors</li> <li>• computer</li> <li>• <i>The Geometer's Sketchpad</i>®</li> <li>• scientific calculator</li> <li>• graphing calculator</li> <li>• computer algebra system</li> </ul>
<b>4.2 Trigonometric Ratios and Special Angles</b> • 60–75 min	211–219	<ul style="list-style-type: none"> <li>• G–1 Grid Paper</li> <li>• G–4 Protractor</li> <li>• T–2 <i>The Geometer's Sketchpad</i>® 4</li> <li>• T–4 The Computer Algebra System (CAS) on the TI-89 Calculator</li> <li>• BLM 4–4 Summary of Special Angles and Trigonometric Ratios</li> <li>• BLM 4–5 Section 4.2 Practice</li> </ul>		<ul style="list-style-type: none"> <li>• scientific calculator</li> <li>• graphing calculator</li> <li>• computer algebra system</li> <li>• compasses</li> <li>• grid paper</li> <li>• protractor</li> <li>• computer</li> <li>• <i>The Geometer's Sketchpad</i>®</li> </ul>
<b>4.3 Equivalent Trigonometric Expressions</b> • 60–75 min	220–227	<ul style="list-style-type: none"> <li>• G–1 Grid Paper</li> <li>• G–2 Placemat</li> <li>• G–4 Protractor</li> <li>• T–2 <i>The Geometer's Sketchpad</i>® 4</li> <li>• BLM 4–6 Summary of Trigonometric Identities</li> <li>• BLM 4–7 Section 4.3 Practice</li> </ul>		<ul style="list-style-type: none"> <li>• grid paper</li> <li>• graphing calculator</li> <li>• compasses and protractor</li> </ul> <p>OR</p> <ul style="list-style-type: none"> <li>• computer with <i>The Geometer's Sketchpad</i>®</li> </ul>
<b>4.4 Compound Angle Formulas</b> • 60–75 min	228–235	<ul style="list-style-type: none"> <li>• BLM 4–8 Section 4.4 Practice</li> </ul>		<ul style="list-style-type: none"> <li>• scientific calculator</li> <li>• graphing calculator</li> </ul>
<b>4.5 Prove Trigonometric Identities</b> • 60–75 min	236–241	<ul style="list-style-type: none"> <li>• BLM 4–9 Section 4.5 Practice</li> </ul>		<ul style="list-style-type: none"> <li>• graphing calculator</li> </ul>
<b>Extension: Use <i>The Geometer's Sketchpad</i>® to Sketch and Manipulate Three-Dimensional Structures in a Two-Dimensional Representation</b> • 30–40 min	242–243	<ul style="list-style-type: none"> <li>• T–2 <i>The Geometer's Sketchpad</i>® 4</li> </ul>		<ul style="list-style-type: none"> <li>• computer</li> <li>• <i>The Geometer's Sketchpad</i>®</li> </ul>
<b>Chapter 4 Review</b> • 60–75 min	244–245	<ul style="list-style-type: none"> <li>• BLM 4–10 Chapter 4 Review</li> </ul>		<ul style="list-style-type: none"> <li>• scientific calculator</li> <li>• graphing calculator</li> </ul>
<b>Chapter 4 Problem Wrap-Up</b> • 20–30 min	245	<ul style="list-style-type: none"> <li>• G–1 Grid Paper</li> </ul>	<ul style="list-style-type: none"> <li>• BLM 4–11 Chapter 4 Problem Wrap-Up Rubric</li> </ul>	<ul style="list-style-type: none"> <li>• grid paper</li> <li>• scientific calculator</li> </ul>
<b>Chapter 4 Practice Test</b> • 40–50 min	246–247		<ul style="list-style-type: none"> <li>• BLM 4–12 Chapter 4 Test</li> </ul>	<ul style="list-style-type: none"> <li>• scientific calculator</li> <li>• graphing calculator</li> </ul>
<b>Chapter 4 Task: Make Your Own Identity</b> • 60–75 min	248	<ul style="list-style-type: none"> <li>• BLM 4–14 BLM Answers</li> </ul>	<ul style="list-style-type: none"> <li>• BLM 4–13 Task: Make Your Own Identity Rubric</li> </ul>	<ul style="list-style-type: none"> <li>• graphing calculator</li> </ul>

