8.2 Products and Quotients of Functions

Student Text Pages
429 to 438

Suggested Timing
75–150 min

Tools
- grid paper
- graphing calculator

Related Resources
- G–1 Grid Paper
- G–3 Four Quadrant Grids
- BLM 8–3 Section 8.2 Practice

Teaching Suggestions

- For the **Investigate** activity, some students may need to review the nature of even and odd functions and the symmetrical properties of their graphs. This activity can be done with or without graphing technology. If done without, students may need a review of the nature of the graphs of power functions.
- Rubric for **Investigate**, **Part B**:

Level 1	Level 2	Level 3	Level 4
Designs and carries out an investigation using a process having limited or no logic.	Designs and carries out an investigation using a somewhat logical process, with one or two errors or omissions.	Designs and carries out an investigation using a logical process.	Designs and carries out an investigation using a particularly insightful or creative process.
Discovers the nature of the symmetric behaviour of the quotient of two symmetric functions with limited accuracy.	Discovers the nature of the symmetric behaviour of the quotient of two symmetric functions with some accuracy.	Discovers the nature of the symmetric behaviour of the quotient of two symmetric functions with considerable accuracy.	Discovers the nature of the symmetric behaviour of the quotient of two symmetric functions with complete accuracy.
Communicates findings with limited clarity.	Communicates findings with some clarity.	Communicates findings with considerable clarity.	Communicates findings with a high level of clarity.

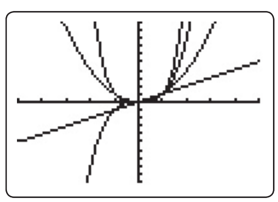
- **Example 1** provides an opportunity for students to apply previously learned algebraic skills to determine a simplified form of a product or quotient of functions. Some review of algebraic concepts (e.g., expanding, factoring, dividing binomial factors, identifying restrictions on variables) may be necessary for some students. Algebraic and graphical reasoning should be applied to identify the hole that must be inserted in the graph of the function in part b).
- When teaching **Example 2**, ask students to predict the nature of the product function in part a) before determining its equation algebraically, and to justify their reasoning. Graphing technology is recommended for producing the graph of the function, which should verify their prediction. When inspecting the graph of the function in part b), ask students to identify the domain for which the mathematical model has meaning, and why it has no meaning outside of this domain. Connections can be made between this function and the probability strand of the Data Management course, which some students may be taking.
- The **Communicate Your Understanding** questions are designed to help gauge students' understanding of the nature of a product or quotient of functions. Question C1 challenges students to identify the component functions of a damped harmonic oscillator. Students can check their answers to questions C2 and C3 using graphing technology.
- Some students may not be familiar with the absolute value notation used in **question 3**. If so, use one or two simple numerical examples to illustrate its meaning.
- For **question 4**, students will need to recall the restrictions that must be placed on rational functions and how these relate graphically to asymptotes and holes.

- Connections can be made between the mathematics of **questions 6** and **7** and environmental science. Graphing technology is recommended for these questions.
- **Question 6** gives students the opportunity to make connections with material they have learned previously, to select the necessary tools to do this, and to reflect upon and reason through strategies that will help them to explain, through their communicating skills, what the crisis point means, how the *t*-intercept for the function in part e) relates to the crisis point and the validity of the mathematical model for *P(t)* for *t*-values greater then the intercept. Representing skills will be used to graph and sketch, and communicating skills will be necessary to give the explanations required in parts a) and b) and to justify their answers in part f).
- **Questions 11** and **12** provide an opportunity for students to recognize how the mathematics of combined functions can be applied to political science and economics. Students intending on pursuing post-secondary studies in these areas may come to appreciate the importance of mathematics in these areas.
- **Question 11** uses reasoning and reflecting skills to determine a trend for the function in part a). Connecting skills will be needed to adjust the window on the graphing calculator so that a clear representation of the graph can be seen. Also, mathematical connections will have to be made to determine an explanation, using communicating skills, for the inequalities in part c).
- **Questions 13** and **14** involve probability concepts that are explored in greater depth (although not necessarily involving combined functions) in the Data Management course and in post-secondary courses in probability and statistics.
- **Questions 15** and **16** provide opportunities to assess students' ability to reason and communicate mathematically.
- For **question 20**, there are a number of pendulum simulations available on the Internet. Go to *www.mcgrawhill.ca/books/functions12* and follow the links.
- Use **BLM 8–3 Section 8.2 Practice** for remediation or extra practice.

