6.3

Transformations of Logarithmic Functions

Student Text Pages
331 to 340

Suggested Timing
75 min

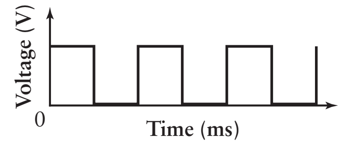
Tools
- grid paper
- computer
- *The Geometer's Sketchpad®*
- graphing calculator

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad®* 4
- BLM 6–4 Section 6.3 Practice

Teaching Suggestions

- The purpose of the **Investigate** is for students to verify, using graphing technology, that standard transformations of functions learned in previous grades hold true for logarithmic functions. **Part A**, which deals with translations, is more scaffolded than **Part B**, which deals with stretches and reflections. Students should take care of placement of brackets when programming functions in *The Geometer's Sketchpad®*. Use **T–2 *The Geometer's Sketchpad®* 4** to support this activity.
- Technology tips for the **Investigate**:
  - For **Part A**, an option is to create sliders or, from the **Custom Tool**, choose **Sliders**, then **Integer horizontal**. Creating sliders for parameters c and d would allow students to experiment with changing values with ease by dragging the slider.
  - For **Part B**, two more sliders for parameters a and k can be created.
- When applying both a vertical and a horizontal translation, as in **Example 1**, it may be advisable to perform these in two steps, keeping track of what is happening to certain key features of the graph (e.g., asymptote, x-intercept) along the way.
- The multiple transformations that are applied in **Example 2** require a methodical stepwise approach. Encourage students to track the behaviour of a couple of "anchor points" as each transformation is made, thus making it easier to generate the final shape of the transformed graph.
- The **Communicate Your Understanding** questions can be used to assess students' ability to make connections between algebraically transformed logarithmic functions and the graphical implications. Some students may benefit from transcribing or writing their own summary of transformations, such as the one appearing in the **Key Concepts**.
- Graphing technology can be used to check **questions 1** through **7**.
- Operational amplifiers, which are discussed in **question 10**, have the capacity to perform virtually any type of mathematical function when converting one or more input signals to an output signal. They are tremendously useful integrated circuit elements that future students of electronics will learn more about in university or college.
- **Questions 11** through **13** provide opportunities to assess students' ability to reason with, represent, and communicate their understanding of logarithmic functions.
- **Question 11** requires students to explain the algebraic reasoning that took place to determine the change in the domain and range of a logarithmic function under a horizontal reflection. Representations in the form of diagrams are needed.
- Students can check their results to **question 16**, using graphing technology.
- Students should be encouraged to use graphing technology to solve and/or check their answers to **questions 17** and **18**.

- **Question 19** provides an opportunity to make connections to digital logic and digital electronics. In most digital circuits, the voltage at any given point has either a high value, 1, or a low value, 0. The waveform for a periodic digital circuit could look something like this (resembling the pulsator):

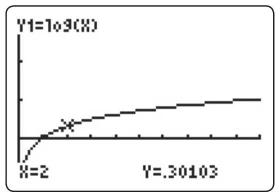


- Use **BLM 6–4 Section 6.3 Practice** for remediation or extra practice.

Investigate Answers (pages 331–332)

Part A

1. Window settings: $x \in [0, 10]$, $y \in [-1, 3]$



2. When $c > 0$, the graph translates up c units. When $c < 0$, the graph translates down c units.

3. a) When $d > 0$, the graph shifts right d units. When $d < 0$, the graph translates left d units.

b) Answers may vary. Sample answer: Since $f(x) = \log x$ is undefined at $x = 0$, then the graph of $f(x) = \log (x - d)$ will be undefined at $x - d = 0$, or $x = d$.

4. a) The function will be translated horizontally and vertically.

b) See answers to steps 2 and 3a).

Part B

1. Answers may vary. Sample answer:
If $a > 1$, the graph stretches vertically by a factor of a.
If $0 < a < 1$, the graph compresses vertically by a factor of a.
If $a = -1$, the graph reflects in the x-axis.
If $a < -1$, the graph reflects in the x-axis and stretches vertically by a factor of $|a|$.
If $-1 < a < 0$, the graph reflects in the x-axis and compresses vertically by a factor of $|a|$.