## Chapter 4 Blackline Masters Checklist

	BLM	Title	Purpose
<b>Prerequisite Skills</b>			
	G-1	Grid Paper	Student Support
	BLM 4-1	Prerequisite Skills	Practice
<b>4.1 Radian Measure</b>			
	G-4	Protractor	Student Support
	T-2	<i>The Geometer's Sketchpad</i> ® 4	Technology
	T-4	The Computer Algebra System (CAS) on the TI-89 Calculator	Technology
	BLM 4-2	Section 4.1 Practice	Practice
	BLM 4-3	Section 4.1 Achievement Check Rubric	Assessment
<b>4.2 Trigonometric Ratios and Special Angles</b>			
	G-1	Grid Paper	Student Support
	G-4	Protractor	Student Support
	T-2	<i>The Geometer's Sketchpad</i> ® 4	Technology
	T-4	The Computer Algebra System (CAS) on the TI-89 Calculator	Technology
	BLM 4-4	Summary of Special Angles and Trigonometric Ratios	Student Support
	BLM 4-5	Section 4.2 Practice	Practice
<b>4.3 Equivalent Trigonometric Expressions</b>			
	G-1	Grid Paper	Student Support
	G-2	Placemat	Student Support
	G-4	Protractor	Student Support
	T-2	<i>The Geometer's Sketchpad</i> ® 4	Technology
	BLM 4-6	Summary of Trigonometric Identities	Student Support
	BLM 4-7	Section 4.3 Practice	Practice
<b>4.4 Compound Angle Formulas</b>			
	BLM 4-8	Section 4.4 Practice	Practice
<b>4.5 Prove Trigonometric Identities</b>			
	BLM 4-9	Section 4.5 Practice	Practice
<b>Extension: Use <i>The Geometer's Sketchpad</i>® to Sketch and Manipulate Three-Dimensional Structures in a Two-Dimensional Representation</b>			
	T-2	<i>The Geometer's Sketchpad</i> ® 4	Technology
<b>Chapter 4 Review</b>			
	BLM 4-10	Chapter 4 Review	Practice
<b>Chapter 4 Problem Wrap-Up</b>			
	G-1	Grid Paper	Student Support
	BLM 4-11	Chapter 4 Problem Wrap-Up Rubric	Assessment
<b>Chapter 4 Practice Test</b>			
	BLM 4-12	Chapter 4 Test	Summative Assessment
<b>Chapter 4 Task: Make Your Own Identity</b>			
	BLM 4-13	Task: Make Your Own Identity Rubric	Assessment
	BLM 4-14	BLM Answers	Answers

# Prerequisite Skills

**Student Text Pages**

200 to 201

**Suggested Timing**

40–50 min

**Tools**

- grid paper
- scientific calculator

**Related Resources**

- G–1 Grid Paper
- BLM 4–1 Prerequisite Skills

**Assessment**

You may wish to use **BLM 4–1 Prerequisite Skills** as a diagnostic assessment. Refer students to the Skills Appendix for examples and further practice of topics.

## Chapter Problem

- The Chapter Problem is introduced on page 201. Have students discuss their understanding of the topic. To stimulate interest, you can find demonstration video clips for Computer Generated Imagery (CGI) on the Internet. You can bookmark clips of interest to make them easy to retrieve in class. The Chapter Problem is revisited in Sections 4.1 (question 18), 4.2 (question 19), 4.3 (question 22), and 4.4 (question 17). These questions are designed to help students move toward the Chapter 4 Problem Wrap-Up on page 245. Alternatively, you may wish to assign the Chapter Problem questions and Chapter Problem Wrap-Up when students have completed the chapter, as part of a summative assessment.