7.3 Product and Quotient Laws of Logarithms

Student Text Pages
378 to 386

Suggested Timing
75–150 min

Tools
- computer
- *The Geometer's Sketchpad®*
- graphing calculator
- computer algebra system

Related Resources
- T–2 *The Geometer's Sketchpad®* 4
- T–4 The Computer Algebra System (CAS) on the TI-89 Calculator
- BLM 7–4 Investigate A: Graphs of Common Logarithms of Products
- BLM 7–5 Section 7.3 Practice

Teaching Suggestions

- The questions posed in the **Introduction** can be assigned as a minds-on activation of prior knowledge, using a Think-Pair-Share or mind-mapping format. The exponent laws are summarized below:

  Product Law
  $a^m \times a^n = a^{m+n}$

  Quotient Law
  $a^m \div a^n = a^{m-n}$

  Power Law
  $(a^m)^n = a^{mn}$

  Power of a Product Law
  $(ab)^m = a^m b^m$

  Power of a Quotient Law
  $(a \div b)^m = a^m \div b^m, b \neq 0$
- The purpose of the **Investigate** is for students to discover the product and quotient laws of logarithms. Graphing technology is recommended as students can visually connect their prior understanding of transformations of logarithmic functions to develop the product and quotient laws. Note that TI Interactive!™ could also be used for this activity, if available. Depending on the technology chosen for the **Investigate**, use **T–2 *The Geometer's Sketchpad®* 4** or **T–4 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this activity.
- Refer to **BLM 7–4 Investigate A: Graphs of Common Logarithms of Products** when taking up Part A of the Investigation.
- Depending on the progress of the class, you may wish to divide the lesson into two periods, as follows:
  - Day 1: Complete the **Investigate**, followed by either **Example 2** or **Example 3**. Then assign only **questions 1** through **6** or **questions 1** through **8**, depending on the break point that is chosen.
  - Day 2: Complete the remaining examples and assign the balance of the exercise questions. Parts A and B, which focus on the product law, are fairly guided and encourage students to examine patterns of their results and apply inductive reasoning. Part C, which focuses on the quotient law, is deliberately less scaffolded, requiring students to choose their own tools and strategies.
- As students apply the product and quotient rules of logarithms to simplify numerical and algebraic expressions, as in **Example 1** and **Example 2**, have them take note of the proper use of brackets, and nested brackets, as needed, and also to be aware of the importance of identifying restrictions on the variables.
- In **Example 3**, students apply the product and quotient laws in reverse. It should be noted, as in part b), that sometimes this can be done in more than one way; note that the second method leads to a simpler result than the first.
- **Example 4** requires students to apply a high level of algebraic reasoning. Students may need some review of exponent laws (for parts a) and b)) and factoring to simplify a rational expression (part c)). Students should also carefully consider the reasoning at the conclusion of part c) determine the restrictions on x.

