5.1 Graphs of Sine, Cosine, and Tangent Functions

Student Text Pages
252 to 260

Suggested Timing
60–75 min

Tools
- scientific calculator
- grid paper
- graphing calculator
- graphing software (optional)
- computer
- *The Geometer's Sketchpad®*

Related Resources
- G–7 Trigonometric Graph Paper
- G–2 Placemat
- T–2 *The Geometer's Sketchpad®* 4
- BLM 5–2 Section 5.1 Practice

Ongoing Assessment

Use **Assessment Masters A–1** to **A–7** to remind students about the Math Processes expectations and how you may be assessing their integrated use of them.

Teaching Suggestions

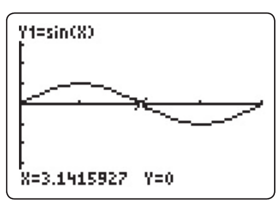
- Although the **Investigate** is a repeat of a grade 11 topic, using radian measure instead of degree measure, it is worth working through both as a review and to consolidate the concept of radian measure. Use **T–2 *The Geometer's Sketchpad*® 4** to support this activity, if needed.
- If using a graphing calculator for the **Investigate**, the following technology tips may be useful:
  - To distinguish between the different trigonometric functions, change the line style by using the left cursor key to move in front of **Y1=** and press ENTER.
  - Students can view these functions in a friendlier window by pressing ZOOM and selecting **7:ZTrig**.
- While working through **Example 1**, Method 2, take a moment to familiarize students with the **ZOOM** option **7:ZTrig**, and especially with the selection of window settings that this option generates. In particular, ensure that students can explain why Xscl = 1.5707963….
- If a computer lab is available, consider using *The Geometer's Sketchpad*® as a possible Method 3 for **Example 1**.
- **Example 1** through **Example 4** revisit concepts from grade 11, but using radian measure. Take the time to consider features such as maximum and minimum values, and the values of x for which they occur.
- After discussing **Communicate Your Understanding** question C1, students can demonstrate the equivalence of the sine function and the phase-shifted cosine function using a graphing calculator to plot each one sequentially, similar to the method used in Chapter 4, Section 5, to demonstrate identities.
- Incorporate the quotient identity $\tan x = \frac{\sin x}{\cos x}$ as part of the discussion for question C2, as preparation for **question 15**.
- Before assigning **question 12**, consider presenting a simple example of the relationship between frequency and period, using a pendulum. Measure the time required for a set number of swings—say, five—and then calculate both the period and the frequency. Show that they are reciprocal values.
- **Question 15** allows students to reflect upon the connection between the graphs of $y = \sin x$ and $y = \cos x$ and the quotient identity, and then apply reasoning skills to determine the shape of the graph of $y = \tan x$. They then will be required to communicate the reflecting and reasoning that was done.
- If students have access to *The Geometer's Sketchpad*® at home, you can assign **question 16** as homework. The question leaves much room for extensions and student exploration.
- You can add interest to **question 17** by showing a video clip of an Octopus ride. Clips of this ride can be found on the Internet.
- For **question 18**, students can learn more about how a wave pool works. Go to *www.mcgrawhill.ca/books/functions12* and follow the links.
- Although transformations of the tangent function are not required to satisfy the expectations of this course, they are useful for students proceeding to more advanced courses in mathematics. Assign **questions 21** and **22** to these students.
- Consider assigning **question 23** as preparation for the next lesson (i.e., illustrating the difference between $\sin^{-1} x$ and $\csc x$).
- Use **BLM 5–2 Section 5.1 Practice** for remediation or extra practice.

Investigate Answers (pages 252–254)

1. a), b)

Angle, x	$y = \sin x$	$y = \cos x$	$y = \tan x$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2} = 0.50$	$\frac{\sqrt{3}}{2} \doteq 0.87$	$\frac{1}{\sqrt{3}} \doteq 0.58$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}} \doteq 0.71$	$\frac{1}{\sqrt{2}} \doteq 0.71$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2} \doteq 0.87$	$\frac{1}{2} = 0.50$	$\sqrt{3} \doteq 1.73$
$\frac{\pi}{2}$	1	0	undefined
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2} \doteq 0.87$	$-\frac{1}{2} = -0.50$	$-\sqrt{3} \doteq -1.73$
$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}} \doteq 0.71$	$-\frac{1}{\sqrt{2}} \doteq -0.71$	−1
$\frac{5\pi}{6}$	$\frac{1}{2} = 0.50$	$-\frac{\sqrt{3}}{2} \doteq -0.87$	$-\frac{1}{\sqrt{3}} \doteq -0.58$
π	0	−1	0
$\frac{7\pi}{6}$	$-\frac{1}{2} = -0.50$	$-\frac{\sqrt{3}}{2} \doteq -0.87$	$\frac{1}{\sqrt{3}} \doteq 0.58$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}} \doteq -0.71$	$-\frac{1}{\sqrt{2}} \doteq -0.71$	1
$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2} \doteq -0.87$	$-\frac{1}{2} = -0.50$	$\sqrt{3} \doteq 1.73$
$\frac{3\pi}{2}$	−1	0	undefined
$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2} \doteq -0.87$	$\frac{1}{2} = 0.50$	$-\sqrt{3} \doteq -1.73$
$\frac{7\pi}{4}$	$-\frac{1}{\sqrt{2}} \doteq -0.71$	$\frac{1}{\sqrt{2}} \doteq 0.71$	−1
$\frac{11\pi}{6}$	$-\frac{1}{2} = -0.50$	$\frac{\sqrt{3}}{2} \doteq 0.87$	$-\frac{1}{\sqrt{3}} \doteq -0.58$
2π	0	1	0

