6.4 Power Law of Logarithms

Student Text Pages
341 to 348

Suggested Timing
75 min

Tools
- grid paper
- computer
- *The Geometer's Sketchpad®*
- graphing calculator
- computer algebra system
- scientific calculator
- spreadsheet software

Related Resources
- G–1 Grid Paper
- T–1 Microsoft® *Excel*
- T–2 *The Geometer's Sketchpad®* 4
- T–4 The Computer Algebra System (CAS) on the TI-89 Calculator
- BLM 6–5 Section 6.4 Practice

Teaching Suggestions

- The purpose of **Investigate 1** is to pose a contextual problem in which the unknown appears as an exponent and which cannot be solved accurately by inspection. This drives the need for the power law of logarithms, which provides a technique to do this algebraically. Students should explore other methods of finding an approximate solution to this problem first (e.g., systematic trial, inspecting a graph using technology), which they can later reflect on after applying the power law in **Example 2**. Depending on technology applied, use **T–2 *The Geometer's Sketchpad*® 4** or **T–4 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this activity.
- In **Investigate 2**, students examine patterns to discover the power law of logarithms inductively. Encourage the use of technology (e.g., spreadsheet or graphing calculator) to facilitate several repeated calculations, if available. A formal proof of the power law is presented after this activity, which students should be exposed to but not expected to reproduce. Use **T–4 The Computer Algebra System (CAS) on the TI-89 Calculator** or **T–1 Microsoft® *Excel*** to support this activity.
- **Example 1** illustrates how the power law of logarithms can be applied to evaluate numerical expressions. In Method 1 of part a), students will need to recall the power law of exponents. In part d), they will need to recall how to write a square root in exponential form.
- In **Example 2**, the initial problem posed in Investigate 1 is revisited, as a practical application of the power law of logarithms. This technique for solving for an unknown exponent algebraically is critical, and students will apply it time and again in future work in mathematics.
- Directly following **Example 2**, an algebraic derivation of the change of base formula is presented. This formula is vital, as it provides a means to calculate logarithms of any base, which is the focus of **Example 3**. When teaching this example, review keystrokes required for performing calculations using a scientific or graphing calculator.
- In **Example 4**, Method 2 illustrates how the change of base formula can be applied in order to graph a logarithmic function having any base using graphing technology. Method 1 shows how this can also be done by reflecting the corresponding inverse exponential function in the line $y = x$. Students should see both methods as they continue to develop their understanding of the logarithmic function, and its relationship to the exponential function.
- Technology tip for **Example 2**, Method 2:
  - Students can also use the graphing calculator to graph the function **Y1** $= 5^x$. Then, press [2nd] [DRAW], and select **8:DrawInv**. Press [VARS], cursor right to the **Y-VARS** menu, select **1:Function…**, and then select **1:Y1**. After pressing [ENTER] students will see the graph of the inverse function.
- **Example 4** gives students the opportunity to select tools and use their connecting skills with material learned previously in order to represent the logarithmic function graphically.
- The purpose of the **Communicate Your Understanding** is to provide an opportunity for students to express their understanding of the power law of logarithms and its applications. The change of base formula is of particular importance as it provides a means for evaluating and graphing logarithms and logarithmic functions having any base. Consider assigning these questions using a think-pair-share technique.

- Students should be able to solve **questions 1** and **2** without using technology. They can check their answers afterward using a calculator.
- For **question 6**, students will need to apply the change of base formula in reverse. Have them look at the formula on page 344 as a hint.
- **Questions 11** and **12** provide opportunities to assess students' reasoning and communicating skills.
- **Question 11** allows students to use their algebraic reasoning skills to make mathematical conjectures and to make connections with previously learned material related to logarithmic functions. They must select the necessary tools to apply graphical reasoning and use communicating skills to explain their findings.
- Students can check their answers to **questions 13** and **14** using a scientific or graphing calculator.
- Encourage students to explore the function in **question 15** using graphing technology. When discussing, ask "Why does this curve have the shape it does? Why is this important for the given problem? What does the asymptote indicate? Is this entirely accurate? Why or why not?"
- Some students should be encouraged to solve and/or check **question 16** using graphing technology.
- As a slightly easier alternative approach to **question 17**, mix up cut strips of paper, each containing one line of the solution, and have students reorganize the strips and write explanatory steps. This would be a good activity to assign to pairs of students.
- As a hint to **question 20**, have students begin by writing an expression for A as a function of t, and then rearranging the formula.
- Use **BLM 6–5 Section 6.4 Practice** for remediation or extra practice.