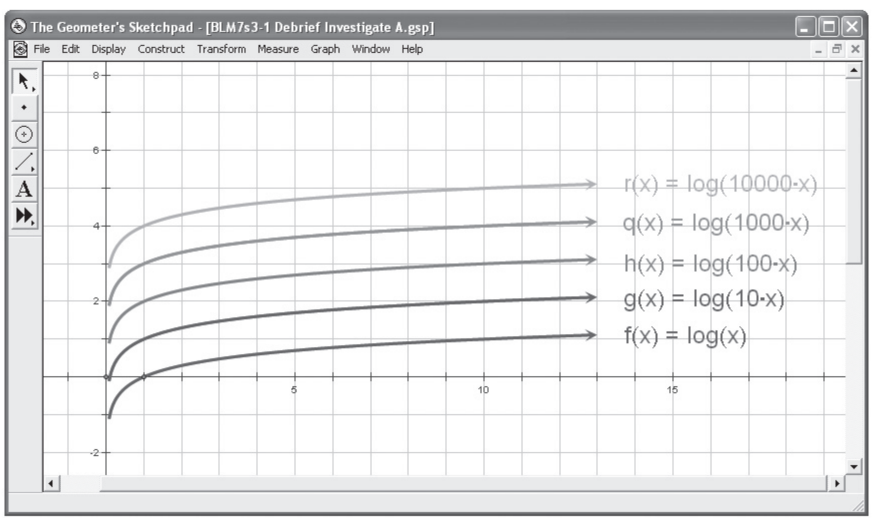
- The **Communicate Your Understanding** questions can be used to assess students' understanding of the product and quotient laws of logarithms. Knowledge of the domain and range of logarithmic functions is also required for the questions that focus on the restrictions on the variables.
- A computer algebra system (CAS) can be used to check **questions 1** through **10**.
- **Question 12** gives students an opportunity to reflect upon how transformations can be used to produce $g(x) = \log(10nx)$ from $f(x) = \log x$. Reasoning and connecting skills will be used to create the two examples required, and an explanation of the examples involves communication skills. Graphical representations will then be needed to illustrate the two examples.
- Students can check their answers to **questions 12** and **13** using graphing technology.
- **Questions 14** and **15** have students connect transformations to the power law of logarithms.
- **Question 15** requires students to reflect on the effectiveness of strategies that could be used to establish a change in the function. Reasoning and connecting skills will also be needed to determine how to make the changes. Graphical representations will be produced and since explanations are required, communicating skills will be involved.
- Encourage students to consider a variety of tools and strategies for **question 16**. When discussing this question, have students share different methods that they used.
- **Question 19** encapsulates many of the key concepts related to the laws of logarithms. Have students review these, as needed.
- **Questions 20** and **21** provide opportunities to assess students' reasoning and communicating skills as well as their ability to select tools and strategies effectively.
- Use **BLM 7–5 Section 7.3 Practice** for remediation or extra practice.

Investigate Answers (pages 378–379)

A:

1. a) 0 **b)** 1 **c)** 2 **d)** 3 **e)** 4

2. a)



b) Answers may vary. Sample answer: There is a vertical translation of 1 unit up for each graph. i.e., $g(x) = f(x) + 1$, $h(x) = g(x) + 1$, $i(x) = h(x) + 1$, and $j(x) = i(x) + 1$

3. a)

Function	Vertical Translation of $f(x)$	Sum of Common Logarithms
$g(x) = \log(10x)$	$g(x) = \log x + 1$	$g(x) = \log x + \log 10$
$h(x) = \log(100x)$	$h(x) = \log x + 2$	$h(x) = \log x + \log 100$
$i(x) = \log(1000x)$	$i(x) = \log x + 3$	$i(x) = \log x + \log 1000$
$j(x) = \log(10\,000x)$	$j(x) = \log x + 4$	$j(x) = \log x + \log 10\,000$

b) $\log(a \times b) = \log a + \log b$

B:

1. Note: Answers may vary for the bottom two rows.

a	b	$\log_2 a$	$\log_2 b$	$\log_2 a + \log_2 b$	$\log_2 (a \times b)$
1	2	$\log_2 1 = 0$	$\log_2 2 = 1$	$0 + 1 = 1$	$\log_2(1 \times 2)$ $= \log_2 2$ $= 1$
2	4	$\log_2 2 = 1$	$\log_2 4 = 2$	$1 + 2 = 3$	$\log_2(2 \times 4)$ $= \log_2 8$ $= 3$
4	4	$\log_2 4 = 2$	$\log_2 4 = 2$	$2 + 2 = 4$	$\log_2(4 \times 4)$ $= \log_2 16$ $= 4$
8	16	$\log_2 8 = 3$	$\log_2 16 = 4$	$3 + 4 = 7$	$\log_2(8 \times 16)$ $= \log_2 128$ $= 7$
16	32	$\log_2 16 = 4$	$\log_2 32 = 5$	$4 + 5 = 9$	$\log_2(16 \times 32)$ $= \log_2 512$ $= 9$
2	64	$\log_2 2 = 1$	$\log_2 64 = 6$	$1 + 6 = 7$	$\log_2(2 \times 64)$ $= \log_2 128$ $= 7$

