7.1

Equivalent Forms of Exponential Equations

Student Text Pages
364 to 369

Suggested Timing
75 min

Tools
- linking cubes
- graphing calculator
- computer
- *The Geometer's Sketchpad®*

Related Resources
- T–2 *The Geometer's Sketchpad®* 4
- BLM 7–2 Section 7.1 Practice

Ongoing Assessment

Use **Assessment Masters A–1** to **A–7** to remind students about the Math Processes Expectations and how you may be assessing their integrated use of them.

Teaching Suggestions

- The concrete models used in the **Investigate** are designed to illustrate how the same exponential growth pattern can be expressed using two different bases. The intent is to lead into the symbolic reasoning required in order to change the base of a power.
- **Example 1** illustrates a variety of techniques for changing the base of a power, in increasing complexity. It should be noted in part c) how an initially complicated expression can be written in a relatively simpler form. While the methods in parts a) through c) will only work given certain circumstances, part d) shows a method that can be extended to relating powers of any bases.
- **Example 2** illustrates one very important application of the change of base of an exponential expression—to solve an equation. Students should see both methods of solution, as they provide a conceptual link between the algebraic and graphical representations.
- The **Communicate Your Understanding** questions are designed to check that students understand how to change the base of a power and also to verify that they understand the power graphing technique in Example 2a), Method 2. It should be noted that this technique can be applied in all sorts of situations involving equations that are difficult to solve algebraically, not just those involving exponentials.
- Students can use a scientific or graphing calculator to check their answers for **questions 1** through **4**.
- **Question 7** allows students to reflect upon the equation given and select tools to use connecting skills to solve the equation in two different ways. Reasoning skills would provide them with the ability to choose their preferred method, and communicating skills would enable them to explain their preference.
- **Questions 8** and **9** provide opportunities to assess students' ability to build concrete mathematical representations and apply reasoning to connect to the given algebraic representation.
- For **question 10**, prompt students to consider whether one base can be written as a power of the other base, in each equation.
- **Questions 11** through **13** provide opportunities for students to exercise their ability to reason algebraically. Some review of concepts (power law, change of base) from Chapter 6 may be required.
- The graphing calculator method in **question 14**, part c), is a powerful one, particularly for equations that arise which cannot be solved easily using algebraic means. Students will encounter such equations in Chapter 8 Combining Functions.
- As a hint for **question 15**, have students graph each side of the inequality as a separate function and apply graphical reasoning. They will learn a number of methods for solving inequalities in Chapter 8 Combining Functions.
- There are connections to rate of change, learned earlier in the course, in **question 16**. Encourage students to experiment with various viewing windows to recognize that some functions appear to be quite similar over a restricted domain, but can quickly diverge once extended beyond that domain. This has profound implications when developing mathematical models using the regression features of graphing technology, and should prompt lively debate when these concepts are revisited in Section 7.5.
- **Use BLM 7–2 Section 7.1 Practice** for remediation or extra practice.

DIFFERENTIATED INSTRUCTION

Continue the **word wall**.

COMMON ERRORS

- Students forget or misapply exponent laws.

R_x Have students review the exponent laws using one or two simple numerical examples to clarify any misconceptions (e.g., $(2^2)^3 = (2^2)(2^2)(2^2) = 2^{2+2+2} = 2^6 = 2^{2\times3}$, etc.)

ONGOING ASSESSMENT

Achievement Check, question 14, on student text page 369.

Investigate Answers (pages 364–365)

1. b) The number of squares in each model illustrates the sequence 4^n, for $n = 0$, 1, and 2: $4^0 = 1$, $4^1 = 4$, and $4^2 = 16$.

2. b) The number of squares in each model illustrates the sequence 2^{2n}, for $n = 0$, 1, and 2: $2^0 = 1$, $2^2 = 4$, and $2^4 = 16$.

3. a) Yes. **b)** Yes.

4. a) $f(x) = g(x)$ **b)** The graphs of f and g are the same. **c)** $4^x = (2^2)^x = 2^{2x}$

Communicate Your Understanding Responses (page 367)

C1. a) 4^4 **b)** Answers may vary. Sample answer: 2^8, $8^{\frac{8}{3}}$

C2. a) $2^x = 4^{x-1}$ **b)** The x-coordinate of the point of intersection, $x = 2$.

C3. Answers may vary. Sample answer:
No. Counterexample: 5 cannot be written as a power of −2.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	12, 14, 15
Reasoning and Proving	5–9, 11–15, 16
Reflecting	7, 12, 14
Selecting Tools and Computational Strategies	1–10, 13, 14, 16
Connecting	1–11, 13–15, 16
Representing	8–10, 14, 15
Communicating	7, 8, 13–15

Achievement Check, question 14, student text page 369

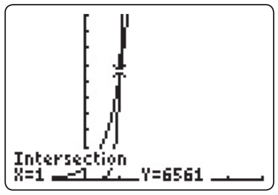
This performance task is designed to assess the specific expectations covered in section 7.1.
The following Math Process Expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

a)
$$\begin{aligned} 81^{2x} &= 9^{x+3} \\ (3^4)^{2x} &= (3^2)^{x+3} \\ 3^{8x} &= 3^{2x+6} \\ 8x &= 2x + 6 \\ 6x &= 6 \\ x &= 1 \end{aligned}$$

b)
$$\begin{aligned} (9^2)^{2x} &= 9^{x+3} \\ 9^{4x} &= 9^{x+3} \\ 4x &= x + 3 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

c) 

d) I prefer the algebraic method. The graphing method can be inaccurate and you need to find an appropriate window for seeing the intersection.

e) Solve:

$$25^{3x-1} = 625^{x+3} \qquad \text{or} \qquad 25^{3x-1} = 625^{x+3}$$
$$(5^2)^{3x-1} = (5^4)^{x+3} \qquad\qquad (25)^{3x-1} = (25^2)^{x+3}$$
$$6x - 2 = 4x + 12 \qquad\qquad 3x - 1 = 2x + 6$$
$$x = 7 \qquad\qquad x = 7$$

Level 3 Notes

Look for the following:

- Complete multiple solutions using different bases
- Complete solutions using technology
- An interesting equation that can be solved in multiple ways

What Distinguishes Level 2

- Multiple solutions using different bases with minor errors
- A solution using technology
- An equation that can be solved in multiple ways

What Distinguishes Level 4

- Complete multiple solutions using different bases
- Complete solutions using technology with explanations where needed
- An interesting equation that can be solved in multiple ways