# 4.5 Prove Trigonometric Identities

**Student Text Pages**
236 to 241

**Suggested Timing**
60–75 min

**Tools**
- graphing calculator

**Related Resources**
- BLM 4–9 Section 4.5 Practice

## Teaching Suggestions

- The **Investigate** revisits two important concepts. The first is the difference between an equation and an identity. The second is what constitutes a proof, and what does not constitute a proof.
- While working through **Example 1**, review proper form for a proof. Revisit the issue of form as appropriate.
- While working through **Examples 2**, **3**, and **4**, point out that it is not always necessary to begin with the left side, and work towards the right side. **Example 3** can be done either way. If time permits, show how **Example 3** can be done starting from the right side.
- Sometimes both sides of an identity are complex. A useful procedure is to simplify the left side, then the right side, and show that both are equal to the same expression.
- In **Example 4**, reasoning and then proving is involved: these two skills will help students make sense of this mathematical problem. To solve this problem the students will have to choose the appropriate tools and then make connections with trigonometric concepts that they have previously learned.
- While working through the **Communicate Your Understanding** questions, encourage students to use technology to confirm whether an equation might be an identity before attempting to prove it. This is a good approach for many of the exercise questions as well.
- If no particular approach suggests itself for attacking a proof, one possible approach is to use identities to render an expression solely in terms of sines and cosines, working on both the left side and the right side, such as in **question 9a**).
- **Question 17** encourages the students to use deductive reasoning to assess the validity of a given conjecture. It will then be necessary for them to solve the identity problem and, if needed, to use a counterexample to disprove the given conjecture (i.e., this is another way in which their reasoning skills will be put to use). In solving this trigonometric problem, computational strategies will have to be used by the students to represent this identity graphically and, in addition, connections will have to be made with previous work learned by the students.
- Although the double angle formula for tangents in **question 22** is not required to satisfy the expectations for this course, it is useful knowledge for students proceeding to the study of mathematics at a higher level.
- Hint for **question 21**: write $\sin^6 x$ as $(\sin^2 x)^3$. Then, use the Pythagorean identity, and expand using the binomial theorem.
- Use **BLM 4–9 Section 4.5 Practice** for remediation or extra practice.

### Differentiated Instruction

Use **concept attainment** to introduce this section.

Use **what-so what double entry** to teach strategy.

Build a **decision tree** to help students prove identities.

Use a **journal entry** entitled "My Favourite Identity and Why."

### Common Errors

- Some students will rearrange an identity algebraically, similar to solving an equation, in an attempt to simplify it before beginning the proof.

$R_x$ Ensure that this error is discussed while working through the lesson or taking up the homework.

### Ongoing Assessment

Achievement Check, question 19, on student text page 241.

**Investigate Answers (page 236–237)**

**1.** Yes.

**2.** No. One valid example does not prove that the equation holds for all values of $x$ for which the functions are defined.

**3.** Answers may vary. For example, $x = \frac{\pi}{4}$.

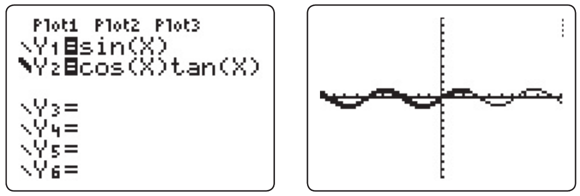
**5. a)** They are different.

**b)** The equation is true at the points where the graphs cross.

**6. a)** Yes.

**b)** No counterexample.

**c)**




**7.** Yes. The graphs appear to be the same.

**Communicate Your Understanding Responses (page 240)**

**C1.** A general equation is true for some values of the variable. An identity is proven for both sides of the equation and is true for all values of the variable for which the functions are defined.

**C2. a)**

$$\sin^2 2\left(\frac{\pi}{4}\right) = 4\sin\frac{\pi}{4}\cos\frac{\pi}{4} - 1$$

$$\sin^2\frac{\pi}{2} = 4\sin\frac{\pi}{4}\cos\frac{\pi}{4} - 1$$

$$1^2 = 4\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) - 1$$

$$1 = 2 - 1$$

$$1 = 1$$

**b)** Test other values. If a counterexample is found, it is not an identity. When $x = \frac{\pi}{3}$ is substituted, the left side of the equation does not equal the right side, so this equation is not an identity.

