8.4 Inequalities of Combined Functions

Student Text Pages
450 to 460

Suggested Timing
75 min

Tools
- graphing calculator
- grid paper

Related Resources
- G–1 Grid Paper
- BLM 8–5 Section 8.4 Practice
- BLM 8–6 Section 8.4 Achievement Check Rubric

Teaching Suggestions

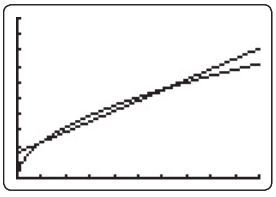
- Students will have encountered some techniques for solving inequalities in Chapter 2. The emphasis in this section is on inequalities related to combined functions, many of which are difficult to graph without technology. A variety of techniques using technology are introduced, each having some unique advantages in the information that is presented and emphasized.
- The interesting thing that students should recognize in a problem such as the one posed in the **Investigate**, is that often there is a *range* of acceptable solutions. In this particular case, for example, there is a range of sold houses for which the contractor will make money.
- Three unique methods of analysing an inequality graphically are illustrated in **Example 1**. In the first case, students simply apply visual inspection, noting the intervals for which one function appears higher on the Cartesian grid than the other. This is a reasonably intuitive technique. The second technique employs the difference function, in which the x-axis is the key referent. This is particularly useful in situations involving money (e.g., profits and losses). The third technique is slightly less intuitive and involves the quotient of two functions. This is particularly useful in situations involving populations where the number of items *per capita* is important. Students should be shown all three methods and encouraged to discuss their relative merits.
- Graphing technology is very helpful for analysing inequalities that are difficult to interpret algebraically, as in the case of **Example 2**. Students should note that the solution for the intersection points delivered by the graphing calculator in this case are approximations only, and that the precise values are very difficult to determine using conventional methods.
- Technology tip for **Example 2**:
  - Students can view the table of values noting the interval for x when $y \square 1$. To set up a table of values, press [2nd] [WINDOW]. Enter the desired x-increment for Δ**Tbl**, such as 0.01. Enter the desired starting x-value for **TblStart**, by viewing the graph to estimate where $y \square 1$ occurs for x. Students can view the table of values by pressing [2nd] [TABLE]. The approximate intervals are (0, 2.70) and (7.32, 8.27).
- **Example 3** provides an application of inequalities involving combined functions that students interested in business should find interesting. Graphing technology is strongly recommended in this case, as well as for the exercise questions that follow. Connections can be made to rational functions which students encountered in some depth in Chapter 3. Another method for graphically illustrating a range of solutions is presented in part b), which uses overlapping graphs of inequalities. Some students may also need to review the application of the quadratic formula when working through Method 2.
- The **Communicate Your Understanding** can be used to have students summarize and explain their understanding of the various ways to solve and interpret inequalities of combined functions.
- Students can use graphing technology to help visualize the impact of altering the conditions of the Investigate scenario, as posed in **questions 1** and **2**.
- Graphing technology is recommended as an exploratory and/or checking tool for most of the Practise questions, particularly **questions 8** and **9**.

- Students should recognize that the function $N(p)$ in **question 10** is a quadratic function opening down from the form of its equation and that its roots are $p = -7$ and $p = 15$. When restricting the practical domain of this function, have students consider the meaning (or lack thereof) of negative values for this function.
- In **question 11**, it may not be immediately obvious to all students that the maximum revenue and maximum number of ticket sales need not coincide. Consider having students discuss this in small groups before debriefing as a class.
- In **question 12**, the scenario becomes increasingly complex as students are now required to distinguish between revenue and profit functions. Students interested in pursuing studies in business should find this set of problems particularly engaging.
- **Question 12** allows students to use their representing skills to graph the function in part a). They will have to use connecting skills to graph the combined function in part b) and to identify the region required in part c). Sudents will also have to reason out and communicate the significance of the inequality in part c) and use their communicating skills to give an explanation for the occurrence of the maxima in the two functions in part d).
- Students could solve **question 16** algebraically and then check using graphing technology.
- **Question 17** provides an opportunity for students to compare how the difference function and the quotient function provide/emphasize different aspects of the same inequality.
- **Question 17** requires students to use reasoning skills along with connecting skills to determine what the functions in parts a) and d) represent. It will be necessary for them to select tools in order to represent the graphs of the functions with sketches. Finally, communicating skills will be needed for the explanations required concerning the different functions in parts a) and d) and the inequality in part c).
- Encourage students to solve and/or check their solutions to **questions 19** to **21** using graphing technology. Some students may need to review interval notation for these problems.
- Use **BLM 8–5 Section 8.4 Practice** for remediation or extra practice.

