7.2 Techniques for Solving Exponential Equations

Student Text Pages
370 to 377

Suggested Timing
75 min

Tools
- graphing calculator

Related Resources
- BLM 7–3 Section 7.2 Practice

Teaching Suggestions

- When using the method shown in part a) of **Example 1**, it is important for students to understand that the power must be isolated before taking the common logarithm of both sides and applying the power law. Careful algebraic manipulation must also be followed so that the variable is systematically solved for using standard equation-solving techniques.
- In part b) of **Example 1**, students should note the careful placement of brackets when entering the function into the graphing calculator.
- Method 1 of **Example 2** illustrates a standard method for solving an equation involving two exponential expressions in the case that one base cannot easily be expressed as a power of the other. Some students may require a quick review of collecting like terms and common factoring.
- Method 2 of **Example 2** provides a quick way to solve and/or check the same question using graphing technology, however the drawback is that it provides an approximate answer only. Students should see both methods and discuss the relative merits of each.
- An optional strategy for **Example 2** is to have students solve the equation $4^{2x-1} - 3^{x+2} = 0$ instead of $4^{2x-1} = 3^{x+2}$. On the graphing calculator, enter Y1 = 4 ^ $(2x - 1)$ − 3 ^ $(x + 2)$, and then use the **Zero** operation to find the solution.
- **Example 3** involves an equation with three terms and requires a special technique to develop an algebraic solution. Being able to recognize that a quadratic equation can be produced by multiplying both sides of the equation by a common term requires some level of algebraic insight. Some students will need to review techniques for solving a quadratic equation, and two approaches are used that differ only in the representation of the variable. Finally, cases must be considered in turn to determine which solution(s) are valid and which are extraneous. Graphing technology is recommended to verify the results. This question is a good example of one that requires algebraic insight and a high level of reasoning and perseverance.
- An optional strategy for **Example 3** is to have students solve the equation $2^x - 2^{-x} - 4 = 0$ instead of $2^x - 2^{-x} = 4$. On the graphing calculator, enter Y1 = 2 ^ x − 2 ^ $-x$ − 4, and then use the **Zero** operation to find the solution.
- **Example 3** requires reflection to establish a strategy to solve the problem given. Students will have to apply reasoning skills to plan a method with which to solve the problem and then make connections with previously learned work and select tools to actually solve the equation which is the given problem. Finally, a graphical representation will be made to verify the solution.
- The **Communicate Your Understanding** questions can be used to assess students' understanding of the nature of exponential equations and the various methods of solving them. Consider assigning these questions in pairs or small groups to facilitate discussion of the concepts before having students transcribe their own individual responses.
- Graphing technology can be used to solve and/or check **questions 1** through **3**. **Question 2** could also be checked using the classical method of substituting the root obtained into the left side and right side of the equation and verifying equality.

- **Questions 6** and **7** are simpler versions of **Example 3**, with additional scaffolding. When helping students, the discussion should come up as to why it is not necessary (or desirable) to multiply both sides of these equations by a common factor as was done in **Example 3**.
- Students may choose to solve **questions 8** and **9** using various tools and strategies (e.g., algebraic reasoning, graphical analysis using technology). Encourage the comparison of various strategies.
- **Question 9** allows students to make mathematical connections to determine the half-life of bismuth-214 and to produce a graphical representation of the amount of bismuth-214 versus time. It will then be necessary to use reasoning skills to determine the effect of changes to the half-life and to the initial sample size and to communicate the conclusions found.
- For **question 10**, students will need to recognize a quadratic equation, its coefficients, and also the significance of how the discriminant of the quadratic formula is related to the nature of the roots of the corresponding quadratic equation. Some review of these previously learned concepts may be required.
- **Question 11** requires students to perform some analysis and algebraic reasoning to solve this diverse collection of equations. Encourage students to try to identify if an equation can be written as a quadratic equation, and if so, what steps need to be applied in each case to express it in standard form.
- Regarding the "get-rich-quick" scheme in **question 13**, platinum is very expensive itself and this particular isotope may not be readily available, especially given its relatively short half-life.
- **Question 14** provides an opportunity to draw connections with concepts in the MDM4U Data Management course, which some students may be taking.
- Students may choose to solve question 15 using various tools and strategies (e.g., algebraic reasoning, graphical analysis using technology). Encourage the comparison of various strategies.
- For **question 18**, encourage students to consider the concept of conservation of mass, so that as one substance (i.e., platinum) disappears, an equal mass of another substance (i.e., gold) is created. There actually is a slight decrease in mass as the fission process occurs, as a tiny amount of mass is converted to a significant amount of energy, but the difference in mass is negligible for this level of analysis. Students who go on to study nuclear physics at university will explore these concepts in greater depth.
- For **question 19**, go to *www.mcgrawhill.ca/books/functions* and follow the links for information about the carbon fusion cycle.
- Use **BLM 7–3 Section 7.2 Practice** for remediation or extra practice.

