5.4 Solve Trigonometric Equations

Student Text Pages
282 to 289

Suggested Timing
60–75 min

Tools
- Special Angles and Trigonometric Ratios table from Chapter 4
- graphing calculator
- computer algebra system

Related Resources
- T–4 The Computer Algebra System (CAS) on the TI-89 Calculator
- BLM 5–8 Section 5.4 Practice

Teaching Suggestions

- This is a good opportunity to remind students of the difference between an identity and an equation.
- While taking up the **Investigate**, you can exploit the graphing method to demonstrate visually why trigonometric equations often have an infinite number of solutions, and consequently, why the domain is usually restricted.
- You can demonstrate the solution to **Example 1** using *The Geometer's Sketchpad*®. Draw a unit circle and a point on the circle. Measure the coordinates of the point, recalling that the coordinates of a point on the unit circle can be written as $(\cos\theta, \sin\theta)$. Drag the point around the circle, looking for values of the ordinate that match the solution to the equation. If needed, use **T–4 The Computer Algebra System (CAS) on the TI-89 Calculator** to support **Example 1**, Method 3.
- You can use the unit circle diagram from **Example 1** to demonstrate the solutions to **Example 2** and **Example 3**.
- For **Example 2**, a CAS can be used to factor the equation $\cos^2 x - \cos x - 2 = 0$. The **solve(** operation can then be used with the factors.
- For **Example 4**, you can demonstrate why there are two answers using a simple toy. Obtain a toy gun that shoots ping pong balls, or similar missiles. Fire a ball at an angle of $\frac{\pi}{4}$ to establish a maximum range for the toy. Then, select a target closer than the maximum range. Show, by repeated trials, that there are two angles that work, one greater than $\frac{\pi}{4}$ and one less than $\frac{\pi}{4}$.
- **Communicate Your Understanding** question C4 draws attention to a common error that students make when solving trigonometric equations. When discussing this question, determine the solutions to the equation, both with and without the division step.
- If a Computer Algebra System (CAS) is available, students may check their solutions to the equations in the exercises by using the **1:solve(** operation from the **F2** menu.
- **Question 23** gives students the opportunity to make connections to solve a problem related to a trigonometric equation. They have to reflect upon the problem and then use reasoning skills to determine whether there is or is not a solution within the domain given.
- The expression on the left side of **question 23** can be simplified by using trigonometric identities to reduce it to an expression strictly in terms of $\sin x$, or alternatively, in terms of $\cos x$. Solutions can be checked using a graphing calculator or a CAS.
- The equations in **questions 26** and **28** can be simplified using appropriate trigonometric identities. Solutions can be checked using a graphing calculator or a CAS.
- Use **BLM 5–8 Section 5.4 Practice** for remediation or extra practice.

DIFFERENTIATED INSTRUCTION

Use **Think-Pair-Share** to teach this section.

Use **what-so what double entry** to reinforce this section.

COMMON ERRORS

- Division by a trigonometric ratio to simplify an equation, such as the one in question C4, is a common error.

R_x Ensure that students understand why the procedure suggested in question C4 is not a valid operation when solving equations. Revisit this concept when appropriate.

ONGOING ASSESSMENT

Achievement Check, question 25, on student text page 289.

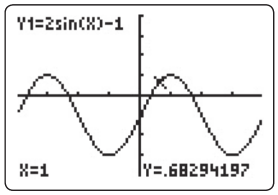
Investigate Answers (pages 282–283)

1. $\sin x = \frac{1}{2}$

2. a) $\frac{\pi}{6}$ **b)** $\frac{5\pi}{6}$

4. Yes.

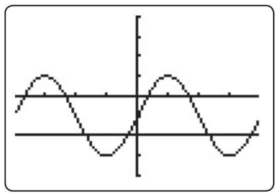
5.



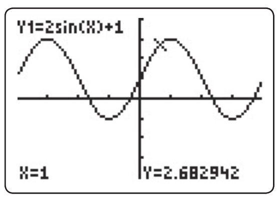
6. b) It is the same.

7. It is the same.

8. No. You could find the solution by graphing both the left side and the right side of the equation $2\sin x - 1 = -2$ and find the x-coordinates of the points of intersection.



Another method is to rearrange the equation so that the right side equals zero, then graph the left side of the equation and find the zeros.



Communicate Your Understanding Responses (page 287)

C1. One; the sine curve reaches a maximum value of 1, once per period.

C2. Four; the solutions will be plus/minus the square root of a value.

C3. The amplitude is 1 and the vertical translation is 2 units downward. The entire graph will lie below the x-axis.

C4. $\cos x$ is a factor.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	23, 26, 28
Reasoning and Proving	14–20, 23–28
Reflecting	23, 25, 27, 28
Selecting Tools and Computational Strategies	2, 4, 6, 8, 21, 22, 25
Connecting	1–3, 5, 7, 9–20, 22–28
Representing	2, 4, 6, 8, 22, 25
Communicating	20, 21, 25, 27

Achievement Check, question 25, student text page 289

This performance task is designed to assess the specific expectations covered in Section 5.4.
The following Math Process Expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

a) The function is expressed using two different trigonometric functions so it cannot be factored.

b) Use $\cos^2 x = 1 - \sin^2 x$ to convert the expression into one trigonometric function only.

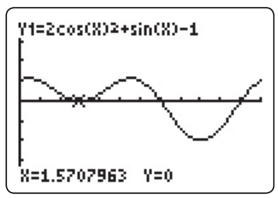
c)
$$2(1 - \sin^2 x) + \sin x - 1 = 0$$
$$-2\sin^2 x + \sin x + 1 = 0$$
$$2\sin^2 x - \sin x - 1 = 0$$

d) $(2\sin x + 1)(\sin x - 1) = 0$

e) $\sin x = -\frac{1}{2}$ or $\sin x = 1$

$x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$ or $x = \frac{\pi}{2}$

f)



The graph has grid marks every $\frac{\pi}{6}$ on the x-axis. The x-intercepts are $\frac{\pi}{2}$, $\frac{7\pi}{6}$,or $\frac{11\pi}{6}$.

Level 3 Notes

Look for the following:

- Explanation as to why the equation cannot be factored in its original form
- A trigonometric identity correctly applied to the equation
- A correct factoring and solution in the interval given
- A reasonable, fully labelled graph

What Distinguishes Level 2

- Some attempt at an explanation as to why the equation cannot be factored in its original form
- A trigonometric identity incorrectly applied to the equation
- A factoring and partial solution in the interval given
- A reasonable graph, not fully labelled

What Distinguishes Level 4

- A thorough explanation as to why the equation cannot be factored in its original form
- A trigonometric identity correctly applied to the equation
- A correct factoring and full solution in the interval given
- A reasonable, fully labelled graph which is used for verification for the solution