6.2 Logarithms

Student Text Pages
323 to 330

Suggested Timing
75 min

Tools
- grid paper
- graphing calculator
- computer
- *The Geometer's Sketchpad®*

Related Resources
- G–3 Four Quadrant Grids
- G–6 Semi-log Graph Paper
- T–2 *The Geometer's Sketchpad®* 4
- BLM 6–3 Section 6.2 Practice

Teaching Suggestions

- In this section, students are introduced to the term *logarithm* and the logarithmic function as the inverse of an exponential function.
- **Example 1** models how to write an exponential relationship in logarithmic form. Some students may wish to transcribe the explanation (or write their own version) of the Connections to provide a general guide for how to do this.
- In **Example 2**, students apply graphical and numerical reasoning to evaluate simple logarithms. This is a vital step toward conceptual understanding of what a logarithm is, which many students have difficulty with at this level.
- **Example 3** models how to write a logarithmic relationship in exponential form and introduces the convention of writing common (base 10) logarithms without the base explicitly indicated.
- It is important for students to experience the process of estimating logarithms using the numerical and two graphical approaches of **Example 4** prior to using the LOG function on a calculator, so that a deeper conceptual understanding of the logarithmic function is established. Graphing technology can efficiently show how a logarithmic function is connected to its corresponding inverse, the exponential function.
- **Example 4** shows three different methods to find the value for each logarithm. The three methods require the students to select tools and apply appropriate representations to solve the problem given. With Method 1, they must use connecting skills and the selecting of tools to estimate an answer; Methods 2 and 3 allow the students to use graphing software to analyse and solve the problem.
- The **Communicate Your Understanding** questions are intended to elicit student understanding that the logarithm is an exponent to which a base must be raised to produce a given value, the relationship between a logarithmic function and its corresponding inverse exponential function, and the key features of its graph.
- Refer to Methods 2 and 3 of **Example 4** for **question 7**.
- For a hint to question 11, have students write the relationship in exponential form.
- **Question 12** allows students to develop and apply reasoning and connecting skills to make a comparison between the rate of change of a logarithmic function and its inverse. They are then required to explain using their communicating skills the similarities and differences found and to represent their explanations with an example.
- Some students may wish to explore **question 12** using graphing technology, such as *The Geometer's Sketchpad®*.
- As an extension, some students could rearrange the equation in **question 13** to express N in terms of t, to show where this equation comes from.
- **Question 17** is designed to elicit from students an understanding that the rate of change of the logarithmic function decreases rapidly as the independent variable increases. This makes it difficult to view much data representing a logarithmic relationship using standard techniques.
- **Questions 18** and **19** illustrate two methods for viewing a logarithmic relationship over a large range of data: one uses a special type of graph paper, and the other uses technology. Students who go on to study physical sciences or engineering at university are likely to encounter one or both of these methods in their lab work.
- Use **BLM 6–3 Section 6.2 Practice** for remediation or extra practice.

Differentiated Instruction

Use **concept attainment** to teach this section.

Common Errors

- Students lack a conceptual understanding of what a logarithm is and rely on memorized algorithmic methods for answering questions.

R_x Students typically understand exponential functions fairly well, since they are more intuitive. Reinforce the inverse relationship between the logarithmic function and its corresponding exponential function. Utilize graphing technology to facilitate a visual representation of this connection.

Ongoing Assessment

Achievement Check, question 16, on student text page 329.

Communicate Your Understanding Responses (page 327)

C1. Yes. The value of $\log_b x$ is equal to the exponent to which the base, b, is raised to produce x.

C2. $y = \log_b x \leftrightarrow x = b^y$

C3. No. The domain is $x > 0$.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	
Reasoning and Proving	11, 12, 14–16, 18, 19
Reflecting	16
Selecting Tools and Computational Strategies	1–7, 10, 17
Connecting	9–16
Representing	5–10, 16–19
Communicating	10, 11, 12, 14, 16–19

Achievement Check, question 16, student text page 329

This performance task is designed to assess the specific expectations covered in Section 6.2. The following Math Process Expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

a) i) $t = \dfrac{\log\left(\dfrac{30}{4}\right)}{\log 3}$

$\doteq 1.834$

It will take almost two days.

ii) $t = \dfrac{\log\left(\dfrac{550}{4}\right)}{\log 3}$

$\doteq 4.48$

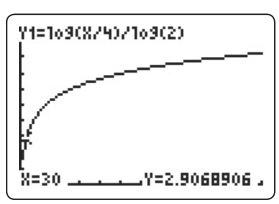
It will take almost 4.5 days.

iii) $t = \dfrac{\log\left(\dfrac{1100}{4}\right)}{\log 3}$

$\doteq 5.113$

It will take a little longer than 5 days.

b)



c) i) If you start with more than four people the rumour will spread faster, and the graph will be steeper.

ii) If you start with fewer than four people the rumour will take longer to spread and the graph will be flatter.

d) If some students hear the rumour more than once, in the above model, we are assuming that the students will persist in telling more people until they find two who haven't heard it before. This is unlikely. The rumour will take longer to spread especially as more people know it and it is difficult to find two people who haven't heard it.

Level 3 Notes

Look for the following:

- Correct calculations of the times involved
- An accurate sketch of the graph
- A reasonable interpretation of the effect of starting with more people or fewer people
- A reasonable explanation of the effect on the model if students hear the rumour more than once

What Distinguishes Level 2

- Some correct calculations of the times involved
- An adequate sketch of the graph
- An interpretation of the effect of starting with more people or fewer people
- A partial explanation of the effect on the model if students hear the rumour more than once

What Distinguishes Level 4

- Correct calculations of all of the times involved
- An accurate, fully labelled graph of the model
- A reasonable interpretation of the effect of starting with more people or fewer people with some justification
- A reasonable explanation of the effect on the model if students hear the rumour more than once with some mathematical support