Chapter 6

Exponential and Logarithmic Functions

Curriculum Expectations

Exponential and Logarithmic Functions

Evaluating Logarithmic Expressions

A1.1 recognize the logarithm of a number to a given base as the exponent to which the base must be raised to get the number, recognize the operation of finding the logarithm to be the inverse operation (i.e., the undoing or reversing) of exponentiation, and evaluate simple logarithmic expressions

A1.2 determine, with technology, the approximate logarithm of a number to any base, including base 10 (e.g., by reasoning that $\log_3 29$ is between 3 and 4 and using systematic trial to determine that $\log_3 29$ is approximately 3.07)

Technology Notes

The technology used in this chapter is primarily graphing calculators, specifically the TI-83 Plus/TI-84 Plus series, a computer algebra system (CAS), specifically the TI-89/89T series, and *The Geometer's Sketchpad®*.

A1.3 make connections between related logarithmic and exponential equations (e.g., $\log_5 125 = 3$ can also be expressed as $5^3 = 125$), and solve simple exponential equations by rewriting them in logarithmic form (e.g., solving $3^x = 10$ by rewriting the equation as $\log_3 10 = x$)

A1.4 make connections between the laws of exponents and the laws of logarithms [e.g., use the statement $10^{a+b} = 10^a 10^b$ to deduce that $\log_{10} x + \log_{10} y = \log_{10} (xy)$], verify the laws of logarithms with or without technology (e.g., use patterning to verify the quotient law for logarithms by evaluating expressions such as $\log_{10} 1000 - \log_{10} 100$ and then rewriting the answer as a logarithmic term to the same base), and use the laws of logarithms to simplify and evaluate numerical expressions

Connecting Graphs and Equations of Logarithmic Functions

A2.1 determine, through investigation with technology (e.g., graphing calculator, spreadsheet) and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, increasing/decreasing behaviour) of the graphs of logarithmic functions of the form $f(x) = \log_b x$, and make connections between the algebraic and graphical representations of these logarithmic functions

A2.2 recognize the relationship between an exponential function and the corresponding logarithmic function to be that of a function and its inverse, deduce that the graph of a logarithmic function is the reflection of the graph of the corresponding exponential function in the line $y = x$, and verify the deduction using technology

A2.3 determine, through investigation using technology, the roles of the parameters d and c in functions of the form $y = \log_{10} (x - d) + c$ and the roles of the parameters a and k in functions of the form $y = a\log_{10} (kx)$, and describe these roles in terms of transformations on the graph of $f(x) = \log_{10} x$ (i.e., vertical and horizontal translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes)

A2.4 pose problems based on real-world applications of exponential and logarithmic functions (e.g., exponential growth and decay, the Richter scale, the pH scale, the decibel scale), and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation

Solving Exponential and Logarithmic Equations

A3.4 solve problems involving exponential and logarithmic equations algebraically, including problems arising from real-world applications

Characteristics of Functions

Understanding Rates of Change

D1.5 recognize examples of instantaneous rates of change arising from real-world situations, and make connections between instantaneous rates of change and average rates of change (e.g., an average rate of change can be used to approximate an instantaneous rate of change)

D1.6 determine, through investigation using various representations of relationships (e.g., tables of values, graphs, equations), approximate instantaneous rates of change arising from real-world applications (e.g., in the natural, physical, and social sciences) by using average rates of change and reducing the interval over which the average rate of change is determined

D1.8 determine, through investigation using a variety of tools and strategies (e.g., using a table of values to calculate slopes of secants or graphing secants and measuring their slopes with technology), the approximate slope of the tangent to a given point on the graph of a function (e.g., quadratic, exponential, sinusoidal) by using the slopes of secants through the given point (e.g., investigating the slopes of secants that approach the tangent at that point more and more closely), and make connections to average and instantaneous rates of change

D1.9 solve problems involving average and instantaneous rates of change, including problems arising from real-world applications, by using numerical and graphical methods (e.g., by using graphing technology to graph a tangent and measure its slope)

Using Function Models to Solve Problems

D3.1 compare, through investigation using a variety of tools and strategies (e.g., graphing with technology; comparing algebraic representations; comparing finite differences in tables of values) the characteristics (e.g., key features of the graphs, forms of the equations) of various functions (i.e., polynomial, rational, trigonometric, exponential, logarithmic)

