4.3 Equivalent Trigonometric Expressions

Student Text Pages
220 to 227

Suggested Timing
60–75 min

Tools
- grid paper
- graphing calculator
- compasses and protractor

OR
- computer with *The Geometer's Sketchpad®*

Related Resources
- G–1 Grid Paper
- G–2 Placemat
- G–4 Protractor
- T–2 *The Geometer's Sketchpad®* 4
- BLM 4–6 Summary of Trigonometric Identities
- BLM 4–7 Section 4.3 Practice

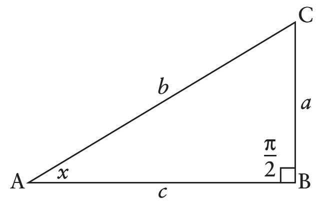
Teaching Suggestions

- Students will begin to assemble a table of trigonometric identities in **Investigate 1**. **Investigate 2** extends this table. Encourage students to continue to add to this table as the lesson progresses. Consider allowing students to use their own tables for tests and exams.
- If *The Geometer's Sketchpad®* (GSP) is used with **Investigate 2**, refer students needing assistance to the Technology Appendix on page 506, the detailed instructions found under the **Help** menu in GSP, or use **T–2 *The Geometer's Sketchpad®* 4**.
- Technology tips for **Investigate 2**:
  - When using GSP to determine equivalent trigonometric expressions, from the **Edit** menu choose **Preferences** and change the **Angle Units** from degrees to radians and vice versa.
  - To create a vertical line from point P to point Q on the x-axis, select point P and the x-axis, then from the **Construct** menu choose **Perpendicular Line**. Then select the perpendicular line and the x-axis and under **Construct** choose **Point of Intersection**. Finally, select points P and Q, then from the **Construct** menu, choose a **Line Segment**. Do the same for point Q and the origin.
  - To find the coordinates of points, select the point and from the **Measure** menu choose **Coordinates**.
- After working through **Example 1** and **Example 2**, take some time to reflect on how to choose a trigonometric identity involving $\frac{\pi}{2} - a$ or $\frac{\pi}{2} + a$ in a particular situation.
- In **Example 3**, students are often tempted to conclude that this method of verification constitutes a proof. Emphasize why it does not.
- Another approach to **Communicate Your Understanding** question C1 is to consider the CAST rule.
- **Questions 15** through **19** develop the remaining equivalent trigonometric expressions for students' summary sheets. Be sure to assign all of these questions, and ensure that the summary sheets contain the correct form for each pair of expressions. A summary table of the trigonometric identities from **Investigate 1**, **Investigate 2**, and **questions 15** through **19** is available as **BLM 4–6 Summary of Trigonometric Identities**.
- **Question 21** is a good example of how more than one identity can be used to obtain an answer.
- **Question 21** allows students to reflect and search for related content knowledge to help Charmaine find the sine of the required angle without using the SIN key on her calculator. They will also have to use their reasoning skills to solve this problem. The students will be required to select the necessary computational strategies and then make connections with past mathematical skills that have been learned.
- Although **question 22** refers to an aircraft, the same equation applies to leaning a bicycle or a motorcycle when making a turn on a road that is not banked.
- **Question 24** asks students to use their critical-thinking skills to evaluate the results of a question that is new to them. To solve this problem, they will have to use their reflecting and reasoning skills. They will also be required to select the appropriate tools necessary for solving this problem and then make connections with concepts that they have learned in their past.
- Use **BLM 4–7 Section 4.3 Practice** for remediation or extra practice.

Investigate Answers (pages 220–222)

Investigate 1

1.



2. $\angle C = \frac{\pi}{2} - x$

3. $\sin x = \frac{a}{b}$

4. $\sin C = \frac{c}{b}$, $\cos C = \frac{a}{b}$, $\tan C = \frac{c}{a}$; $\sin x = \cos\left(\frac{\pi}{2} - x\right)$

5. a) No, the relationship is true for all values of x. The graphs of $\sin x$ and $\cos\left(\frac{\pi}{2} - x\right)$ look the same.

b) $\frac{\pi}{2}$; complementary angles

6. $\cos x = \sin\left(\frac{\pi}{2} - x\right)$, $\tan x = \cot\left(\frac{\pi}{2} - x\right)$

7. $\csc x = \sec\left(\frac{\pi}{2} - x\right)$, $\sec x = \csc\left(\frac{\pi}{2} - x\right)$, $\cot x = \tan\left(\frac{\pi}{2} - x\right)$

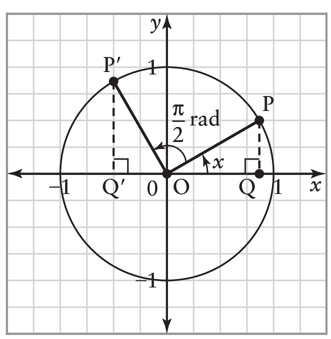
8.

