4.1 Radian Measure

Student Text Pages
202 to 210

Suggested Timing
60–75 min

Tools
- a large wheel, such as a bicycle wheel
- a length of heavy string
- masking tape
- protractor
- scissors
- computer
- *The Geometer's Sketchpad®*
- scientific calculator
- graphing calculator
- computer algebra system

Related Resources
- G–4 Protractor
- T–2 *The Geometer's Sketchpad®* 4
- T–4 The Computer Algebra System (CAS) on the TI-89 Calculator
- BLM 4–2 Section 4.1 Practice
- BLM 4–3 Section 4.1 Achievement Check Rubric

Teaching Suggestions

- If sufficient wheels are not available for the **Investigate**, consider working through Method 1 as a class group, with students taking turns performing steps or making measurements.
- Technology tips for Method 2 of the **Investigate**:
  – To turn on the grid display, from the **Graph** menu choose **Show Grid**.
  – Adjust the scale on the axes by dragging the **Unit Point** on the x-axis.
  – When using *The Geometer's Sketchpad®* get rid of unwanted points, lines, and labels by using the **Hide** features from the **Display** menu.
  – Change the label of a point by double clicking on that letter.
  – To calculate the ratio of the length of the arc to the length of the radius, from the **Measure** menu choose **Calculate**. Select the measurement of the length of the arc by clicking on it (this will appear on the calculator), then the division sign followed by the measurement of the length of the radius.
- While working through the **Investigate**, some students may need a refresher on how to perform elementary operations with *The Geometer's Sketchpad®*. Refer students to the Technology Appendix on page 506, the detailed instructions found under the **Help** menu in *The Geometer's Sketchpad®*, or use **T–2 *The Geometer's Sketchpad®* 4**.
- When taking up the **Investigate**, consider demonstrating the use of the **Transform** menu to perform step 11 of Method 2. Set the **Preferences** to measure angles in radians. Then, start with point C on the positive x-axis and repeatedly rotate it about the origin in steps of one radian. Point out that six radians are not quite enough for one complete revolution about the origin, and relate this fact to 2π.
- Before working through **Example 1**, have students estimate the answer using the fact that one radian is a little less than 60°. Hence, an angle of 30° should be about half a radian.
- Method 2 of **Example 1** makes use of a computer algebra system (CAS). The instructions are detailed enough so that students who have not used a CAS before can perform the calculations. You can also direct students to the Technology Appendix on page 506. If needed, use **T–4 The Computer Algebra System (CAS) on the TI-89 Calculator** to support **Example 1**. If a CAS is not readily available at all times, consider waiting until you have encountered several CAS activities, and then have students perform them all at once.
- Technology tip for Method 2 of **Example 1**:
  – On the TI-89 calculator, change the **MODE** and ensure that the **Angle** measure is set to **RADIAN**. Scroll down to change the **Exact/Approx** so that it is set to **AUTO**. To save these settings, select **ENTER**. Students at times select **ESC**, which does not save the current settings but cancels the save.
- Before working through **Example 2**, have students estimate the answer using the fact that one radian is a little less than 60°. $\frac{\pi}{4}$ is about $\frac{3}{4}$ radians, or about 45°.
- Before introducing **Example 3**, use the bicycle wheel as a visual aid in demonstrating why the formula for arc length works.
- Before introducing **Example 4**, use the bicycle wheel to demonstrate angular velocity. Attach a piece of masking tape to the rim, and spin the wheel. Have students time five revolutions, and then make an estimate of the angular velocity in both degrees and radians.

Ongoing Assessment

Use **Assessment Masters A–1** to **A–7** to remind students about the Math Processes expectations and how you may be assessing their integrated use of them.