Investigate Answers (pages 450–451)

1. $C(n) = 8 + 0.065n$

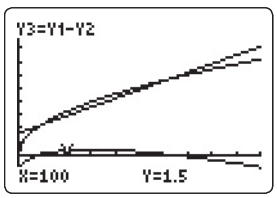
2. Window variables: $x \in [0, 500]$, Xscl = 50, $y \in [0, 50]$, Yscl = 5



3. $R(n) > C(n)$ between the points of intersection. The developer will make a profit in this region.

4. Approximately (49, 11.165 288) and (311, 28.219 327).
When 49 houses are sold the Cost and Revenue equal $11 165 288.
When 311 houses are sold the Cost and Revenue equal $28 219 327.

5. a)



Common Errors

- Students misunderstand the significance of a quotient of functions as it pertains to an inequality.

R_x Have students consider this as a comparison or rate function (e.g., food per person).

Ongoing Assessment

Achievement Check, question 17, on student text page 460.

b) Answers may vary. Sample answer:

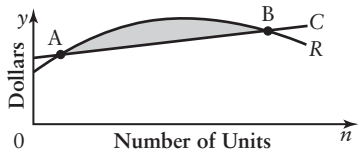
This function represents profit. So, the developer will make a profit if she sells between 49 and 311 houses. The developer will lose money if she sells less than 49 houses of more than 311 houses. A maximum profit of approximately $1.85 million is reached when 152 houses are sold.

6. Answers may vary.

Communicate Your Understanding Responses (page 457)

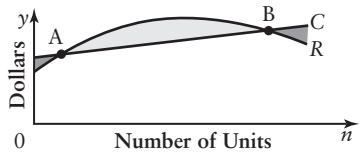
C1. Methods are compare the functions visually to see when one graph is greater than another, analyse the difference function to see where the graph is positive, or analyse the quotient function to see when it is greater than one.

C2. a)



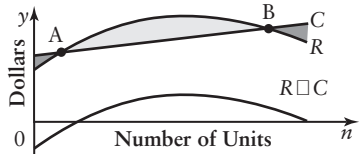
The shaded region represents profit.

b)



The additional shaded region represents loss.

C3. a)



b) Answers may vary. Sample answer:

If slope of C is increased, the interval for which the graph $y \square R \square C$ is positive will be reduced and profit will decrease.

If slope of C is decreased, the interval for which the graph $y \square R \square C$ is positive will be increased and profit will increase.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	18–21
Reasoning and Proving	7, 10–12, 14–21
Reflecting	11, 15, 21
Selecting Tools and Computational Strategies	1–6, 8–11, 15–17, 20, 21
Connecting	3, 5, 6, 12–21
Representing	1–5, 7–12, 15–17
Communicating	1, 2, 4, 7, 10–12, 15, 17, 20

Achievement Check, question 17, student text page 460

This performance task is designed to assess the specific expectations covered in Section 8.4.

The following Math Process Expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

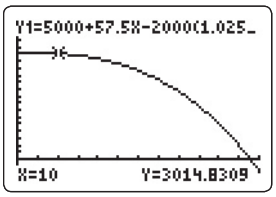
Achievement Chart Category	Related Math Processes
Knowledge and Understanding	Selecting tools and computational strategies
Thinking	Problem solving Reasoning and proving Reflecting
Communication	Communicating, Representing
Application	Selecting tools and computational strategies Connecting

Sample Solution

Provide students with **BLM 8–6 Section 8.4 Achievement Check Rubric** to help them understand what is expected.

a) $y = N(t) - P(t)$

$= 5000 + 57.5t - 2000(1.025)^t$



b) The above function represents the number of extra people who could be supplied by local services.

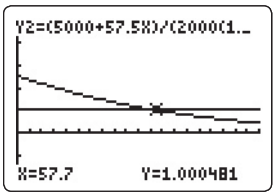
c) $N(t) - P(t) > 0$

$t > 57.7$

It takes approximately 58 years before the population grows larger than the number of people who can be supplied by local services.

d) $y = \dfrac{N(t)}{P(t)}$

$= \dfrac{5000 + 57.7t}{2000(1.025)^t}$



e) The above function shows the percent of the population that can be supplied by local services. After about 58 years, services will not be able to supply the entire (100%) of the population.