Chapter **8**

Combining Functions

Curriculum Expectations

Characteristics of Functions

Combing Functions

D2.1 determine, through investigation using graphing technology, key features (e.g., domain, range, maximum/minimum points, number of zeros) of the graphs of functions created by adding, subtracting, multiplying, or dividing functions [e.g., $f(x) = 2^{-x} \sin 4x$, $g(x) = x^2 + 2^x$, $h(x) = \frac{\sin x}{\cos x}$], and describe factors that affect these properties

D2.2 recognize real-world applications of combinations of functions (e.g., the motion of a damped pendulum can be represented by a function that is the product of a trigonometric function and an exponential function; the frequencies of tones associated with the numbers on a telephone involve the addition of two trigonometric functions), and solve related problems graphically

D2.3 determine, through investigation, and explain some properties (i.e., odd, even, or neither; increasing/ decreasing behaviours) of functions formed by adding, subtracting, multiplying, and dividing general functions [e.g., $f(x) + g(x)$, $f(x)g(x)$]

D2.4 determine the composition of two functions [i.e., $f(g(x))$] numerically (i.e., by using a table of values) and graphically, with technology, for functions represented in a variety of ways (e.g., function machines, graphs, equations), and interpret the composition of two functions in real-world applications

D2.5 determine algebraically the composition of two functions [i.e., $f(g(x))$], verify that $f(g(x))$ is not always equal to $g(f(x))$ [e.g., by determining $f(g(x))$ and $g(f(x))$, given $f(x) = x + 1$ and $g(x) = 2x$], and state the domain [i.e., by defining $f(g(x))$ for those x-values for which $g(x)$ is defined and for which it is included in the domain of $f(x)$] and the range of the composition of two functions

D2.6 solve problems involving the composition of two functions, including problems arising from real-world applications

D2.7 demonstrate, by giving examples for functions represented in a variety of ways (e.g., function machines, graphs, equations), the property that the composition of a function and its inverse function maps a number onto itself [i.e., $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$ demonstrate that the inverse function is the reverse process of the original function and that it undoes what the function does]

D2.8 make connections, through investigation using technology, between transformations (i.e., vertical and horizontal translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes) of simple functions $f(x)$ [e.g., $f(x) = x^3 + 20$, $f(x) = \sin x$, $f(x) = \log x$] and the composition of these functions with a linear function of the form $g(x) = A(x + B)$

Using Function Models to Solve Problems

D3.1 compare, through investigation using a variety of tools and strategies (e.g., graphing with technology; comparing algebraic representations; comparing finite differences in tables of values) the characteristics (e.g., key features of the graphs, forms of the equations) of various functions (i.e., polynomial, rational, trigonometric, exponential, logarithmic)

D3.2 solve graphically and numerically equations and inequalities whose solutions are not accessible by standard algebraic techniques

D3.3 solve problems, using a variety of tools and strategies, including problems arising from real-world applications, by reasoning with functions and by applying concepts and procedures involving functions (e.g., by constructing a function model from data, using the model to determine mathematical results, and interpreting and communicating the results within the context of the problem)

