5.5 Making Connections and Instantaneous Rate of Change

Student Text Pages
290 to 299

Suggested Timing
60–75 min

Tools
- computer
- *The Geometer's Sketchpad®*
- graphing calculator

Optional
- graph of sine function and mathematics construction set

Related Resources
- G–1 Grid Paper
- G–7 Trigonometric Graph Paper
- T–2 *The Geometer's Sketchpad®* 4
- BLM 5–9 Section 5.5 Practice
- BLM 5–10 Section 5.5 Achievement Check Rubric

Teaching Suggestions

- The **Investigate** moves students toward the realm of calculus, as applied to sinusoidal functions. It reinforces the concept of a limit, introduced in earlier chapters, without the formalism of a true limiting procedure. Use **T–2 *The Geometer's Sketchpad®* 4** to support this activity, if needed.
- Technology tip for the **Investigate**:
  - When plotting points on a function using *The Geometer's Sketchpad®*, highlight the function then choose **Point on Function Plot** from the **Construct** menu. The **Point on Function Plot** only appears if there is a function drawn on the grid.
- **Example 1** is an algebraic analogue of the limiting process introduced geometrically in the **Investigate**. It is worth taking the time to compare and contrast the two approaches toward determining an instantaneous rate of change.
- In **Example 2**, two models are developed for a set of data; the first is by using pencil and paper and the second by using a graphing calculator to perform a sinusoidal regression. Refer students to the 5.3 Extension on pages 280–281 or the Technology Appendix on page 506.
- **Example 2** allows for reasoning skills to be used when establishing whether a sinusoidal function is an appropriate model in this case. When justifying the conclusion reached, communicating skills will be necessary. The students will have to make connections with work done previously to make the representations of a model in the form of a sine function and scatter plot of the data. Connecting skills will also be necessary to check that the model is accurate by using a sinusoidal regression.
- In **Example 3**, one population is modelled using a sine function, while the other is modelled using a cosine function. When finished with the mathematics, take some time to consider why there is a phase shift in the models of predator/prey populations.
- You can demonstrate the varying rate of change of the sine function in **Communicate Your Understanding** question C1 using *The Geometer's Sketchpad®*. Graph the sine function, and plot a point on the function. Measure the ordinate. Animate the point, and watch the changes in the ordinate as the point moves along the function with a constant speed. The slow rate of change near the maximum or minimum becomes apparent, as does the much faster rate of change halfway between a maximum and the next minimum.
- Data such as those presented in **question 4** are available from the U.S. Naval Observatory web site. Go to *www.mcgrawhill.ca/books/functions12* and follow the links to this site.
- Before assigning **question 6**, consider demonstrating the sinusoidal oscillation of a weight on a spring. You can use a Slinky® or similar spring, and suspend it in a stairwell for effect.
- You can add interest to **question 10** by showing a video clip of a real speed wobble, leading to an accident. These can be found on the Internet.
- **Question 13** directs students to E-STAT, which is a good source for modelling data, sinusoidal and otherwise. If you have not used E-STAT before, you will need a login and a password. You can obtain these from your site administrator or mathematics consultant.

DIFFERENTIATED INSTRUCTION

Use **timed retell** to teach this section.

Use **four corners** to reinforce this section.

Use a **journal entry**. Give the topic as "Explain How the Instantaneous Rate of Change of $y = \sin x$ Varies Over at Least Two Periods of the Curve."

ONGOING ASSESSMENT

Achievement Check, question 15, on student text page 299.

- Graphs of inverse trigonometric relations are not required to satisfy the expectations of this course, but are useful extensions for those who will be studying mathematics at a higher level. Consider recommending **question 16** for these students.
- Use **BLM 5–9 Section 5.5 Practice** for remediation or extra practice.

Investigate Answers (pages 290–291)

1. Answers may vary.

2. b) 0.79

c) The secant line becomes a tangent line. The slope of the tangent line is the approximate instantaneous rate of change.

d) 0.71

3. a)–c)

Angle x	$f(x) = \sin x$	Instantaneous Rate of Change
0	0.00	1.00
$\frac{\pi}{6}$	0.50	0.87
$\frac{\pi}{4}$	0.71	0.71
$\frac{\pi}{3}$	0.87	0.50
$\frac{\pi}{2}$	1.00	0.00
$\frac{2\pi}{3}$	0.87	−0.50
$\frac{3\pi}{4}$	0.71	−0.71
$\frac{5\pi}{6}$	0.50	−0.87
π	0.00	−1.00
$\frac{7\pi}{6}$	−0.50	−0.87
$\frac{5\pi}{4}$	−0.71	−0.71
$\frac{4\pi}{3}$	−0.87	−0.50
$\frac{3\pi}{2}$	−1.00	0.00
$\frac{5\pi}{3}$	−0.87	0.50
$\frac{7\pi}{4}$	−0.71	0.71
$\frac{11\pi}{6}$	−0.50	0.87
2π	0.00	1.00

4. The graph of the instantaneous rates of change is the cosine curve.

5. Answers may vary.

Communicate Your Understanding Responses (page 296)

C1. The instantaneous rate of change of the sine function decreases from 1 to 0 as the sine function increases from 0 to 1.

C2. $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$; the tangent line is horizontal at these values.

C3. Maximum of 1 at $x = 0$ and $x = 2\pi$, and minimum of -1 at $x = \pi$. The sine function has x-intercepts at these values.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	
Reasoning and Proving	3–5, 9–15
Reflecting	12, 15, 16
Selecting Tools and Computational Strategies	2, 4–18
Connecting	1–18
Representing	1, 2, 4, 8, 11, 13, 15–18
Communicating	2–4, 7–9, 12–18

Achievement Check, question 15, student text page 299

This performance task is designed to assess the specific expectations covered in Section 5.5.

The following Math Process Expectations can be assessed.

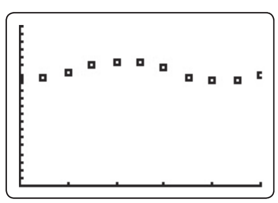
- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Achievement Chart Category	Related Math Processes
Knowledge and Understanding	Selecting tools and computational strategies
Thinking	Problem solving Reasoning and proving Reflecting
Communication	Communicating, Representing
Application	Selecting tools and computational strategies Connecting

Sample Solution

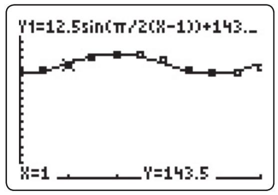
Provide students with **BLM 5–10 Section 5.5 Achievement Check Rubric** to help them understand what is expected.

a)



b) The period is about 4 units, so the k-value is $\frac{\pi}{2}$. The phase shift is 1 unit to the right. The amplitude is $\frac{156 - 131}{2} = 12.5$. The vertical shift is about 143.5 units up.

$V = 12.5\sin\left(\frac{\pi}{2}(t - 1)\right) + 143.5$

c) 

The phase shift should be a little more to the right, otherwise it looks like a good fit.

d) The calculator gave a regression equation of $y \doteq 12.4\sin(1.5x - 1.8) + 143.8$, which can be factored to $y \doteq 12.4\sin(1.5(x - 1.2)) + 143.8$. Since $\frac{\pi}{2} \doteq 1.6$, this equation is very close to my own. The phase shift is a little larger, as I suggested, and the vertical translation is almost right on.

e) If the cat breathed at a faster rate, the period would be shorter. If the cat increased the volume of air, the amplitude would be larger.