Chapter 6 Planning Chart

Section Suggested Timing	Student Text Page(s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 6 Opener • 10 min	307			
Prerequisite Skills • 40–75 min	308–309	• G–3 Four Quadrant Grids • BLM 6–1 Prerequisite Skills		• grid paper • graphing calculator
6.1 The Exponential Function and Its Inverse • 75–150 min	310–322	• G–1 Grid Paper • G–2 Placemat • G–3 Four Quadrant Grids • T–2 *The Geometer's Sketchpad*® 4 • BLM 6–2 Section 6.1 Practice		• grid paper • computer • *The Geometer's Sketchpad*® 4 • graphing calculator (optional)
6.2 Logarithms • 75 min	323–330	• G–3 Four Quadrant Grids • G–6 Semi-log Graph Paper • T–2 *The Geometer's Sketchpad*® 4 • BLM 6–3 Section 6.2 Practice		• grid paper • graphing calculator • computer • *The Geometer's Sketchpad*® 4
6.3 Transformations of Logarithmic Functions • 75 min	331–340	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad*® 4 • BLM 6–4 Section 6.3 Practice		• grid paper • computer • *The Geometer's Sketchpad*® 4 • graphing calculator
6.4 Power Law of Logarithms • 75 min	341–348	• G–1 Grid Paper • T–1 Microsoft® *Excel* • T–2 *The Geometer's Sketchpad*® 4 • T–4 The Computer Algebra System (CAS) on the TI-89 Calculator • BLM 6–5 Section 6.4 Practice		• grid paper • computer • *The Geometer's Sketchpad*® 4 • graphing calculator • computer algebra system • scientific calculator • spreadsheet software
6.5 Making Connections: Logarithmic Scales in the Physical Sciences • 75 min	349–355	• BLM 6–6 Section 6.5 Practice	• BLM 6–7 Section 6.5 Achievement Check Rubric	• graphing calculator
Chapter 6 Review • 60–75 min	356–357	• G–1 Grid Paper • BLM 6–8 Chapter 6 Review		• grid paper • graphing calculator
Chapter 6 Problem Wrap-Up • 40–75 min	357		• BLM 6–9 Chapter 6 Problem Wrap-Up Rubric	• computer • Internet • library
Chapter 6 Practice Test • 60–75 min	358–359	• G–1 Grid Paper	• BLM 6–10 Chapter 6 Test	• grid paper • graphing calculator
Chapter 6 Task: Not Fatal • 60–75 min	360	• BLM 6–12 BLM Answers	• BLM 6–11 Task: Not Fatal Rubric	• 100 coins • graphing calculator

Chapter 6 Blackline Masters Checklist

	BLM	Title	Purpose
Prerequisite Skills			
	G–3	Four Quadrant Grids	Student Support
	BLM 6–1	Prerequisite Skills	Practice
6.1 The Exponential Function and Its Inverse			
	G–1	Grid Paper	Student Support
	G–2	Placemat	Student Support
	G–3	Four Quadrant Grids	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	BLM 6–2	Section 6.1 Practice	Practice
6.2 Logarithms			
	G–3	Four Quadrant Grids	Student Support
	G–6	Semi-log Graph Paper	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	BLM 6–3	Section 6.2 Practice	Practice
6.3 Transformations of Logarithmic Functions			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	BLM 6–4	Section 6.3 Practice	Practice
6.4 Power Law of Logarithms			
	G–1	Grid Paper	Student Support
	T–1	Microsoft® *Excel*	Technology
	T–2	*The Geometer's Sketchpad*® 4	Technology
	T–4	The Computer Algebra System (CAS) on the TI-89 Calculator	Technology
	BLM 6–5	Section 6.4 Practice	Practice
6.5 Making Connections: Logarithmic Scales in the Physical Sciences			
	BLM 6–6	Section 6.5 Practice	Practice
	BLM 6–7	Section 6.5 Achievement Check Rubric	Assessment
Chapter 6 Review			
	G–1	Grid Paper	Student Support
	BLM 6–8	Chapter 6 Review	Practice
Chapter 6 Problem Wrap-Up			
	BLM 6–9	Chapter 6 Problem Wrap-Up Rubric	Assessment
Chapter 6 Practice Test			
	G–1	Grid Paper	Student Support
	BLM 6–10	Chapter 6 Test	Summative Assessment
Chapter 6 Task: Not Fatal			
	BLM 6–11	Task: Not Fatal Rubric	Assessment
	BLM 6–12	BLM Answers	Answers

Prerequisite Skills

Student Text Pages
308 to 309

Suggested Timing
40–75 min

Tools
- grid paper
- graphing calculator

Related Resources
- G–3 Four Quadrant Grids
- BLM 6–1 Prerequisite Skills

Assessment
You may wish to use **BLM 6–1 Prerequisite Skills** as a diagnostic assessment. Refer students to the Skills Appendix for examples and further practice of topics.

Chapter Problem

- The Chapter Problem is introduced on page 309. This collection of problems, related to stars and space travel, should particularly appeal to students who are considering a post-secondary program in aeronautical engineering or the physical sciences. Explain that as they work through the Chapter Problem, in Section 6.1 (question 22), Section 6.2 (question 15), Section 6.4 (question 15), and Section 6.5 (question 15), students will solve a variety of problems related to stars and interstellar travel. In the Chapter Problem Wrap-Up, students will explore a mathematical model that is related to how a star is powered. Explain that they will learn more about such stars and stellar structure if they study courses in astronomy or astrophysics.