5.2 Graphs of Reciprocal Trigonometric Functions

Student Text Pages
261 to 269

Suggested Timing
60–75 min

Tools
- scientific calculator
- grid paper
- graphing calculator
- graphing software (optional)

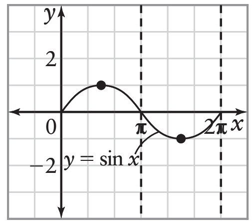
Related Resources
- G–7 Trigonometric Graph Paper
- BLM 5–3 Graph of Sine and Cosecant
- BLM 5–4 Graph of Cosine and Secant
- BLM 5–5 Graph of Tangent and Cotangent
- BLM 5–6 Section 5.2 Practice

Teaching Suggestions

- After working through the **Investigate**, you can demonstrate the process dynamically using *The Geometer's Sketchpad*® (GSP). Go to *www.mcgrawhill.ca/books/functions12* and follow the links to the GSP sketches for sine, cosine, and tangent functions.
- For **Example 1**, students can also enter $\frac{1}{\mathbf{Y1}}$ in **Y2** since $\sin x$ is in **Y1**.
- In the absence of technology, **Examples 1** and **2** can be done using grid paper. Use **BLM 5–3 Graph of Sine and Cosecant, BLM 5–4 Graph of Cosine and Secant**, and **BLM 5–5 Graph of Tangent and Cotangent** to support this activities.
- After working through **Example 3**, ask students to predict how the model would change if the assumptions in part e) had not been made and the real atmosphere and surface of the Earth were used.
- **Example 3** requires representation to take place with the drawing of a graph. Once the graph is completed the students are asked to communicate their understanding of different components of the graph, the reason why they made these statements, and the assumptions that they took into account. Finally, the students must make connections with different elements of their previous knowledge in order to determine the expression required in part a) and the graph required in part b).
- Use **Example 4** to consolidate students' knowledge of the difference between $\sin^{-1}x$ and $\csc x$. Use a graph or graphing technology to demonstrate the multiple answers when using $\sin^{-1}x$.
- After discussion of the **Communicate Your Understanding** questions, use technology to obtain the points of intersection.
- After completing **question 7**, use graphing technology to demonstrate that the correct transformation has been applied.
- You can model the situation in **question 9** using *The Geometer's Sketchpad*®. Then, you can determine whether there is a point P such that the time taken to reach the swimmer is a minimum. Go to *www.mcgrawhill.ca/books/functions12* and follow the links to this GSP sketch.
- You can add interest to **question 11** by showing a video clip of a swing ride. Examples can be found on the Internet.
- **Question 13** allows students to select tools and to use reasoning skills to find a method of determining the relation required. Once the relation is found, the students have to make connections with concepts learned previously to determine exact and approximate values for the distance asked for in part b). Then, a representation is to be made by graphing an interval of the relation found in part a).
- For **question 19**, you can use *The Geometer's Sketchpad*® to sketch graphs of equations in polar form. From the **Graph** menu, select **Grid Form** then **Polar Grid**. Use the calculator box to enter relations in polar form. Try some more complicated plots, such as $r = 2\cos(4\theta)$, or combinations such as $r = 2\sin(4\theta) + 3\cos(3\theta)$.
- Use **BLM 5–6 Section 5.2 Practice** for remediation or extra practice.

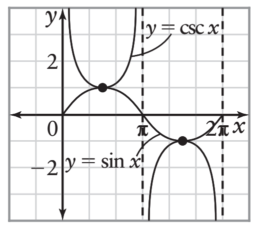
Investigate Answers (pages 261–263)

1. a), b)



c) $x = 0, x = \pi, x = 2\pi$

2. a) decreases, increases **b)** decreases, increases **c)** increases, decreases **d)** increases, decreases



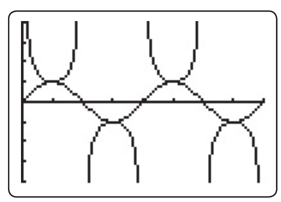
3. a) Domain: $\{x \in \mathbb{R}, x \neq n\pi, n \text{ is an integer}\}$
Range: $\{y \in \mathbb{R}, y \leq -1, y \geq 1\}$
Period: 2π

b) $x = n\pi$, n is an integer. The asymptotes occur where $\sin x = 0$.

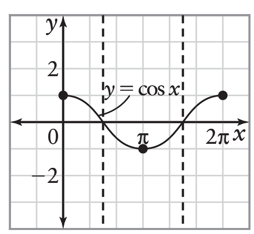
c) The slopes have opposite signs. This is not true for $x = \frac{n\pi}{2}$, where n is an odd integer.

d) decrease, increase, opposite, always true

4. $x = 2\pi, x = 3\pi, x = 4\pi$
Window variables: $x \in [0, 4\pi]$, Xscl $= \frac{\pi}{2}$, $y \in [-4, 4]$

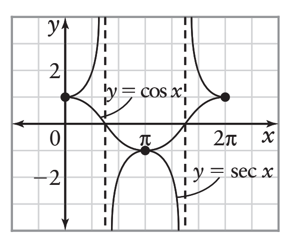


5. Step 1: a), b)



c) $x = \frac{\pi}{2}, x = \frac{3\pi}{2}$

Step 2: a) increases, decreases
b) decreases, increases
c) decreases, increases
d) increases, decreases



Step 3: a) Domain: $\{x \in \mathbb{R}, x \neq \frac{n\pi}{2}, n \text{ is an odd integer}\}$
Range: $\{y \in \mathbb{R}, y \leq -1, y \geq 1\}$
Period: 2π

