8.1 Sums and Differences of Functions

Student Text Pages
416 to 428

Suggested Timing
75 min

Tools
- coloured tiles or linking cubes
- graphing calculator or graphing software
- grid paper

Related Resources
- G–3 Four Quadrant Grids
- T–2 *The Geometer's Sketchpad®* 4
- BLM 8–2 Section 8.1 Practice

ONGOING ASSESSMENT

Use **Assessment Masters A–1** to **A–7** to remind students about the Math Processes expectations and how you may be assessing their integrated use of them.

Teaching Suggestions

- The concrete models used in the **Investigate** are designed to provide a physical illustration of various growing patterns: linear, quadratic, exponential, and the sum of these. Students should note which parts of the growing pattern are growing fastest initially, and then again in later stages.
- Alternate instructions for steps 4 and 5a), using *The Geometer's Sketchpad®* (GSP) are given:
  – First graph the functions Y_1, Y_2, and Y_3 (note that GSP may rename these, e.g.,. $f(x)$, $g(x)$, and $q(x)$). From the **Graph** menu, choose **Plot New Function**. Enter the function and click **OK**. Repeat for the other functions.
  – Then, plot Y_4. From the **Graph** menu, choose **Plot New Function**. Click on the equation for Y_1, then +, then the equation for Y_2, then +, then the equation for Y_3, and click **OK**.
  – Select the graph of the function Y_4. From the **Construct** menu, choose **Point On Function Plot**. From the **Measure** menu, choose **Coordinates**. Click and drag the constructed point to determine the required values for Y_4.

  Use **T–2 *The Geometer's Sketchpad®* 4** to support this activity.
- When teaching **Example 1**, encourage students to use different colours or line styles for sketching the graphs of various functions and their superpositions, to make it easier to distinguish them. For part b), some students may need to review the characteristics of the graph of a quadratic function. Connections to transformation concepts should be a part of the class discussion.
- Although **Example 2** could be done without graphing technology, this is a good point to introduce its usage so that the graphical and algebraic results can be easily compared. Use of graphing technology becomes increasingly important as the chapter progresses. It is important for students to apply number sense and practical sense when interpreting the solution to the linear system (i.e., one must round the number of T-shirts to the nearest whole number; the domain and range must be restricted to positive integral values).
- The **Communicate Your Understanding** questions, C1 and C2, are designed to check students' understanding of how the superposition principle can be used to generate the graph of a sum or difference of two functions. Questions C3 and C4 can be used to see if students understand the connection between a mathematical model and the practical situation that it represents.
- Linking cubes and/or colour tiles are recommended for **questions 1** and **2**.
- Students can check their work on **questions 3** to **5** using graphing technology or a computer algebra system (CAS).
- For **question 10**, the symbolic notation representing the three functions may confuse some students. Suggest writing a word above or beneath each equation as a reference, for example, above *P(h)*, write the word *Profit*, etc.
- **Question 10** requires reasoning strategies along with the use of connecting skills to determine equations for the total cost and revenue and to determine what the break-even point coordinates mean. Representation will occur by graphing the two functions in part b) and by developing models both algebraically and graphically for the profit function. To develop and graph the required functions, the appropriate tools will have to be selected. Communicating skills will be required to explain the meaning of the coordinates of the break-even point and to discuss the domain and range within the context of this problem.

- Graphing technology may be used in **question 11**.
- For **question 12**, some students may need to review the characteristics of the graphs of sinusoidal functions.
- **Question 13** draws out the point that exponential functions grow very rapidly as one moves into the positive direction of its domain, to the point that the effects of the quadratic and linear components of the combined function become negligible.
- Some students may benefit from exploring **question 14** using graphing technology.
- **Question 14** allows students to apply algebraic reasoning to make a conclusion about the commutative property of the sum and difference of two functions. Tools will be selected and connections with previously learned mathematical concepts will be made to enable them to communicate a conclusion concerning the commutative property.
- In **question 15**, students explore the superposition of two sinusoidal functions when they are both in and out of phase. GSP is the recommended tool to use here, as the motion controller can illustrate the concepts nicely.
- Technology tips for using the graphing calculator for **question 15**:
  - Use the **Y=** editor to enter $Y_1 = \sin(x)$ and $Y_2 = \sin(x - c)$. In the home window, press **0** [STO▸] [ALPHA] **C** to store the value of C as zero.
  - Change the line style for Y_2. Refer to the Technology Appendix on page 506.
  - Use the **Y=** editor to enter $Y_3 = Y_1 + Y_2$ to see the superposition principle applied to create a new equation.
- Students could solve or check their answer to **question 16** using graphing technology.
- **Question 17** gives students an opportunity to reason through an algebraic argument that illustrates the superposition principle. Tools will be selected and connections with concepts from their mathematical past will enable them to develop the argument. The actual illustration will involve representing skills.
- GSP is recommended for **questions 20** and **21**. Advise students that the concepts of concavity and inflection points will be further explored in the Calculus and Vectors course.
- Graphing technology is recommended for **questions 22** and **23**. Students will explore products and quotients of functions, and even and odd combined functions in further detail in the next section.
- Use **BLM 8–2 Section 8.1 Practice** for remediation or extra practice.

