7.5 Making Connections: Mathematical Modelling With Exponential and Logarithmic Equations

Student Text Pages
393 to 407

Suggested Timing
75 min

Tools
- graphing calculator
- computer
- *Fathom*™
- *The Geometer's Sketchpad*®

Related Resources
- T–1 Microsoft® Excel
- T–2 *The Geometer's Sketchpad*® 4
- T–3 *Fathom*™
- BLM 7–7 Investigate the Population Growth of Decimal Point
- BLM 7–8 Section 7.5 Practice
- BLM 7–9 Section 7.5 Achievement Check Rubric

Teaching Suggestions

- The focus of this section is on mathematical modelling with exponential and logarithmic functions. It is important for students to see that sometimes more than one model can be determined that fit a set of points well. However, a superior model will be more reliable when extrapolating beyond the given data. Accordingly, a good model will be based on both mathematical and contextual reasoning.
- **Example 1** could be done as an investigation. Refer to **BLM 7–7 Investigate the Population Growth of Decimal Point** for this option.
- In **Example 1**, Method 1, students use graphing technology to compare two possible models for population growth. While a quadratic regression yields a well-fit curve, the model must be rejected as it does not represent continuous growth, which is revealed upon deeper analysis of the curve. The exponential model makes sense in this context because it generates a function that represents continuous growth. If students are using graphing calculators, a brief discussion of non-linear regression may be helpful. **Example 1**, Method 2, uses sliders in *Fathom*™ to approximate the curve of best fit (*Fathom*™ will not perform non-linear regression). Use **T–3 *Fathom*™** to support this method. If available, Microsoft® *Excel* can be used to perform non-linear regression by using the Trendline feature. Use **T–1 Microsoft® *Excel*** to support this activity. If students are able to explore both methods of solution, they should notice that the graphing calculator and *Fathom*™ solutions each give different looking equations for part c). However, the calculations done in parallel in part e) confirm to some extent that these are essentially different forms of the same relationship; students should recall that any given exponential relation can be expressed using any base.
- Technology tip for **Example 1**, Method 1:
  - To enter the data quickly in list **L1**, scroll to the top of **L1**, press [2nd] [LIST], cursor over to the **OPS** menu, and select **5:seq**(. Type x,x,0,30,5) which will enter the data starting from 0 to 30 in increments of 5.
- In **Example 2**, *The Geometer's Sketchpad*® is the tool of choice to explore and compare various compound interest scenarios. While other tools could be used (e.g., graphing calculator, *Fathom*™), it is valuable for students to be exposed to a variety of tools. In part d), students should make the connection between the algebraic representation of the penalty factor and a vertical shift of the graphical model. The problem wraps up with a discussion that takes the level of reasoning beyond the pure mathematical analysis. As in real life, students should recognize that mathematics can be thought of as one set of reasoning tools that complements and is applied in conjunction with other contextual factors. Use **T–2 *The Geometer's Sketchpad*® 4** to support this activity.
- **Example 2** requires students to reflect on the effectiveness of different strategies that may be used to solve the problem of constructing two algebraic models. They will have to make connections with mathematical concepts learned previously to construct the algebraic models and to compare the two investment options. They will have to apply reasoning skills to answer questions related to the models; they will use their representing skills to illustrate these investments graphically. It will then be necessary for them to communicate their understanding of the results found.