2. Answers may vary. Sample answer:
If $k > 1$, the graph compresses horizontally by a factor of $\left|\frac{1}{k}\right|$.
If $0 < k < 1$, the graph stretches horizontally by a factor of $\left|\frac{1}{k}\right|$.
If $k = -1$, the graph reflects in the y-axis.
If $k < -1$, the graph reflects in the y-axis and compresses horizontally by a factor of $\left|\frac{1}{k}\right|$.
If $-1 < k < 0$, the graph reflects in the y-axis and stretches horizontally by a factor of $\left|\frac{1}{k}\right|$.

3. Answers may vary. Sample answer:

Function	Effect of Parameter
$y = \log x + c$	if $c > 0$, translate up c units if $c < 0$, translate down c units
$y = \log (x - d)$	if $d > 0$, translate right d units if $d < 0$, translate left d units
$y = a\log x$	if $\|a\| > 1$, stretch vertically by a factor of $\|a\|$ if $\|a\| < 1$, compress vertically by a factor of $\|a\|$ if $a < 0$, reflect in the x-axis
$y = \log kx$	if $\|k\| > 1$, compress horizontally by a factor of $\left\|\frac{1}{k}\right\|$ if $\|k\| < 1$, $k \neq 0$, stretch horizontally by a factor of $\left\|\frac{1}{k}\right\|$ if $k < 0$, reflect in the y-axis

Differentiated Instruction

Use **jigsaw** to teach this section.

Use **group graph** and a **gallery walk** to teach this section.

Common Errors

- Students get mixed up when trying to illustrate multiple transformed graphs on the same grid.

R_x Suggest the use of multiple colours and/or line styles to help distinguish the various graphs.

Ongoing Assessment

Achievement Check, question 16, on student text page 340.

Communicate Your Understanding Responses (page 337)

C1. a) Translate the graph of $y = \log x$ to the right 2 units and up 7 units.

b) Vertically stretch the graph of $y = \log x$ by a factor of 3 and then reflect it in the x-axis.

c) Horizontally compress the graph of $y = \log x$ by a factor of $\frac{1}{3}$, reflect it in the y-axis, and then translate it down 5 units.

C2. a) The graph of g is a reflection of f in the x-axis. They are not inverses because they are not reflected in the line $y = x$.

b) The graph of h is a reflection of f in the y-axis. They are not inverses because they are not reflected in the line $y = x$.

c) No, they are reflected in the x-axis and the y-axis.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	
Reasoning and Proving	10–12
Reflecting	18
Selecting Tools and Computational Strategies	9, 13–18
Connecting	3–18
Representing	3, 6, 7, 9–18
Communicating	5, 10–12, 15, 18

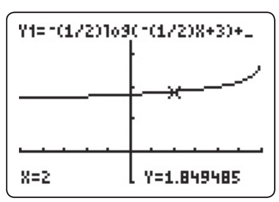
Achievement Check, question 16, student text page 340

This performance task is designed to assess the specific expectations covered in Section 6.3. The following Math Process Expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

a)

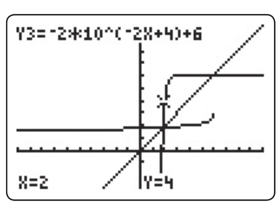


b) i) Domain: $x < 6$

ii) Range: $y \in \mathbb{R}$

iii) Vertical asymptote is $x = 6$.

c)



Level 3 Notes

Look for the following:

- Reasonable, fully labelled graphs of the function and its inverse
- Correct analysis of the domain, range and asymptote

What Distinguishes Level 2

- Reasonable graphs of the function and its inverse
- Some analysis of the domain, range and asymptote

What Distinguishes Level 4

- Reasonable, fully labelled graphs of the function and its inverse with some justification
- Correct analysis of the domain, range and asymptote with full explanations