- As students consider the **Communicate Your Understanding** questions, have them make connections to physical objects, such as the bicycle wheel or a carousel.
- *The Geometer's Sketchpad*® (GSP) is a useful tool for students to have at home. Consider taking advantage of the home use licence provided by the Ministry of Education to allow students to do questions such as **question 12** at home.
- Before assigning **question 13**, consider spending a minute or two with a globe reviewing the terms *latitude* and *longitude* and visually demonstrating the meaning of a nautical mile.
- **Question 13** allows the students to reflect and clarify their understanding of this particular question while they apply reasoning skills to make conjectures and to justify conclusions concerning the radius of Earth versus the nautical mile. This question also gives the students the opportunity to select the tools necessary and then connect methods from their past learning to determine the radian measure and the length of the nautical mile needed. Finally, communicating the reasoning and the reflecting that was done is an important component of this question as well.
- You can add interest to **question 17** by playing a video clip of a turbine engine. Go to *www.mcgrawhill.ca/books/functions12* and follow the links to find out how the engine works.
- You can demonstrate the operation of a geostationary satellite, such as the one in **question 19**, using GSP. Draw a circle with the centre at the origin. Draw a point on the circle. Mark the origin as a centre. Dilate the point on the circle using a factor of 2:1. Animate the point on the circle. You can turn tracing on to display the trail left by the satellite. Go to *www.mcgrawhill.ca/books/functions12* and follow the links to this GSP file.
- **Question 19** allows students to reason and reflect concerning angular velocities of two different elements. The students are then asked to communicate these thoughts and make connections among mathematical concepts learned in their past concerning one complete revolution around Earth and the calculation of angular velocity.
- Before assigning **question 24**, consider a visual demonstration using a globe.
- For **question 26**, you can demonstrate a polar grid using GSP. From the Graph menu, select **Grid Form**, and then **Polar Grid**. Print a blank grid for student use.
- Use **BLM 4–2 Section 4.1 Practice** for remediation or extra practice.

Investigate Answers (pages 202–204)

METHOD 1

4. The sector angle appears to be about 60°. Using a protractor, a student should be able to determine a more accurate measurement of 57° plus or minus 1°.
5. Using the result from step 4, one radian is about 57°.
6. To go around the circle, slightly more than six arc lengths are required. A good estimate is 6.3, plus or minus 0.1.
7. Using a string, a student should be able to arrive at an answer very close to 6.3 arc lengths.
8. The circumference of a circle is given by $C = 2\pi r$. The circumference has a length of about 6.3 radii. This compares favourably with the measurement in step 7.

METHOD 2

9. To the nearest hundredth, one radian is approximately equal to 57.30°.
10. To go around the circle, slightly more than six arc lengths are required. A good estimate is 6.3, plus or minus 0.1.
11. To the nearest hundredth, you need 6.28 radii to go around the circle.
12. The circumference of a circle is given by $C = 2\pi r$. The circumference has a length of about 6.28 radii. This compares favourably with the measurement in step 11.

Differentiated Instruction

Use a **journal entry**. Give the topic as "What is a radian?"

- Students often find radian measure confusing, especially in the early stages.

R_x Coach students to think in terms of radians, without converting to degrees first. For example, ask students to demonstrate an angle such as $\frac{\pi}{3}$ using a thumb and index finger. When converting from degree measure to radian measure, or vice versa, train students to estimate the expected answer first.

Ongoing Assessment

Achievement Check, question 21, on student text page 210.

Communicate Your Understanding Responses (page 208)

C1. Since a complete revolution measures 2π radians, π radians are half of a revolution, or 180°.

C2. A right angle is $\frac{1}{4}$ of a complete revolution. So, the radian measure of a right angle is $\frac{2\pi}{4}$, or $\frac{\pi}{2}$.

C3. Radian measure does not depend on the division of a complete revolution into an arbitrary number of units, such as 360. It makes use of a natural quantity, the arc length, compared to the radius of the circle.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	
Reasoning and Proving	13, 19, 22–25
Reflecting	13
Selecting Tools and Computational Strategies	1–9, 13, 20
Connecting	10–23, 25
Representing	12, 14
Communicating	13, 19, 24

Achievement Check, question 21, student text page 210

This performance task is designed to assess the specific expectations covered in Section 4.1. The following Math Process Expectations can be assessed.

- Reflecting
- Representing
- Communicating
- Selecting Tools
- Connecting
- Problem Solving

Achievement Chart Category	Related Math Processes
Knowledge and Understanding	Selecting tools and computational strategies
Thinking	Problem solving Reasoning and proving Reflecting
Communication	Communicating, Representing
Application	Selecting tools and computational strategies Connecting

Sample Solution

Provide students with **BLM 4–3 Section 4.1 Achievement Check Rubric** to help them understand what is expected.

a) $\frac{5}{30}(2\pi) = \frac{\pi}{3}$

A passenger will travel $\frac{\pi}{3}$ rad in 5 min.

b) $a = r\theta$

$= 67.5\left(\frac{\pi}{3}\right)$

$\doteq 70.7$

A passenger travels about 70.7 m in 5 min.

c) It will take $30\left(\frac{2}{2\pi}\right)$, or about 9.5 min to travel 2 rad.

d) The angular velocity of a passenger is $\frac{2\pi}{30 \times 60}$, or $\frac{\pi}{900}$ rad/s.

e) The angular velocity of a passenger is 0.2°/s.