- The **Communicate Your Understanding** questions provide opportunities for students to reflect on the worked examples with a critical eye toward evaluating the strength and limitations of a mathematical model. Consider having students discuss these questions in pairs or small groups before transcribing individual responses and sharing with the class.
- **Questions 1** and **2** require students to extrapolate forward and backward in time. It may be interesting for students to note that according to the rejected quadratic model, the population of the town never was at 100, and in fact was never less than 500!
- For **question 3**, students could graph both functions on the same grid, using technology, and see how close to each other the two models are.
- **Question 4** gives students the opportunity to reflect upon and reason through how waiving the early withdrawal penalty would affect the town's investment decision. Making mathematical connections will be necessary to come to a conclusion, and then communicating the information found is required.
- **Questions 4** and **5** could be solved using a variety of tools and strategies (e.g., algebraically, graphing technology). Encourage students to pursue and share different methods of solution.
- For **question 6**, have students consider why each model would or would not make sense, from a geometric perspective. They may need to recall that $A = \pi r^2$ for a circle. Limitations of the model might include the finite source of oil for the spill and how the oil was spilled.
- **Question 9** allows students to reflect upon a prediction and apply algebraic reasoning to determine the best model that should be used for this situation of surface area versus time. Selecting tools will be required and connecting previously learned mathematical concepts will be needed to create a scatter plot and use the model to answer the required questions. Finally, communicating skills will be necessary to describe the best model to be used and any assumptions made.
- A temperature probe and a graphing calculator are recommended for **question 9**, if available. If not available, then a thermometer and clock can suffice. An optional strategy for collecting data for a cooling liquid uses a CBL™ (Calculator Based Laboratory) with CHEM/BIO or CHEM/PHYSICS App program for the TI-83 Plus/ TI-84 Plus and a Temperature Probe or using Vernier LabPro® technology.
- For **question 10**, students may wish to refer back to the decibel scale that was introduced in Section 6.5.
- For the regression work in **question 11**, part b), students will need to use either a graphing calculator or software such as Microsoft® *Excel*.
- For **questions 12** through **14**, students will need to use either a graphing calculator or software such as Microsoft® *Excel*. Internet access is also recommended.
- Use **BLM 7–8 Section 7.5 Practice** for remediation or extra practice.

Communicate Your Understanding Responses (page 404)

C1. a) Answers may vary. Sample answer:
When extrapolating back in time, an anomaly occurs (larger population than at year 0).

b) Yes.

c) Answers may vary. Sample answer:
No, when extrapolating back in time an anomaly occurs (negative population).

C2. Answers may vary.

C3. $A = 50\ 000(1.0625)^t + 0.02(50\ 000),\ t > 10$

DIFFERENTIATED INSTRUCTION

Use **blastoff** to introduce this section.

Use **graffiti** to teach this section.

Use **Think-Pair-Share** to reinforce this section.

ONGOING ASSESSMENT

Achievement Check, question 11, on student text page 406.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	12, 13
Reasoning and Proving	3–7, 9–14
Reflecting	3, 5, 12–14
Selecting Tools and Computational Strategies	1, 2, 4, 5, 10, 12
Connecting	3, 6, 7, 9–13
Representing	1, 6–9, 11
Communicating	3–6, 8–14

Achievement Check, question 11, student text page 406

This performance task is designed to assess the specific expectations covered in Section 7.5.

The following Math Process Expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

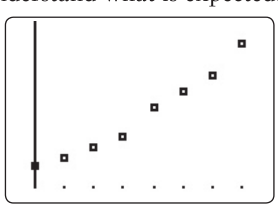
The following Math Process Expectations can be assessed.

Achievement Chart Category	Related Math Processes
Knowledge and Understanding	Selecting tools and computational strategies
Thinking	Problem solving Reasoning and proving Reflecting
Communication	Communicating, Representing
Application	Selecting tools and computational strategies Connecting

Sample Solution

Provide students with **BLM 7-9 Section 7.5 Achievement Check Rubric** to help them understand what is expected.

a)



b) **i)** $y \doteq 4.87x + 12.8$ is the linear regression.

ii) $y \doteq 0.4x^2 + 2.1x + 15.6$ is the quadratic regression.

iii) $y \doteq 15.4(1.18)^x$ is the exponential regression.

c) The linear is the worst fit at the beginning. The quadratic turns back up three months prior, which is unrealistic. The exponential is best since it will never go negative or increase as you move back in time.

d) The exponential model will reach 100 just after 11 months.

e) This trend will not continue indefinitely as a scarcity of food or living space would eventually limit population growth.