**C3.** Answers may vary.

## Mathematical Process Expectations

| Process Expectation | Selected Questions |
|---|---|
| Problem Solving | 9–13, 15–17, 19–21 |
| Reasoning and Proving | 9–13, 15–22 |
| Reflecting | |
| Selecting Tools and Computational Strategies | 1–8, 14, 17, 19, 22 |
| Connecting | 1–22 |
| Representing | 9, 17, 19, 22 |
| Communicating | 14, 18 |

**Achievement Check, question 19, student text page 241**

This performance task is designed to assess the specific expectations covered in Section 4.5. The following Math Process Expectations can be assessed.

- Reflecting
- Representing
- Communicating
- Selecting Tools
- Connecting
- Reasoning and Proving

**Sample Solution**

**a)** $3x = 2x + x$

**b)** $\sin 3x = \sin(2x + x)$

**c)** $\sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x$

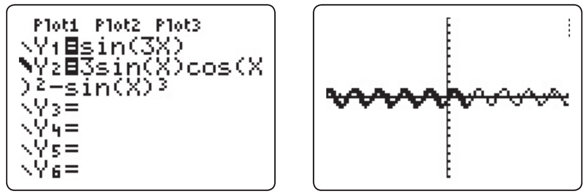
**d)** $2 \sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x = 3 \sin x \cos^2 x - \sin^3 x$

**e)** Let $x = \pi$.

$\sin(2\pi + \pi) = \sin 3\pi = 0$

$3 \sin \pi \cos^2 \pi - \sin^3 \pi = 3(0)(-1)^2 - 0^3 = 0$

**f)**




## Level 3 Notes

Look for the following:

- Evidence of fairly accurate solution
- Correct choice of addition formula to expand part c)
- Correct choice of double angle formula to expand part d)
- Use of formulas in part c) and part d) is fairly accurate
- Mostly simplified answer in part d)
- Appropriate verification of answer in part e) demonstrating algebraic justification
- Fairly accurate illustrations of identity
- Consistent evidence of proper form

## What Distinguishes Level 2

- Evidence of somewhat accurate solution
- Correct choice of addition formula to expand part c)
- Correct choice of double angle formula to expand part d)
- Use of formulas in part c) and part d) is somewhat accurate
- Somewhat simplified answer in part d)
- Some verification of answer in part e)
- Somewhat accurate illustration of identity
- Some evidence of proper form

## What Distinguishes Level 4

- High degree of accuracy in solution
- Correct choice of addition formula to expand part c)
- Correct choice of double angle formula to expand part d)
- Formulas in part c) and part d) are applied accurately
- Highly simplified answer in part d)
- Verification of answer in part e) demonstrating high degree of reasoning and justification
- Accurate illustration of identity
- Answers consistently supported with explanation
- High degree of proper form evident

# Extension

**Student Text Pages**
242 to 243

**Suggested Timing**
30–40 min

**Tools**
- computer
- *The Geometer's Sketchpad®*

**Related Resources**
- T–2 *The Geometer's Sketchpad®* 4

## Use *The Geometer's Sketchpad®* to Sketch and Manipulate Three-Dimensional Structures in a Two-Dimensional Representation

### Teaching Suggestions

- This extension allows students to get a feel for the production of Computer Generated Imagery (CGI) that is prevalent in the entertainment industry, using a familiar software package.
- *The Geometer's Sketchpad®* (GSP) allows students to create fairly sophisticated representations of motion in three dimensions while confined to a two-dimensional workspace.
- Ensure that students understand that the complex calculations involving trigonometric expressions are being done by the software behind the scenes, and that they are instructing the software as to what kinds of motions they want to create.
- This is an open-ended activity with tremendous potential for extension, including the use of colour, morphing, and other enhancements. Some students can extend this activity into a full-fledged project.
- Use **T–2 *The Geometer's Sketchpad®* 4** to support this activity, if needed.
- If you have little experience in using action buttons, you may wish to spend a little time in the GSP Help file to ensure that you understand what these do. Look under Animate, Action Buttons, Presentation, and Movement.