2. $\log_2 (a \times b) = \log_2 a + \log_2 b$

3. $\log_b (m \times n) = \log_b m + \log_b n$

4. Answers may vary. Prediction is correct.

5. a) i) $\log_b (m \times n) = \log_b m + \log_b n$

ii) The logarithm of a product with any base, b, can be written as the sum of the logarithms of each factor, m and n, with the same base.

b) i) $b > 0, b \neq 1, m > 0, n > 0$

ii) b is greater than 0 and does not equal 1, m and n are greater than 0.

C:

1. Yes. Answers may vary.

2. Answers may vary. Sample answer:

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$, for $b > 0, b \neq 1, m > 0, n > 0$

The logarithm of a quotient with any base, b, can be written as the difference between the logarithm of the dividend, m, and the logarithm of the divisor, n.

Examples:

$$\log_3 15 - \log_3 5 = \log_3\left(\frac{15}{5}\right) = \log_3 3 = 1$$

$$\log_4 16384 - \log_4 1024 = \log_4\left(\frac{16\,384}{1024}\right) = \log_4 16 = 2$$

$$\log 200 - \log 2 = \log\left(\frac{200}{2}\right) = \log 100 = 2$$

Differentiated Instruction

Use **graffiti** to reinforce this section.

Use a **Frayer model** to summarize this section.

Common Errors

- Students misapply the product and quotient laws, such as in question C2.

R_x Have students consider the concept from the point of view of transformations, as in the Investigate, to determine whether or not two expressions involving logarithmic functions are equal.

Ongoing Assessment

Achievement Check, question 19, on student text page 386.

Communicate Your Understanding Responses (page 384)

C1. No. Answers may vary. Sample answer:
No, because $\log x + \log y = \log(xy)$ by the product rule of logarithms.

C2. No. Answers may vary. Sample answer:
No, because $\log\left(\frac{a}{b}\right) = \log a - \log b$ by the quotient rule of logarithms.

C3. Answers may vary. Sample answer: So that all the logarithmic expressions in the equation are defined.

C4. Answers may vary.

C5. Answers may vary. Sample answer: $10^y > 0$, 10^y cannot be negative.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	16
Reasoning and Proving	8, 9, 12, 15–17, 20, 21
Reflecting	8, 12, 15
Selecting Tools and Computational Strategies	1, 3–7, 9, 16, 18, 20, 21
Connecting	1–9, 11, 12, 15, 17, 18, 20, 21
Representing	5, 11, 12, 14–16
Communicating	8, 12–16, 18

Achievement Check, question 19, student text page 386

This performance task is designed to assess the specific expectations covered in Section 7.3. The following Math Process Expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

a) $$\begin{aligned}\log_4 192 - \log_4 3 &= \log_4 \frac{192}{3}\\ &= \log_4 64\\ &= 3\end{aligned}$$

b) $$\begin{aligned}\log_5 35 - \log_5 7 + \log_5 25 &= \log_5\left(\frac{35 \times 25}{7}\right)\\ &= \log_5 125\\ &= 3\end{aligned}$$

c) $$\begin{aligned}\log_a(ab) - \log_a(a^3b) &= \log_a\left(\frac{ab}{a^3b}\right)\\ &= \log_a a^{-2}\\ &= -2\end{aligned}$$

d) $$\begin{aligned}\log(xy) + \log\frac{y}{x} &= \log\left(xy\left(\frac{y}{x}\right)\right)\\ &= \log y^2\\ &= 2\log y,\ y > 0\end{aligned}$$

Level 3 Notes

Look for the following:

- Accurate and complete solutions with few minor errors
- Solutions in simplest form

What Distinguishes Level 2

- Complete solutions with some minor errors
- Most of the solutions in simplest form

What Distinguishes Level 4

- Accurate and complete solutions with no errors
- Solutions in simplest form