Investigate Answers (page 341–342)

Investigate 1

1. Answers may vary.

2. a) Answers may vary. Sample answer: Use systematic trial.

b) approximately 14 years

c) Answers may vary.

3. Answers may vary. Sample answer:
The answer doesn't change, since the equation that needs to be solved is still $2 = 1.05^t$.

4. $t = \log_{1.05}\left(\frac{A}{100}\right)$

Investigate 2

1. a) **i)** 0.3 **ii)** 0.6 **iii)** 0.9 **iv)** 1.2 **v)** 1.5

b) They are multiples of $\log 2$.

i) 1.8 **ii)** 3

2. $\log 2^n = n\log 2$

3. a) **i)** 0.477 **ii)** 0.954 **iii)** 1.431

b) $\log 3^n = n\log 3$

4. a) $\log b^n = n\log b$, for $b > 0$

Differentiated Instruction

Use **Think-Pair-Share** to teach this section.

Common Errors

- Students reverse the numerator and denominator of the change of base formula.

R_x Have students recognize that log *b* is a constant and that log *m* is the variable. Therefore if log *m* is in the numerator, a logarithmic function will result; if, however, log *m* is in the denominator, then a different type of function will be produced. Have students explore this using graphing technology to confirm the reasoning.

Ongoing Assessment

Achievement Check, question 16, on student text page 348.

Communicate Your Understanding Responses (page 346)

C1. a) $\dfrac{\log 10}{\log 2} \doteq 3.32$

b) Answers may vary. Sample answer:
Graph $y = 2^x$ and estimate the x-value that gives a y-value of 10.

C2. a) Answers may vary. Sample answer:
To evaluate and/or graph logarithms with any base.

b) Answers may vary. Sample answer:
$b \neq 1$ because $\log 1 = 0$ (division by zero).
$b > 0$ because $\log b\ m$ is only defined for base $b > 0$.
$m > 0$ because the domain for $\log m$ is $\{m \in \mathbb{R}, m > 0\}$.

C3. a) Answers may vary. Sample answer: Use the change of base formula and then take the inverse.

b) $\dfrac{\log 3}{\log 7} \doteq 0.56$

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	14, 17
Reasoning and Proving	11–17, 19
Reflecting	
Selecting Tools and Computational Strategies	1–3, 5, 6, 8, 11, 12
Connecting	4–7, 9–20
Representing	7, 8, 16, 20
Communicating	10–12, 13, 15, 16

Achievement Check, question 16, student text page 348

This performance task is designed to assess the specific expectations covered in Section 6.4.

The following Math Process Expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

a) $A = P(1.08)^t$

b) i) The investment doubles in 9 years 2.5 days.

ii) The investment triples in 14 years 3.5 months.

c) i) $(1.08)^5 = 1.4693$ Therefore, there is an increase of 46.9% in 5 years.

ii) $(1.08)^{10} = 2.1589$ Therefore, there is an increase of 116% in 10 years.

d) In part b) the equation looks like $2P = P(1.08)^t$ and the principal can be factored out of both sides. In part c) the comparison looks like $\dfrac{P(1.08)^5}{P}$ and the principal divides out. In both of these cases the initial principal is irrelevant to the calculation.

Level 3 Notes

Look for the following:

- An accurate formula to model the compound interest
- Accurate calculations of the times and increases
- A complete explanation of the fact that the initial principal does not affect the calculations

What Distinguishes Level 2

- A formula to model the compound interest
- Calculations of the times and increases with minor errors
- A partial explanation of the fact that the initial principal does not affect the calculations

What Distinguishes Level 4

- An accurate formula to model the compound interest
- Accurate calculations of the times and increases with explanations when needed
- A complete explanation of the fact that the initial principal does not affect the calculations, justified by some mathematical support