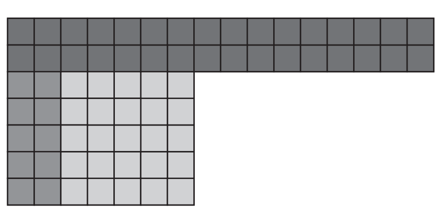
Investigate Answers (pages 416–418)

1., 3. b), 5. b) and **c)**

Stage Number, n	Blue Tiles, Y_1	Green Tiles, Y_2	Yellow Tiles, Y_3	Total Number of Tiles, $Y_1 + Y_2 + Y_3$
1	2	2	1	5
2	4	4	4	12
3	8	6	9	23
4	16	8	16	40
:	:	:	:	:
n	2^n	$2n$	n^2	$2^n + 2n + n^2$

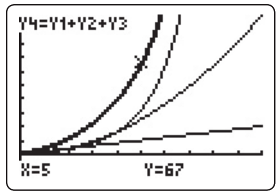
2. a) 32 blue, 10 green, and 25 yellow

b)



3. a) **i)** green **ii)** yellow **iii)** blue

4.



5. a) Answers may vary. Sample answer: They are the total number of tiles for stages 1 to 4.

6. a) Answers may vary. Sample answer: The graph looks exponential.

b) Answers may vary. Sample answer: physical model, table of values, graph, algebraic equation

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Communicate Your Understanding Responses (page 423)

C1. a) To produce the graph of $y = f(x) + g(x)$ you can add the y-coordinates at each point along the x-axis or graph the sum of the two functions, $y = -x + 5$.

b) To produce the graph of $y = f(x) - g(x)$ you can subtract the y-coordinates at each point along the x-axis or graph the difference of the two functions, $y = 3x - 5$.

C2. Answers may vary. Sample answer: The parabola is shifted up by x (variable) instead of a constant.

C3. Answers may vary. Sample answers:

a) n represents the number of T-shirts sold, which cannot be negative, and you cannot sell part of a T-shirt.

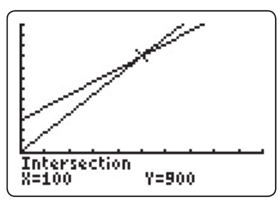
b) Since the fixed cost is $200, C cannot be less than that. P can be less than zero because you can lose money.

C4. The break-even point is where the cost and revenue are equal. Above this point is a profit, below is a loss. Examples may vary. Sample shown:

Revenue: $y = 9x$

Cost: $y = 300 + 6x$

Window variables: $x \in [0, 200]$, Xscl = 20, $y \in [-300, 1200]$, Yscl = 100



A profit is made when x is greater than 100 and the revenue is greater than the cost.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	
Reasoning and Proving	10, 11, 14–18, 20–23
Reflecting	18, 22, 23
Selecting Tools and Computational Strategies	1, 2, 6–21, 23
Connecting	1, 6–8, 10, 12, 14, 16, 17, 22, 23
Representing	1, 2, 6–10, 12, 13, 15–18, 20–22
Communicating	8, 10, 11, 13–16, 18, 22, 23

Differentiated Instruction

Use an **anticipation guide** to introduce this chapter.

Use a **Frayer model** to summarize the concepts in this section.

Use **jigsaw** to teach this section.

Common Errors

- Students make mechanical errors when adding or subtracting functions algebraically.

R_x This typically happens when integers are involved (e.g., subtracting a negative). Most of these errors can be self-diagnosed and corrected by encouraging students to visualize, or sketch, the resultant sum or difference function and consider whether it seems reasonable, based on the original component functions.

Ongoing Assessment

Achievement Check, question 19, on student text page 427.

Achievement Check, question 19, student text page 427

This performance task is designed to assess the specific expectations covered in Section 8.1.

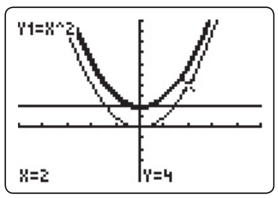
The following Math Process Expectations can be assessed.

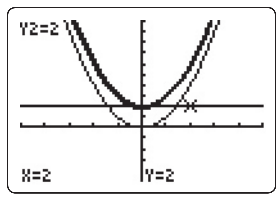
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

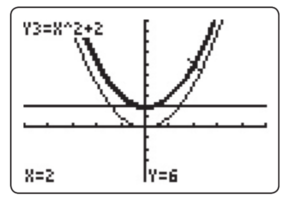
Sample Solution

a) $Y_3 = Y_1 + Y_2$
$= x^2 + 2$

x	$Y_1 = x^2$	$Y_2 = 2$	$Y_3 = Y_1 + Y_2$
0	0	2	2
1	1	2	3
2	4	2	6
3	9	2	11
4	16	2	18
5	25	2	27







The two functions are added and the original parabola is moved up 2 units.

b) The range of the function is $\{y \in \mathbb{R}, y \geq 2\}$.

Level 3 Notes

Look for the following:

- All three types of representations of the sum of the two functions
- An accurate range stated

What Distinguishes Level 2

- Two of the three types of representations of the sum of the two functions
- Range does not include 2

What Distinguishes Level 4

- All three types of representations of the sum of the two functions with accurate graphs using technology
- An accurate range stated in an acceptable form