Trigonometric Identities
$\sin x = \cos\left(\frac{\pi}{2} - x\right)$
$\cos x = \sin\left(\frac{\pi}{2} - x\right)$
$\tan x = \cot\left(\frac{\pi}{2} - x\right)$
$\csc x = \sec\left(\frac{\pi}{2} - x\right)$
$\sec x = \csc\left(\frac{\pi}{2} - x\right)$
$\cot x = \tan\left(\frac{\pi}{2} - x\right)$

9. They are cofunctions because their angle relationships are complementary.

Investigate 2

1.–4.



$P\left(\cos\frac{\pi}{2}, \sin\frac{\pi}{2}\right)$ and $P'\left(\cos\left(x + \frac{\pi}{2}\right), \sin\left(x + \frac{\pi}{2}\right)\right)$

5. PO = OP′
$\angle$POQ = $\angle$OP′Q′
$\angle$Q = $\angle$Q′
Therefore, $\triangle$PQO ≅ $\triangle$OQ′P′ (AAS).

Differentiated Instruction

Use a **matching game** to reinforce this section.

Use **placemat** to teach this section.

Common Errors

- Students often confuse an identity with an equation.

R_x Establish the meaning of an identity early in the lesson, preferably in the **Investigate**. Revisit the concept as appropriate throughout this chapter.

- Question C2 illustrates a common error, $\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2} - \cos x$, made by students.

R_x Emphasize that this is not an identity. Revisit the concept from time to time throughout this chapter.

Ongoing Assessment

Achievement Check, question 23, on student text page 226.

6. Due to a rotation of $\frac{\pi}{2}$, (x, y) becomes $(-y, x)$.

$P'\left(\cos\left(x + \frac{\pi}{2}\right), \sin\left(x + \frac{\pi}{2}\right)\right) = P(-\sin x, \cos x)$

7. $\sin\left(x + \frac{\pi}{2}\right) = \cos x$

8. $\tan\left(x + \frac{\pi}{2}\right) = -\cot x$

9. $\csc\left(\frac{\pi}{2} + x\right) = \sec x$, $\sec\left(\frac{\pi}{2} + x\right) = -\csc x$, $\cot\left(\frac{\pi}{2} + x\right) = -\tan x$

10.

Trigonometric Identities Featuring $\frac{\pi}{2}$			
Cofunction Identities			
$\sin x = \cos\left(\frac{\pi}{2} - x\right)$	$\cos x = \sin\left(\frac{\pi}{2} - x\right)$	$\sin\left(x + \frac{\pi}{2}\right) = \cos x$	$\cos\left(x + \frac{\pi}{2}\right) = -\sin x$
$\tan x = \cot\left(\frac{\pi}{2} - x\right)$	$\cot x = \tan\left(\frac{\pi}{2} - x\right)$	$\tan\left(x + \frac{\pi}{2}\right) = -\cot x$	$\cot\left(x + \frac{\pi}{2}\right) = -\tan x$
$\csc x = \sec\left(\frac{\pi}{2} - x\right)$	$\sec x = \csc\left(\frac{\pi}{2} - x\right)$	$\csc\left(x + \frac{\pi}{2}\right) = \sec x$	$\sec\left(x + \frac{\pi}{2}\right) = -\csc x$

11. $\cos(\pi - x) = -\cos x$

Communicate Your Understanding Responses (page 225)

C1. No. The graphs are reflections of each other in the x-axis. $-\cos(\pi - x) = \cos x$

C2. No. Since $\cos\left(\frac{\pi}{2}\right) = 0$, then $\cos\left(\frac{\pi}{2} - x\right) \neq \cos x$.

C3. No. The graphs are reflections of each other in the x-axis. $\cos\left(\frac{\pi}{2} - x\right) = \cos\left(x - \frac{\pi}{2}\right)$

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	21, 24–26
Reasoning and Proving	21–27
Reflecting	21, 22, 24–26
Selecting Tools and Computational Strategies	1–4, 20–22
Connecting	5–27
Representing	17–19
Communicating	

Achievement Check, question 23, student text page 226

This performance task is designed to assess the specific expectations covered in Section 4.3. The following Math Process Expectations can be assessed.

- Reflecting
- Representing
- Communicating
- Selecting Tools
- Connecting

Sample Solution

a) $\sec x = \csc\left(\frac{\pi}{2} - x\right)$

b) $x + \frac{\pi}{2} - x = \frac{\pi}{2}$

c) $\frac{\pi}{2} = a + 3a$

$\frac{\pi}{2} = 4a$

$a = \frac{\pi}{8}$

d) $\sec\frac{\pi}{8} = \csc\frac{3\pi}{8} \doteq 1.0823$

Level 3 Notes

Look for the following:

- Correct answers, with explanation where necessary
- Left side versus right side reasoning is used to check the solution in part d)
- Consistent use of proper mathematical form

What Distinguishes Level 2

- Somewhat correct answers, with some explanation where necessary
- Left side versus right side reasoning is not used to check the solution in part d)
- Some use of proper mathematical form

What Distinguishes Level 4

- Correct answers, with high degree of justification provided
- Left side versus right side reasoning is used to check the solution in part d)
- High degree and consistent use of proper mathematical form