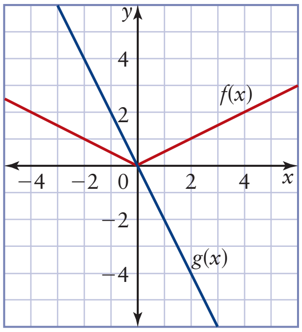
Chapter 8 REVIEW

8.1 Sums and Differences of Functions

1. a) Copy the graph.



b) Use the superposition principle to generate a graph of each function.

i) $y = f(x) + g(x)$

ii) $y = f(x) - g(x)$

iii) $y = g(x) - f(x)$

2. Let $f(x) = x - 2$, $g(x) = x^2 + 3x - 3$, and $h(x) = 2^x$. Determine an algebraic and a graphical model for each combined function. Identify the domain and range in each case.

a) $y = f(x) + g(x)$

b) $y = f(x) + g(x) + h(x)$

c) $y = f(x) - h(x)$

3. Use Technology Use graphing technology to check your answers to question 2.

4. Max can earn \$6/h as a waiter, plus an additional \$9/h in tips.

a) Graph Max's earnings from wages as a function of hours worked.

b) Graph Max's earnings from tips as a function of hours worked.

c) Develop an algebraic and a graphical model for Max's total earnings.

d) How much can Max earn if he works 52 h in one week?

8.2 Products and Quotients of Functions

5. Let $u(x) = x^2$ and $v(x) = \cos x$. Work in radians.

a) What type of symmetry do you predict the combined function $y = u(x)v(x)$ will have? Explain your reasoning.

b) Use Technology Use graphing technology to check your prediction.

6. Let $f(x) = \sin x$ and $g(x) = \cos x$.

a) Graph $f(x)$ and $g(x)$ on the same set of axes.

b) Sketch a graph of the combined function $y = \frac{f(x)}{g(x)}$.

c) Identify the domain and range of this function.

d) Use your understanding of trigonometric identities to identify the graph of $y = \frac{f(x)}{g(x)}$.

7. Refer to question 6.

a) Graph the combined function $y = \frac{g(x)}{f(x)}$.

b) Identify the domain and range of this function.

c) How is $y = \frac{g(x)}{f(x)}$ related to $y = \frac{f(x)}{g(x)}$ in terms of transformations?

8.3 Composite Functions

8. Let $f(x) = x^2 + 3x$ and $g(x) = 2x - 5$. Determine an equation for each composite function, graph the function, and give its domain and range.

a) $y = f(g(x))$

b) $y = g(f(x))$

c) $y = g(g(x))$

d) $y = g^{-1}(g(x))$

9. Assume that a function $f(x)$ and its inverse $f^{-1}(x)$ are both defined for $x \in \mathbb{R}$.

a) Give a geometric interpretation of the composite function $y = f(f^{-1}(x))$.

b) Illustrate your answer to part a) with two examples.

8.4 Inequalities of Combined Functions

10. Let $f(x) = 1.2^x$ and $g(x) = 0.92^x + 5$.

a) Identify the region for which

i) $f(x) > g(x)$

ii) $g(x) > f(x)$

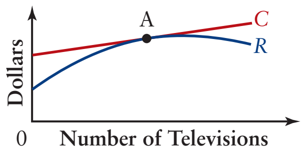
b) Illustrate this inequality graphically in two different ways.

11. Refer to question 10.

a) Write a real-world scenario that these functions could model.

b) Pose and solve two problems based on your scenario.

12. The cost, C, and revenue, R, as functions of the number of televisions sold by an electronics store are shown on the graph.



a) Identify the region(s) for which

i) $C > R$

ii) $R > C$

b) What can you conclude about this business venture?

c) What suggestions would you give to the store owner in order to help him or her improve the situation?

8.5 Making Connections: Modelling with Combined Functions

Refer to the chromatic music scale on page 463.

13. A D-minor chord is formed by striking the following notes together:

D F A high D

a) Double the frequency of D in the table to determine the frequency of high D.

b) Graph the combined function formed by these four notes. Describe the waveform.

CONNECTIONS

Minor chords tend to have a sad sound to them. They combine with major chords (which sound happier) to create musical tension.

14. Experiment with various note combinations from the chromatic scale.

a) Identify two chords that you think would make a good sound.

b) Identify two chords that you think would make a discordant (unpleasant) sound.

c) Use mathematical reasoning to justify your choices. Then, test your theories using a well-tuned guitar or piano. You may need to do a little research to identify the correct notes.

CHAPTER

PROBLEM WRAP-UP

The number, S, in thousands, of Funky Teddy Bears that can be supplied by Funky Stuff as a function of price, p, in dollars, can be modelled by the function $S(p) = p + 3$.

The demand, D, for the bears can be modelled by the function $D(p) = -0.1(p + 8)(p - 12)$.

a) For what interval is $D(p) > S(p)$? What does this imply about the availability of Funky Teddy Bears?

b) For what interval is $D(p) < S(p)$? What does this imply about the availability of Funky Teddy Bears?

c) Graph these functions on the same set of axes. Identify their point of intersection. Explain what the coordinates of this point mean.

d) Graph the function $y = S(p) - D(p)$ and explain what it shows.