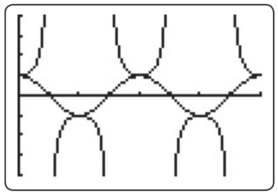
b) $x = \frac{n\pi}{2}$, n is an odd integer. The asymptotes occur where $\cos x = 0$.

c) The slopes have opposite signs. This is not true for $x = n\pi$, n is an integer.

d) decrease, increase, opposite, always true

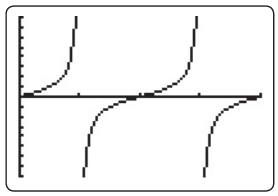
Step 4: $x = \frac{5\pi}{2}$, $x = \frac{7\pi}{2}$

Window variables: $x \in [0, 4\pi]$, Xscl $= \frac{\pi}{2}$, $y \in [-4, 4]$



6. They have the same period, shape, and range. One is the phase shift of the other.

7. a) Window variables: $x \in [0, 4\pi]$, Xscl $= \frac{\pi}{2}$, $y \in [-8, 8]$



b) $x = 0$, $x = \pi$, $x = 2\pi$

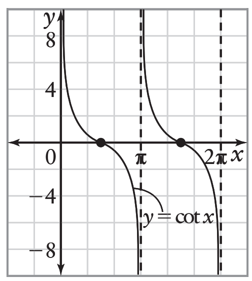
c) $\frac{\pi}{2}$, $\frac{3\pi}{2}$

d) increases, decreases

e) increases, decreases

f) increases, decreases

g) increases, decreases



8. a) Domain: $\{x \in \mathbb{R}, x \neq n\pi, n \text{ is an integer}\}$

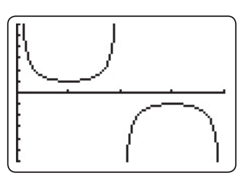
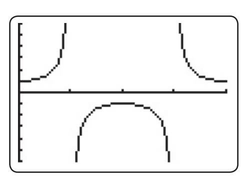
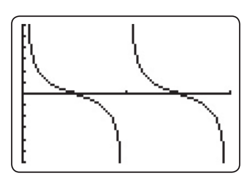
Range: $\{y \in \mathbb{R}\}$

Period: π

b) $x = n\pi$, where n is an integer. The asymptotes occur where $\tan x = 0$.

9. They have the same period and range. One appears to be a reflection and phase shift of the other.

10.

Property	$y = \csc x$	$y = \sec x$	$y = \cot x$
Domain	$\{x \in \mathbb{R}, x \neq n\pi,$ n is an integer$\}$	$\{x \in \mathbb{R}, x \neq \frac{n\pi}{2},$ n is an odd integer$\}$	$\{x \in \mathbb{R}, x \neq n\pi,$ n is an integer$\}$
Range	$\{y \in \mathbb{R}, y \leq -1, y \geq 1\}$	$\{y \in \mathbb{R}, y \leq -1, y \geq 1\}$	$\{y \in \mathbb{R}\}$
Period	2π	2π	π
Equations of asymptotes on the interval $x \in [0, 2\pi]$	$x = 0, x = \pi, x = 2\pi$ In general: $x = n\pi$, n is an integer	$x = \pi/2, x = \frac{3\pi}{2}$ In general: $x = \frac{n\pi}{2}$, n is an odd integer	$x = 0, x = \pi, x = 2\pi$ In general: $x = n\pi$, n is an integer
Sketch of graph Window variables: $x \in [0, 2\pi]$, Xscl $\frac{\pi}{2}$, $y \in [-6, 6]$			

Differentiated Instruction

Use **concept attainment jigsaw** or **group graph** and **gallery walk** to teach reciprocal trigonometric functions.

Ongoing Assessment

Achievement Check, question 14, on student text page 269.

Communicate Your Understanding Responses (page 267)

C1. Answers may vary. $0, \pi, 2\pi$

C2. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

C3. $x = 0, x = \frac{\pi}{2}, x = \pi, x = \frac{3\pi}{2}, x = 2\pi$

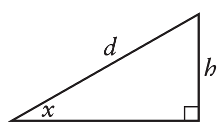
The asymptotes of $y = \csc 2x$ correspond to the zeros of $y = \sin 2x$.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	18
Reasoning and Proving	8, 10–18
Reflecting	10, 13
Selecting Tools and Computational Strategies	1–3, 8–12, 14
Connecting	4–18
Representing	1–3, 8–12, 14–16
Communicating	4–8, 10, 11, 13–17

Achievement Check, question 14, student text page 269

a)



b) $\sin x = \frac{h}{d}$

$d = \frac{h}{\sin x}$

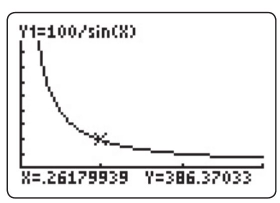
$d = h \csc x$

c) $d = 100 \csc 0.3$

$\doteq 338.4$

The road is 338.4 m in length.

d)



e) As the angle approaches zero, the triangle still has a height of 100 m so the beginning of the hill must be back an infinite distance and the road's length becomes infinitely large.

Level 3 Notes

Look for the following:

- Reasonable, fully labelled sketches
- A proof of the relationship between the length of the road and the angle of inclination
- Correct calculation of the situation given
- Interpretation of the vertical asymptote at $x = 0$

What Distinguishes Level 2

- Reasonable sketches not fully labelled
- An inadequate proof of the relationship between the length of the road and the angle of inclination
- Calculation of the situation given with only minor errors
- An attempt at an interpretation of the vertical asymptote at $x = 0$

What Distinguishes Level 4

- Reasonable, fully labelled sketches
- A complete proof of the relationship between the length of the road and the angle of inclination
- Correct calculation of the situation given
- A thorough interpretation of the vertical asymptote at $x = 0$