Differentiated Instruction

Use **Think-Pair-Share** to teach this section.

Common Errors

- Students perform algebraic errors or get stuck in the middle of a solution without being able to complete it.

R_x Often this is due to weak skills in algebraic manipulation within previously learned concepts, such as collecting like terms, factoring, and applying techniques to solve a quadratic equation. Sometimes confusion is increased when variables and expressions involve exponentials and logarithms. In addition to reviewing algebraic basics as needed, it may be helpful to use variable substitutions that simplify the appearance of an equation, with a reverse substitution applied after the mechanical operations are performed. See Example 3, Method 2 for an illustration of this technique.

Ongoing Assessment

Achievement Check, question 17, on student text page 377.

Communicate Your Understanding Responses (page 374)

C1. a) Answers may vary. Sample answer:
$A(t)$ is an exponential function that decreases by a factor that contains the fraction $\frac{1}{2}$.

b) Answers may vary. Sample answer:
The amount of radioactive material $A(t)$ decreases by $\frac{1}{2}$ during the half-life of the material.

C2. a)–b)

$$
\begin{aligned}
5^{3x} &= 4^{x+1} \\
\log 5^{3x} &= \log 4^{x+1} \\
3x\log 5 &= (x+1)\log 4 && \text{Apply the power law of logarithms.} \\
3x\log 5 &= x\log 4 + \log 4 && \text{Apply the distributive property.} \\
3x\log 5 - x\log 4 &= \log 4 && \text{Collect variable terms on one side of the equation.} \\
x(3\log 5 - \log 4) &= \log 4 && \text{Factor } x \text{ on the left side.} \\
x &= \frac{\log 4}{3\log 5 - \log 4}
\end{aligned}
$$

C3. No. Answers may vary. Sample answer: The power law of logarithms cannot be applied.

C4. a) An extraneous root is an invalid solution.

b) Answers may vary. Sample answer: The roots must be checked to ensure that they are valid solutions to the original equation.

c) Answers may vary.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	16, 19
Reasoning and Proving	6, 7, 9, 11, 13–19
Reflecting	16, 17, 19
Selecting Tools and Computational Strategies	1, 2, 4–11, 14, 17, 19
Connecting	3, 4, 6–11, 13–19
Representing	8, 9, 12, 16, 17
Communicating	3, 9, 10, 13, 16, 19

Achievement Check, question 17, student text page 377

This performance task is designed to assess the specific expectations covered in section 7.2.
The following Math Process Expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

a) $N = 50(2)^t$

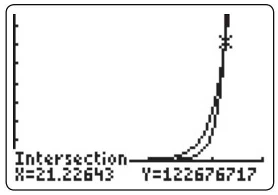
b) $N = 20(3)^{t_2}$, where $t_2 = t - 7$.

c)

$$
\begin{aligned}
50(2)^t &= 20(3)^{t-7} \\
2.5 &= \frac{3^{t-7}}{2^t} \\
5467.5 &= 1.5^t \\
\log 5467.5 &= \log 1.5^t \\
\log 54676.5 &= t\log 1.5 \\
t &= \frac{\log 5467.5}{\log 1.5} \\
t &\doteq 21.2
\end{aligned}
$$

After a little more than 21 days, the number of people who have seen the video is the same number as those who know it was a hoax. (These do not have to be the same people.)

d) It is difficult to get an accurate graphical solution as the numbers are so large. When the expressions are equal, 122 670 000 people have seen the video and the same number have read that it is a hoax.



Level 3 Notes

Look for the following:

- Appropriate equations to model the situations
- An acknowledgment of the relationship between the two times represented
- Some method for solving when the two expressions are equal

What Distinguishes Level 2

- Needing considerable assistance to produce appropriate equations to model the situations
- Using the same variable for the two times represented
- Inaccurately solving when the two expressions are equal

What Distinguishes Level 4

- Appropriate equations to model the situations with some explanation
- An acknowledgment of the relationship between the two times represented
- Both methods for fully solving when the two expressions are equal