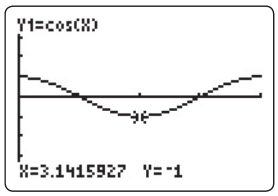
2. a) Window variables: $x \in [0, 2\pi]$, Xscl $= \frac{\pi}{2}$, $y \in [-4, 4]$



2. b), 3. b), 7. b)

Function, $x \in [0, 2\pi]$	$y = \sin x$	$y = \cos x$	$y = \tan x$
Maximum value	1	1	none
Minimum value	−1	−1	none
Amplitude	1	1	none
Period (radians)	2π	2π	π
Zeros in $x \in [0, 2\pi]$	$0, \pi, 2\pi$	$\frac{\pi}{2}, \frac{3\pi}{2}$	$0, \pi, 2\pi$
y-intercept	0	1	0
Domain	$[0, 2\pi]$	$[0, 2\pi]$	$\left[\left(0, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{3\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)\right]$
Range	$[-1, 1]$	$[-1, 1]$	$(-\infty, \infty)$

3. a) Window variables: $x \in [0, 2\pi]$, Xscl $= \frac{\pi}{2}$, $y \in [-4, 4]$



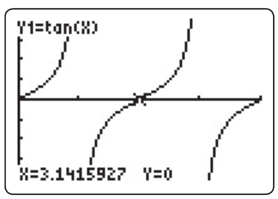
4. a) 4π **b)** -2π

5. a) 4π **b)** -2π

6. a) Some values are defined because of division by 0.

b) As $x \to \frac{\pi}{2}^-$ the graph $\to \infty$. As $x \to \frac{\pi}{2}^+$ the graph $\to -\infty$.

7. a) Window variables: $x \in [0, 2\pi]$, Xscl $= \frac{\pi}{2}$, $y \in [-4, 4]$



The vertical dotted lines are asymptotes.

8. c) The graph passes the vertical line test.

Communicate Your Understanding Responses (page 257)

C1. $\sin\left(x + \frac{\pi}{2}\right) = \cos x$, $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

There is more than one way to map the cosine and the sine function onto each other. For example, add one period, 2π, to the phase shift to get a phase shift of $\frac{5\pi}{2}$. More than one period can be added or subtracted.

C2. The tangent function has asymptotes at the same x-values where zeros occur on the cosine function.

DIFFERENTIATED INSTRUCTION

Use **placemat** to review trigonometry.

COMMON ERRORS

- A simple but common error is having the mode of a calculator set to degrees when the desired answer is to be in radians.

R_x Have students check the mode each time, before using a calculator to perform calculations.

ONGOING ASSESSMENT

Achievement Check, question 19, on student text page 260.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	
Reasoning and Proving	12–16, 18–23
Reflecting	15, 18, 23
Selecting Tools and Computational Strategies	4, 6, 8–11, 13, 18, 20, 21, 23
Connecting	3, 5, 7, 9–15, 17–23
Representing	2, 4, 6, 8–11, 13, 18, 20, 21, 23
Communicating	14–16, 18–23

Achievement Check, question 19, student text page 260

This performance task is designed to assess the specific expectations covered in Section 5.1. The following Math Process Expectations can be assessed.

- Connecting
- Representing
- Communicating

Sample Solution

a) The maximum value is 5 and the minimum value is 1.

b) The amplitude is 2 (half of the range 5 to 1).

c) The graph has been moved vertically up 3 units.

d) The period is $\frac{2\pi}{3}$.

e) $k = 3$

f) Since the graph is periodic, there are an infinite number of phase shifts that will work. Three of them are $-\frac{\pi}{3}$, $\frac{\pi}{3}$, and π or in general $\frac{(2k + 1)\pi}{3}$, $k \in \mathbb{Z}$.

Level 3 Notes

Look for the following:

- Correct answers with explanations where necessary
- Recognition of the nature of a periodic function so there are an infinite number of possible phase shifts

What Distinguishes Level 2

- Mostly correct answers with some explanations where necessary
- An insufficient explanation of there being an infinite number of possible phase shifts

What Distinguishes Level 4

- Correct answers with thorough explanations where necessary
- Recognition of the nature of a periodic function so there are an infinite number of possible phase shifts
- A general solution for the phase shift