Chapter 8 Planning Chart

Section Suggested Timing	Student Text Page(s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 8 Opener • 10 min	413			
Prerequisite Skills • 60–75 min	414–415	• G–1 Grid Paper • G–3 Four Quadrant Grids • BLM 8–1 Prerequisite Skills		• linking cubes or colour tiles • grid paper • graphing calculator
8.1 Sums and Differences of Functions • 75 min	416–428	• G–3 Four Quadrant Grids • T–2 *The Geometer's Sketchpad®* 4 • BLM 8–2 Section 8.1 Practice		• coloured tiles or linking cubes • graphing calculator or graphing software • grid paper
8.2 Products and Quotients of Functions • 75–150 min	429–438	• G–1 Grid Paper • G–3 Four Quadrant Grids • BLM 8–3 Section 8.2 Practice		• grid paper • graphing calculator
8.3 Composite Functions • 75–150 min	439–449	• G–1 Grid Paper • BLM 8–4 Section 8.3 Practice		• graphing calculator or graphing software • grid paper
8.4 Inequalities of Combined Functions • 75 min	450–460	• G–1 Grid Paper • BLM 8–5 Section 8.4 Practice	• BLM 8–6 Section 8.4 Achievement Check Rubric	• grid paper • graphing calculator
8.5 Making Connections: Modelling With Combined Functions • 75 min	461–471	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad®* 4 • T–3 *Fathom™* • T–5 Using the CBR™ (Calculator Based Ranger) • BLM 8–7 Section 8.5 Practice		• graphing calculator • computer • *Fathom™* • *The Geometer's Sketchpad®* • grid paper
Chapter 8 Review • 60–75 min	472–473	• G–1 Grid Paper • BLM 8–8 Chapter 8 Review		• grid paper • graphing calculator
Chapter 8 Problem Wrap-Up • 20–30 min	473	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad®* 4 • T–3 Fathom™	• BLM 8–9 Chapter 8 Problem Wrap-Up Rubric	• grid paper • graphing calculator • computer • *The Geometer's Sketchpad®* • *Fathom™*
Chapter 8 Practice Test • 60–75 min	474–475	• G–1 Grid Paper	• BLM 8–10 Chapter 8 Test	• grid paper • graphing calculator
Chapters 6 to 8 Review • 60–75 min	4 76–477	• G–1 Grid Paper		• grid paper • graphing calculator
Chapter 8 Task: Modelling a Damped Pendulum • 60–75 min	478	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad®* 4 • T–3 *Fathom™* • BLM 8–12 BLM Answers	• BLM 8–11 Task: Modelling a Damped Pendulum Rubric	• grid paper • graphing calculator (optional) • computer • *The Geometer's Sketchpad®* • *Fathom™*
Course Review • 120 min	479–483	• G–1 Grid Paper • G–3 Four Quadrant Grids • G–5 Number Lines • G–7 Trigonometric Graph Paper		• grid paper • graphing calculator

Chapter 8 Blackline Masters Checklist

	BLM	Title	Purpose
Prerequisite Skills			
	G–1	Grid Paper	Student Support
	G–3	Four Quadrant Grids	Student Support
	BLM 8–1	Prerequisite Skills	Practice
8.1 Sums and Differences of Functions			
	G–3	Four Quadrant Grids	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	BLM 8–2	Section 8.1 Practice	Practice
8.2 Products and Quotients of Functions			
	G–1	Grid Paper	Student Support
	G–3	Four Quadrant Grids	Student Support
	BLM 8–3	Section 8.2 Practice	Practice
8.3 Composite Functions			
	G–1	Grid Paper	Student Support
	BLM 8–4	Section 8.3 Practice	Practice
8.4 Inequalities of Combined Functions			
	G–1	Grid Paper	Student Support
	BLM 8–5	Section 8.4 Practice	Practice
	BLM 8–6	Section 8.4 Achievement Check Rubric	Assessment
8.5 Making Connections: Modelling With Combined Functions			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	T–3	*Fathom™*	Technology
	T–5	Using the CBR™ (Calculator Based Ranger)	Technology
	BLM 8–7	Section 8.5 Practice	Practice
Chapter 8 Review			
	G–1	Grid Paper	Student Support
	BLM 8–8	Chapter 8 Review	Practice
Chapter 8 Problem Wrap-Up			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	T–3	*Fathom™*	Technology
	BLM 8–9	Chapter 8 Problem Wrap-Up Rubric	Assessment
Chapter 8 Practice Test			
	G–1	Grid Paper	Student Support
	BLM 8–10	Chapter 8 Test	Summative Assessment
Chapters 6 to 8 Review			
	G–1	Grid Paper	Student Support
Chapter 8 Task: Modelling a Damped Pendulum			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Technology
	T–3	*Fathom™*	Technology
	BLM 8–11	Task: Make Your Own Slide Rule Rubric	Assessment
	BLM 8–12	BLM Answers	Answers
Course Review			
	G–1	Grid Paper	Student Support
	G–3	Four Quadrant Grids	Student Support
	G–5	Number Lines	Student Support
	G–7	Trigonometric Graph Paper	Student Support

Chapter Problem

The Chapter Problem is introduced on page 415, in which a recent graduate
of business is faced with a number of marketing optimization scenarios. Ask
students what they know about marketing. "How can a company increase sales
by adjusting price? Is this always a profitable strategy?" etc. Explain that the
analysis required in setting price points can typically involve a number of factors.
These can be modelled using various combinations of functions, which students
will learn about as they work through the Chapter Problem in Section 8.1
(question 18), Section 8.3 (question 15), and Section 8.4, (question 15).
Explain that they will learn more about costs, revenues, profits, and other
related concepts if they decide to pursue a career in the world of business.