7.4

Techniques for Solving Logarithmic Equations

Student Text Pages
387 to 392

Suggested Timing
75 min

Tools
- grid paper
- graphing calculator

Related Resources
- G–1 Grid Paper
- BLM 7–6 Section 7.4 Practice

Teaching Suggestions

- The purpose of the **Investigate** is to prompt students to think about how an equation involving one or more logarithms could be solved. Most students should be able to obtain at least an approximate solution to the equation given by graphing the left side and right side as separate functions and identifying their point of intersection using graphing technology. Because of the functional nature of a logarithmic relationship, one can equate the two algebraic expressions that are obtained at the end of step 2, as prompted in step 3. This algebraic strategy that is more fully developed in the examples that follow.
- In **Example 1**, part a), both algebraic (Method 1) and graphical (Method 2) reasoning are used to solve a simple equation involving a logarithm. In the former, the equation is rewritten in exponential form and then solved using basic algebra. In the latter, a transformation and intersection of functions approach verifies the algebraically derived result.
- In **Example 1**, part b), the equation is rewritten so that both sides are a logarithm of the same base. Due to the functional nature of the logarithm (one and only one input leading to a unique output), the arguments of the two logarithmic expressions are therefore set equal, which leads to the solution.
- In **Example 2**, previously learned algebraic concepts—such as the product and power laws of logarithms, solving a quadratic equation by factoring, and checking for extraneous roots—are applied. Some students may require review of such concepts as they arise, and a computer algebra system (CAS) may help to focus attention on the algebraic reasoning, as opposed to the algebraic mechanics of each step. On the other hand, problems such as these provide a good opportunity for review and remediation of these important skills. Depending on the needs of the students, you may decide to use a CAS during the solution, or afterward as a means of checking. Also, when assigning the exercise questions, the relative use of technology support should be carefully considered based on students' needs.
- Use the **Communicate Your Understanding** to assess students' readiness to answer the questions that follow. They are designed to help determine whether students understand the techniques illustrated in the examples. Consider assigning these to pairs of students, and encourage them to reflect on the examples before discussing and transcribing individual responses.
- Students can also check their answers to **questions 1** through **4** by substituting roots into the left side and the right side of each equation and verifying equality.
- For **question 5**, suggest that students write the radicals in exponential form and then apply the power law of logarithms.
- For **question 6**, suggest that students think about how the purely numeric terms could be written as a logarithm having the same base as the other logarithms in each equation, and then apply product or quotient laws to simplify.
- Students can also check their answers to **questions 5** through **7** by substituting roots into the left and the right side of each equation and verifying equality.
- An optional strategy for **question 7** is to have students solve the equation $\log(x + 2) - 2 + \log(x) = 0$ instead of $\log(x + 2) = 2 - \log(x)$. On the graphing calculator enter $Y1 = \log(x + 2) - 2 + \log(x)$, and then use the **Zero** operation to find the solution.
- Students may wish to review the decibel scale, which was introduced in Section 6.5, when answering **question 8**.

Differentiated Instruction

Use **what–so what double entry** to teach this section.

Common Errors

- Students have difficulty writing numbers in the form of a logarithm (e.g., $2 = \log 100$), or recognizing when it is advantageous to do so.

R_x Consider using one or two of the exercise questions as additional partially led worked examples (e.g., question 1, part b). Have students engage in Think-Pair-Share to outline and then carry out the sequence of steps. Repeat this type of approach, gradually reducing the scaffolding, until students have acquired confidence with these skills.

- For **question 9**, encourage students to reflect on the domain of a logarithmic function, as well as the laws of logarithms. A transformational approach, using graphing technology for part b), is another possibility.
- Students who are interested in photography or who are planning to study optics in university should find **question 10** particularly interesting.
- For **question 11**, students can refer to Example 2, part b), for support.
- **Question 11** allows students to select tools and to apply reasoning and connecting skills to solve the equation algebraically for any extraneous roots. The second method of solving involves making a graphical representation to verify that the two solutions match.
- Multiple strategies could be used for **question 12** (e.g., algebraic, graphical, systematic trial). Encourage sharing and comparing of these.
- **Question 13** allows students to select tools and to apply reasoning and connecting skills to show that the statement is true. This question requires a deep level of understanding of logarithms; encourage students to consider a variety of strategies, such as variable elimination, rewriting logarithmic equations in exponential form, etc.
- Use **BLM 7–6 Section 7.4 Practice** for remediation or extra practice.

Investigate Answers (page 387)

1. Answers may vary. Sample answer:

a) Graph the left side of the equation and the right side of the equation. The x-coordinate of the point of intersection is the solution to the given equation.

b) $x = 4$

2. a) $\log(x + 5) = \log(x - 1)^2$

b) $\log(x + 5) = \log(x^2 - 2x + 1)$

3. a) They are equal when $x = 4$.

b) Answers may vary. Sample answer: It allows you to equate the arguments and solve for x.

Communicate Your Understanding Responses (page 391)

C1. a) All expressions are equivalent.

b) Use $\log_4 16$, because it has the same base as the expression on the left side.

C2. a) $5\log(x - 3) = 2$

b) The solution is $x \doteq 5.5$, the x-coordinate of the point of intersection.

C3. Answers may vary. Sample answer:
Disagree. Counterexample: see Section 7.4, Example 2, part b).

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	13
Reasoning and Proving	3, 6, 9–13
Reflecting	9, 13
Selecting Tools and Computational Strategies	4, 5, 7, 11, 13
Connecting	1–3, 5, 6, 8–13
Representing	4, 5, 7, 11
Communicating	9, 12