Chapter 5

Trigonometric Functions

Curriculum Expectations

Trigonometric Functions

Connecting Graphs and Equations of Trigonometric Functions

B2.1 sketch the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ for angle measures expressed in radians, and determine and describe some key properties (e.g., period of 2π, amplitude of 1) in terms of radians

B2.2 make connections between the tangent ratio and the tangent function by using technology to graph the relationship between angles in radians and their tangent ratios and defining this relationship as the function $f(x) = \tan x$, and describe key properties of the tangent function

B2.3 graph, with technology and using the primary trigonometric functions, the reciprocal trigonometric functions (i.e., cosecant, secant, cotangent) for angle measures expressed in radians, determine and describe key properties of the reciprocal functions (e.g., state the domain, range, and period, and identify and explain the occurrence of asymptotes), and recognize notations used to represent the reciprocal functions [e.g., the reciprocal of $f(x) = \sin x$ can be represented using $\csc x$, $\frac{1}{f(x)}$, or $\frac{1}{\sin x}$, but not using $f^{-1}(x)$ or $\sin^{-1} x$, which represent the inverse function]

B2.4 determine the amplitude, period, and phase shift of sinusoidal functions whose equations are given in the form $f(x) = a\sin(k(x - d)) + c$ or $f(x) = a\cos(k(x - d)) + c$, with angles expressed in radians

B2.5 sketch graphs of $y = a\sin(k(x - d)) + c$ and $y = a\cos(k(x - d)) + c$ by applying transformations to the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ with angles expressed in radians, and state the period, amplitude, and phase shift of the transformed functions

B2.6 represent a sinusoidal function with an equation, given its graph or its properties, with angles expressed in radians

B2.7 pose problems based on applications involving a trigonometric function with domain expressed in radians (e.g., seasonal changes in temperature, heights of tides, hours of daylight, displacements for oscillating springs), and solve these and other such problems by using a given graph or a graph generated with or without technology from a table of values or from its equation

Solving Trigonometric Equations

B3.4 solve linear and quadratic trigonometric equations, with and without graphing technology, for the domain of real values from 0 to 2π, and solve related problems

Characteristics of Functions

Understanding Rates of Change

D1.5 recognize examples of instantaneous rates of change arising from real-world situations, and make connections between instantaneous rates of change and average rates of change (e.g., an average rate of change can be used to approximate an instantaneous rate of change)

D1.6 determine, through investigation using various representations of relationships (e.g., tables of values, graphs, equations), approximate instantaneous rates of change arising from real-world applications (e.g., in the natural, physical, and social sciences) by using average rates of change and reducing the interval over which the average rate of change is determined

D1.8 determine, through investigation using a variety of tools and strategies (e.g., using a table of values to calculate slopes of secants or graphing secants and measuring their slopes with technology), the approximate slope of the tangent to a given point on the graph of a function (e.g., quadratic, exponential, sinusoidal) by using the slopes of secants through the given point (e.g., investigating the slopes of secants that approach the tangent at that point more and more closely), and make connections to average and instantaneous rates of change

D1.9 solve problems involving average and instantaneous rates of change, including problems arising from real-world applications, by using numerical and graphical methods (e.g., by using graphing technology to graph a tangent and measure its slope)

Using Function Models to Solve Problems

D3.3 solve problems, using a variety of tools and strategies, including problems arising from real-world applications, by reasoning with functions and by applying concepts and procedures involving functions (e.g., by constructing a function model from data, using the model to determine mathematical results, and interpreting and communicating the results within the context of the problem)

Technology Notes

- The expectations for this course mandate heavy use of technology. Strong support for technology use has been included in the lesson design.
- To avoid the necessity for multiple learning curves, the primary technologies in use are
  - graphing calculators, specifically the TI-83 Plus/TI-84 Plus series
  - computer algebra system (CAS), specifically for the TI-89/TI-89T series
  - *The Geometer's Sketchpad®*
- *The Geometer's Sketchpad®* is licensed in Ontario for use by students at home. Consider providing each student with a copy to install on a home computer. This greatly expands the kinds of homework you can assign, and gets around some of the access problems you may run into using school computers. Ensure that students without home computers have an alternative. One possibility is to pair up with a classmate who is willing to share a computer. Another is availability of the software on public use computers in the school, such as library computers.
- A scientific calculator provides strong support for students, and is intended to be a standard tool available to the student at all times.