Investigate Answers (pages 429–430)

Investigate A

1. a) Window variables: $x \in [-5, 5]$, $y \in [-10, 10]$



b) i) odd **ii)** even **iii)** odd **iv)** even

3.

Identity	Symmetry of Factor Functions	Symmetry of Product Function
a) $f(x)g(x) = p(x)$	$f(x) = x$ (odd) $g(x) = x^2$ (even)	$p(x) = x^3$ (odd)
b) $f(x)p(x) = q(x)$	$f(x) = x$ (odd) $p(x) = x^3$ (odd)	$q(x) = x^4$ (even)
c) $f(x)f(x) = g(x)$	$f(x) = x$ (odd) $f(x) = x$ (odd)	$g(x) = x^2$ (even)
d) $[g(x)]^2 = q(x)$	$g(x) = x^2$ (even) $g(x) = x^2$ (even)	$q(x) = x^4$ (even)
$(x^2)(x^3) = x^5$	x^2 (even) x^3 (odd)	x^5 (odd)
$(x^3)(x^3) = x^6$	x^3 (odd) x^3 (odd)	x^6 (even)

4. a) The product of two even functions is an even function

b) The product of two odd functions is an even function.

c) The product of an odd function and an even function is an odd function.

5. b) The product of two like (both even or both odd) functions is even. The product of two different (one odd one even) functions is odd.

Differentiated Instruction

Use **jigsaw** to teach this section.

Common Errors

- Graphs do not make sense when trigonometric functions are involved.

R_x Remind students to ensure that the graphing technology is operating in radian mode.

Ongoing Assessment

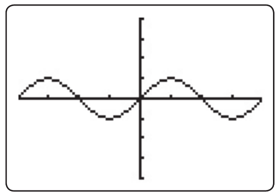
Achievement Check, question 18, on student text page 438.

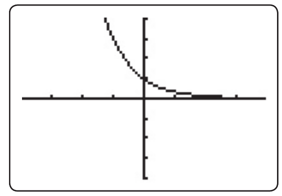
Investigate B

Answers may vary. Sample answer: The quotient of two like (both even or both odd) functions is even. The quotient of two different (one odd one even) functions is odd. When the degree of the function in the numerator is less than that of the denominator an asymptote occurs. When the degree of the function in the numerator is equal to or greater than the denominator a hole occurs.

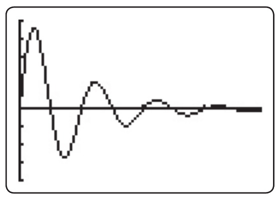
Communicate Your Understanding Responses (page 435)

C1. a) $f(x) = A\sin(kt)$ and $g(x) = 0.5^{ct}$





b)



C2. a) rational function with a different shape, $y = x + 5, x \neq -3$

b) hole at $x = -3$

C3. Answers may vary. Sample answers:

a) If the ticket price is reduced quicker, the slope changes causing the graph of $P(g)$ to decrease faster.

b) The owners would expect the number of fans in attendance to increase so that the graph of $N(g)$ would increase faster.

c) The owners may not change the number of baseball caps they give away to increase profits.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	
Reasoning and Proving	6–8, 10–16, 19, 20
Reflecting	6, 8, 11–14, 20
Selecting Tools and Computational Strategies	3–20
Connecting	1, 2, 4–7, 9–11, 13–20
Representing	4–11, 13–15, 20
Communicating	1, 2, 6–17, 19, 20

Achievement Check, question 18, student text page 438

This performance task is designed to assess the specific expectations covered in Section 8.2.

The following Math Process Expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

a) **i)** $y = f(x)g(x)$
$= (x + 2)(x^2 + 5x + 6)$
$= x^3 + 7x^2 + 16x + 12$

ii) $y = \dfrac{f(x)}{g(x)}$
$= \dfrac{x + 2}{x^2 + 5x + 6}$
$= \dfrac{1}{x + 3}$

b) The domain of $y = \dfrac{f(x)}{g(x)}$ is $\{x \in \mathbb{R}, x \neq -2, x \neq -3\}$ and the range of $y = \dfrac{f(x)}{g(x)}$ is $\{y \in \mathbb{R}, y \neq 0, y \neq 1\}$.

Level 3 Notes

Look for the following:

- A complete simplified expression for each function
- An accurate domain and range stated

What Distinguishes Level 2

- An unsimplified expression for each function
- An accurate domain or range stated

What Distinguishes Level 4

- A complete simplified expression for each function
- An accurate domain and range stated in an acceptable form