8.3 Composite Functions

Student Text Pages
439 to 449

Suggested Timing
75–150 min

Tools
- graphing calculator or graphing software
- grid paper

Related Resources
- G–1 Grid Paper
- BLM 8–4 Section 8.3 Practice

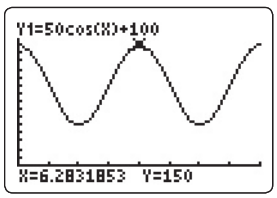
Teaching Suggestions

- The **Investigate** poses a predator-prey situation involving composition of functions that students interested in biology or environmental science may find to be particularly interesting. Graphing technology is recommended for this activity. When debriefing, students may notice that the wolf population cycle lags, or follows the rabbit population cycle. Ask students why they think this might make sense.
- The explanation in the highlight box between the **Investigate** and **Example 1** can be used to teach students the proper way to describe a composite function, verbally and in written form. While the alternate notation $(f \circ g)(x)$ is presented for student awareness, the more conventional nested bracket notation $f(g(x))$ is primarily used throughout the chapter.
- A computer algebra system (CAS) and/or graphing technology can be used to check the results of **Example 1**. Note the connections to transformations that can be made in some cases (e.g., parts b) and d)).
- A CAS and/or graphing technology can be used to check the results of **Example 2**. When comparing the two methods shown in part a), some discussion as to the relative advantages of each could be undertaken. For instance, it may be noted that Method 2 involves simpler numerical computations, whereas Method 1 may be preferable if a number of different values of the composite function are to be found, requiring repeated calculations.
- The context of **Example 3** should appeal to students who are interested in music. Ask students why they think that CD sales and cassette sales may exhibit these trends over time. Some students will recognize that this is a natural effect due to the introduction of new technologies (CDs replacing tapes and records, then being somewhat replaced themselves by MP3 players, Internet music file sharing, etc.). Some students will correctly identify the 1.04^t of the revenue function as an inflationary factor that is natural in a normal Canadian economy (this is asked for in question C3). When viewing the graph of this function, ask students if they think it is quadratic or not and to justify their reasoning based on the algebraic and graphical properties of the function.
- The **Communicate Your Understanding** questions can be used to see if students understand the nature of a composite function both in and out of context. Question C1 illustrates a fine point regarding notation. Question C2 illustrates that predator-prey relationships can take on different mathematical forms, with serious implications regarding extinction. In question C3, students are challenged to deconstruct a complicated composite function and identify the meaning of its various components.
- A CAS can be used to check **questions 1** and **4**.
- Connections can be made, in **question 4**, part c), between the restrictions on the variable and the corresponding features of the graph of the composite function.
- Graphing technology is recommended for most of the Connect and Apply questions.
- **Questions 5** to **7** provide a political context that should appeal to students who may be interested in this area. Assumptions to be discussed should include that polls are not always accurate predictors of electoral outcomes. **Question 6**, part d), and **question 7**, parts b) and c), in particular can provide opportunities to assess students' ability to apply mathematical reasoning to the contextual situation.

- **Question 7** uses thinking skills of reflecting and reasoning and proving to solve the problem of who will win the election if there are three or four parties running in an election. Students will have to show how they can select tools and connect mathematical skills that they have previously learned to solve this problem. They will show a graphical representation of the composite function from part a) and use communicating skills to describe the trend that is indicated when looking at the graph and to explain the solution to the problem.
- **Question 8** provides an opportunity to pose the usage of a counter-example as a reasoning and proving tool.
- **Question 9** requires students to select tools and make mathematical connections to help them find the expected inverses. They will then have to use their communicating skills to compare answers and to give a description of what they observe.
- **Questions 9** and **10** should prompt students to see that a function and its inverse (if defined) counteract each other.
- Students interested in environmental science should find **question 13** to be of interest.
- **Question 11** provides an opportunity to assess students' reasoning skills. Encourage students to explore these problems using graphing technology.
- **Question 14** gives students the opportunity to show their representing skills by giving their own examples when they verify the two equalities. They will have to use their reasoning skills, select tools and make connections with mathematical concepts that they have previously learned in order to create the two examples, and then communicate whether the equations are true or not.
- In working through **question 15**, students should recognize that selling the most number of units does not necessarily maximize revenue, nor does setting the highest possible price.
- Some students may need to review the characteristics of logarithmic and sinusoidal functions. In particular, that the logarithmic function is undefined for $x \leq 0$, which gives rise to the interesting periodic nature and asymptotic behaviour of these combined functions.
- Technology tips for **question 18**:
  - Using the TI-83 Plus or TI-84 Plus, students can graph the composite functions as **Y4** = **Y1**(**Y3**) and **Y4** = **Y1**(**Y3**), where **Y1** = $\log(x)$, **Y2** = $\sin(x)$, and **Y3** = $\cos(x)$.
  - To adjust the window settings, press ZOOM and select **7:ZTrig** or for a friendlier window select **4:ZDecimal**.
- There are connections between **question 20** and Physics and Vectors. Encourage students to draw a diagram to start this problem.
- Use **BLM 8–4 Section 8.3 Practice** for remediation or extra practice.