Chapter 5 Planning Chart

Section Suggested Timing	Student Text Page(s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 5 Opener • 10 min	249			
Prerequisite Skills • 40–50 min	250–251	• G–7 Trigonometric Graph Paper • BLM 5–1 Prerequisite Skills		• grid paper • scientific calculator
5.1 Graphs of Sine, Cosine, and Tangent Functions • 60–75 min	252–260	• G–7 Trigonometric Graph Paper • G–2 Placemat • T–2 *The Geometer's Sketchpad®* 4 • BLM 5–2 Section 5.1 Practice		• scientific calculator • grid paper • graphing calculator • graphing software (optional) • computer • *The Geometer's Sketchpad®*
5.2 Graphs of Reciprocal Trigonometric Functions • 60–75 min	261–269	• G–7 Trigonometric Graph Paper • BLM 5–3 Graph of Sine and Cosecant • BLM 5–4 Graph of Cosine and Secant • BLM 5–5 Graph of Tangent and Cotangent • BLM 5–6 Section 5.2 Practice		• scientific calculator • grid paper • graphing calculator • graphing software (optional)
5.3 Sinusoidal Functions of the Form $f(x) = a\sin[k(x - d)] + c$ and $f(x) = a\cos[k(x - d)] + c$ • 60–75 min	270–279	• G–1 Grid Paper • G–7 Trigonometric Graph Paper • T–2 *The Geometer's Sketchpad®* 4 • BLM 5–7 Section 5.3 Practice		• 2-L plastic bottle • measuring cup or graduated cylinder • flexible tube • sink or basin with water • grid paper or graphing technology • graphing calculator • computer • *The Geometer's Sketchpad®* **Optional** • spirometer • computer with capture and graphing software such as Logger Pro® installed OR • graphing calculator with CBL 2™ Calculator-Based Laboratory
Extension: Use a Graphing Calculator to Fit a Sinusoidal Regression to Given Data • 30–40 min	280–281			• graphing calculator
5.4 Solve Trigonometric Equations • 60–75 min	282–289	• T–4 The Computer Algebra System (CAS) on the TI-89 Calculator • BLM 5–8 Section 5.4 Practice		• Special Angles and Trigonometric Ratios table from Chapter 4 • graphing calculator • computer algebra system
5.5 Making Connections and Instantaneous Rate of Change • 60–75 min	290–299	• G–1 Grid Paper • G–7 Trigonometric Graph Paper • T–2 *The Geometer's Sketchpad®* 4 • BLM 5–9 Section 5.5 Practice	• BLM 5–10 Section 5.5 Achievement Check Rubric	• computer • *The Geometer's Sketchpad®* • graphing calculator **Optional** • graph of sine function and mathematics construction set
Chapter 5 Review • 60–75 min	300–301	• G–7 Trigonometric Graph Paper • BLM 5–11 Chapter 5 Review		• grid paper • graphing calculator
Chapter 5 Problem Wrap-Up • 20–30 min	301	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad®* 4	• BLM 5–12 Chapter 5 Problem Wrap-Up Rubric	• grid paper • graphing calculator • computer • *The Geometer's Sketchpad®*
Chapter 5 Practice Test • 40–50 min	302–303	• G–7 Trigonometric Graph Paper	• BLM 5–13 Chapter 5 Test	• grid paper • graphing calculator
Chapters 4 and 5 Review • 60–75 min	304–305	• G–7 Trigonometric Graph Paper		• grid paper • graphing calculator
Chapter 5 Task: Predators and Prey • 60–75 min	306	• G–1 Grid Paper • BLM 5–15 BLM Answers	• BLM 5–14 Task: Predators and Prey Rubric	• grid paper • graphing calculator

