4.4 Compound Angle Formulas

Student Text Pages
228 to 235

Suggested Timing
60–75 min

Tools
- scientific calculator
- graphing calculator

Related Resources
- BLM 4–8 Section 4.4 Practice

Teaching Suggestions

- While working through the **Investigate**, ensure that students make the distinction between a verification for specific values and a proof.
- *The Geometer's Sketchpad*® can also be used to verify that the compound angle formula $\cos(x + y) = \cos x \cos y - \sin x \sin y$ in the **Investigate** is valid for all angles, as follows:
  - Construct a unit circle on a grid. Construct a point A on the unit circle and drag that point in the first quadrant. Construct a line segment from the origin to point A.
  - Construct a point B on the unit circle and drag that point in the fourth quadrant. Construct a line segment from the origin to point B.
  - Construct a line segment from the origin to the unit point. (The unit point can be labelled C.)
  - Measure the angles of ∠BOC and ∠AOC. Let b represent ∠BOC and let a represent ∠AOC.
  - Select line segments OA and OB. Choose **Rotate** from the **Transform** menu to rotate counterclockwise about the origin through angle b (∠BOC). You will have two new line segments. One of them is the terminal arm of angle $(a + b)$ with point A′ being the point where the line segment intersects the unit circle. The other is on the x-axis with point B′ being the point where this line segment intersects the unit circle.
  - **Construct** line segments AB and A′B′.
  - Now investigate the coordinates:
    A$(\cos a, \sin a)$
    A′$(\cos(a + b), \sin(a + b))$
    B′$(1, 0)$
    B$(\cos(2\pi - b), \sin(2\pi - b))$
- Before beginning the development of the Addition Formula for Cosine, you may want to revisit why the coordinates of a point on the unit circle can be written in the form $(\cos\theta, \sin\theta)$.
- When working through **Example 1**, emphasize proper form for this kind of verification question.
- **Example 2** requires the proper choice of special angles whose sum or difference is equal to the desired angle. Much of the time, the proper choice is apparent from inspection. However, sometimes some trial and error is needed.
- It is well worth spending some time on **Communicate Your Understanding** question C1, and demonstrating the concept using a unit circle. You can use *The Geometer's Sketchpad*® for this purpose, or you can make a unit circle with moveable arms using Bristol board and push pins.
- Before assigning **question 18**, consider spending some time with a globe demonstrating the concepts in the question. Many students have a poor understanding of these phenomena.
- **Question 18** encourages students to reflect and then reason out various situations that arise from different angles of latitude. In addition, in order to justify their answers they will have to make connections with concepts that they have learned previously and then communicate in writing to clarify their ideas.
- **Question 20** allows students to reflect upon and then reason out a selection of computational strategies which will help them develop new trigonometric expressions that they have not seen before. In doing so, they will also make connections with various trigonometric expressions that they have used previously.

DIFFERENTIATED INSTRUCTION

Use **timed retell** to reinforce this section.

Use a **word wall/information wall** to summarize this section.

COMMON ERRORS

- The assumption that the relation in question C2, $\sin(x + y) = \sin x + \sin y$, is correct is a common error.

R_x Ensure that students understand why it is not correct, and revisit the concept from time to time in this chapter.

ONGOING ASSESSMENT

Achievement Check, question 19, on student text page 234.

- The tangent identities in **questions 20** and **21**, the sum formula in **question 23**, and the half-angle formulas in **question 24** are not required to satisfy the expectations for this course but are useful to know for students contemplating the study of mathematics at a higher level.
- Use **BLM 4–8 Section 4.4 Practice** for remediation or extra practice.

Investigate Answers (page 228)

1. $\cos\left(\frac{\pi}{6} + \frac{\pi}{3}\right) = \cos\frac{\pi}{6}\cos\frac{\pi}{3} - \sin\frac{\pi}{6}\sin\frac{\pi}{3}$

   $\cos\frac{\pi}{2} = \cos\frac{\pi}{6}\cos\frac{\pi}{3} - \sin\frac{\pi}{6}\sin\frac{\pi}{3}$
2. Yes.
3. b) Yes.
4. Yes.
5. No, three valid examples is not a proof. You must be able to prove algebraically for all angles.
6. Answers may vary. Substituting $x = \frac{\pi}{6}$ and $y = \frac{\pi}{3}$ shows that the left side does not equal the right side.

Communicate Your Understanding Responses (page 232)

C1. Drawing angle $(a + b)$ in the second quadrant would have no effect on the development of the compound angle formula, since the coordinates of the points will not change.

C2. Answers may vary. For example, by substituting $x = \frac{\pi}{6}$ and $y = \frac{\pi}{3}$, the left side equals 1 but the right side equals $\frac{1 + \sqrt{3}}{2}$. Sine is not a multiplication operator.

C3. Yes. Answers may vary. For example, by substituting $x = \frac{\pi}{2}$ and $y = 0$, both the left side and the right side equal 1.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	22
Reasoning and Proving	8–14, 17–24
Reflecting	17–24
Selecting Tools and Computational Strategies	1–7, 15, 16, 20–22
Connecting	2–24
Representing	16, 19
Communicating	18

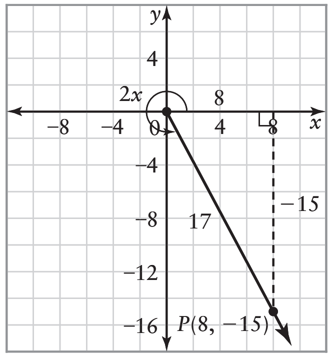
Achievement Check, question 19, student text page 234

This performance task is designed to assess the specific expectations covered in Section 4.4. The following Math Process Expectations can be assessed.

- Reflecting
- Representing
- Communicating
- Selecting Tools
- Connecting
- Reasoning and Proving

Sample Solution

a)



b) Angle x is in the second quadrant.

c) Use the formula $\cos 2x = 2\cos^2 x - 1$.

$$\frac{8}{17} = 2\cos^2 x - 1$$
$$\frac{25}{17} = 2\cos^2 x$$
$$\frac{25}{34} = \cos^2 x$$
$$\cos x = \pm\sqrt{\frac{25}{34}}$$
$$\cos x = \pm\frac{5}{\sqrt{34}}$$

Since angle x is in the second quadrant, $\cos x = -\frac{5}{\sqrt{34}}$.

d) The calculator returns a measure of approximately 0.54 rad. However, angle x is in the second quadrant, so $x = \pi - 0.54$, or 2.60 rad.

e) $\cos 2.60 \doteq -0.857$, which is approximately the same as the result in part c).

Level 3 Notes

Look for the following:

- Clearly labelled sketch provided in part a)
- Most answers are accurate
- Answer given in exact value or radians where required
- Consistent evidence of proper form

What Distinguishes Level 2

- Sketch in part a) is somewhat labelled
- Most answers are accurate
- Answer not given in exact value or radians where required
- Some evidence of proper form

What Distinguishes Level 4

- Clearly labelled and detailed sketch provided in part a)
- Answers may contain minor errors
- Answers given in exact value or radians where required
- High degree of proper form evident
- Answers supported with explanation