Investigate Answers (page 439)

1. a) periodic



b) i) $\{t \in \mathbb{R}, t \geq 0\}$; the time from year 0.

ii) $\{R \in \mathbb{R}, 50 \leq R \leq 150\}$; the number of rabbits varies from 50 to 150.

iii) 2π, or approximately 6.28 years

iv) $\frac{1}{2\pi}$, or approximately 0.16 of a period.

Differentiated Instruction

Use **timed retell** to teach this section.

Common Errors

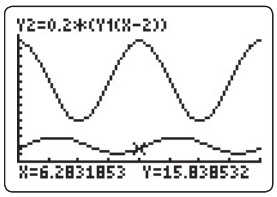
- Students mix up the order in a composite function.

R_x Remind students to follow the order of operations when working with nested brackets, that is to work from the inside out (e.g., to evaluate $f(g(x))$, x is first substituted into $g(x)$, and the resulting expression or value is then substituted into $f(x)$.

Ongoing Assessment

Achievement Check, question 17, on student text page 448.

2. a)



b) i) $\{t \in \mathbb{R}, t \geq 0\}$; the time from year 0.

ii) $\{W \in \mathbb{R}, 10 \leq W \leq 30\}$; the number of wolves varies from 10 to 30.

iii) 2π, or approximately 6.28 years

iv) $\frac{1}{2\pi}$, or approximately 0.16 of a period.

3. $W(t) = 10\cos(t - 2) + 20$

4. a) i) The graphs of the two functions are periodic with the same domain, period, and frequency.

ii) The graphs of the two functions differ in maximum and minimum values (i.e. different ranges) that occur at different times, the amplitude, the vertical translation, and the phase shift.

b) $W(t)$ depends on $R(t)$.

Communicate Your Understanding Responses (page 446)

C1. No. $f(g(x))$ means $g(x)$ is substituted into $f(x)$; $f(x)g(x)$ means $f(x)$ multiplied by $g(x)$. Example may vary.

C2. a) No. The two functions only decrease until the predator reaches zero.

b) The cat population continually declined until it reached 0. The mouse population continually declined to a point where it could not sustain the cat population.

c) With no predators remaining, it will increase.

C3. a) 3 times the number of CD sold and 2 times the number of tapes sold

C3. a) The price of CDs to price of tapes is 3:2.

b) Answers may vary. Sample answer:
Price per unit varies every year by a factor of 1.04^t, where t is time, in years, since the store opened.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	5–7
Reasoning and Proving	5–8, 10, 14, 15, 18, 20
Reflecting	5–7, 10, 20
Selecting Tools and Computational Strategies	2, 3, 5–7, 9, 11–20
Connecting	1, 4–7, 9–20
Representing	2, 5–7, 11–16, 18
Communicating	5–12, 14, 15, 18

Achievement Check, question 17, student text page 448

This performance task is designed to assess the specific expectations covered in Section 8.3.

The following Math Process Expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

a) i) $f(g(x)) = (x - 2)^2$

$= x^2 - 4x + 4$

ii) $h(g(x)) = \dfrac{1}{x - 2}, x \neq 2$

iii) $g^{-1}(h(x)) = \dfrac{1}{x} + 2, x \neq 0$

b) $f(h(2)) = \left(\dfrac{1}{2}\right)^2$

$= \dfrac{1}{4}$

Level 3 Notes

Look for the following:

- A complete simplified expression for each function
- An accurate evaluation

What Distinguishes Level 2

- An unsimplified expression for each function
- An inaccurate evaluation

What Distinguishes Level 4

- A complete simplified expression for each function with the restriction mentioned for ii)
- An accurate evaluation