Chapter 5 Blackline Masters Checklist

	BLM	Title	Purpose
Prerequisite Skills			
	G–7	Trigonometric Graph Paper	Student Support
	BLM 5–1	Prerequisite Skills	Practice
5.1 Graphs of Sine, Cosine, and Tangent Functions			
	G–7	Trigonometric Graph Paper	Student Support
	G–2	Placemat	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	BLM 5–2	Section 5.1 Practice	Practice
5.2 Graphs of Reciprocal Trigonometric Functions			
	G–7	Trigonometric Graph Paper	Student Support
	BLM 5–3	Graph of Sine and Cosecant	Student Support
	BLM 5–4	Graph of Cosine and Secant	Student Support
	BLM 5–5	Graph of Tangent and Cotangent	Student Support
	BLM 5–6	Section 5.2 Practice	Practice
5.3 Sinusoidal Functions of the Form $f(x) = a\sin[k(x - d)] + c$ and $f(x) = a\cos[k(x - d)] + c$			
	G–1	Grid Paper	Student Support
	G–7	Trigonometric Graph Paper	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	BLM 5–7	Section 5.3 Practice	Practice
Extension: Use a Graphing Calculator to Fit a Sinusoidal Regression to Given Data			
5.4 Solve Trigonometric Equations			
	T–4	The Computer Algebra System (CAS) on the TI-89 Calculator	Technology
	BLM 5–8	Section 5.4 Practice	Practice
5.5 Making Connections and Instantaneous Rate of Change			
	G–1	Grid Paper	Student Support
	G–7	Trigonometric Graph Paper	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	BLM 5–9	Section 5.5 Practice	Practice
	BLM 5–10	Section 5.5 Achievement Check Rubric	Assessment
Chapter 5 Review			
	G–7	Trigonometric Graph Paper	Student Support
	BLM 5–11	Chapter 5 Review	Practice
Chapter 5 Problem Wrap-Up			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	BLM 5–12	Chapter 5 Problem Wrap-Up Rubric	Assessment
Chapter 5 Practice Test			
	G–7	Trigonometric Graph Paper	Student Support
	BLM 5–13	Chapter 5 Test	Summative Assessment
Chapters 4 and 5 Review			
	G–7	Trigonometric Graph Paper	Student Support
Chapter 5 Task: Predators and Prey			
	G–1	Grid Paper	Student Support
	BLM 5–14	Task: Predators and Prey Rubric	Assessment
	BLM 5–15	BLM Answers	Answers

Prerequisite Skills

Student Text Pages
250 to 251

Suggested Timing
40–50 min

Tools
- grid paper
- scientific calculator

Related Resources
- G–7 Trigonometric Graph Paper
- BLM 5–1 Prerequisite Skills

Assessment
You may wish to use **BLM 5–1 Prerequisite Skills** as a diagnostic assessment. Refer students to the Skills Appendix for examples and further practice of topics.

Chapter Problem

- The Chapter Problem is introduced on page 251. Have students discuss their understanding of the topic. To stimulate interest, show demonstration video clips of theme park rides. Go to *www.mcgrawhill.ca/books/functions12* and follow the links to find such items. The Chapter Problem is revisited in Sections 5.1 (question 17), 5.2 (question 11), 5.3 (question 18), 5.4 (question 22), and 5.5 (question 11). These questions are designed to help students move toward the Chapter 5 Problem Wrap-Up on page 301. Alternatively, you may wish to assign the Chapter Problem questions and Chapter Problem Wrap-Up when students have completed the chapter, as part of a summative assessment.