8.5 Making Connections: Modelling With Combined Functions

Student Text Pages
461 to 471

Suggested Timing
75 min

Tools
- computer
- *Fathom*™
- graphing calculator
- *The Geometer's Sketchpad*®
- grid paper

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad*® 4
- T–3 *Fathom*™
- T–5 Using the CBRTM (Calculator Based Ranger)
- BLM 8–7 Section 8.5 Practice

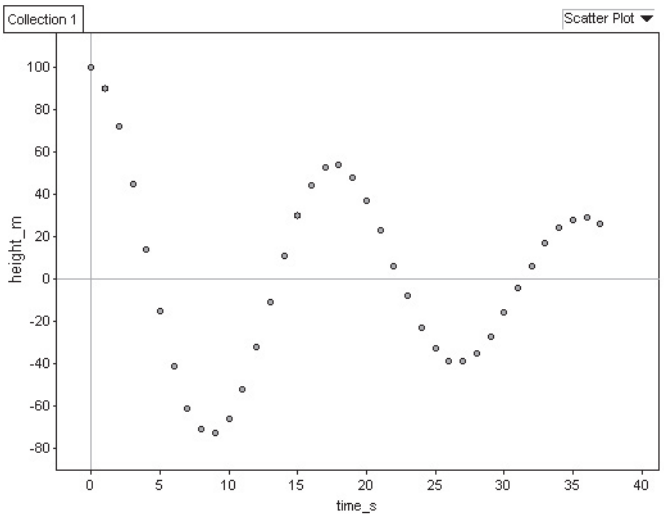
Teaching Suggestions

- The scenario posed in the **Investigate** is an example of damped harmonic motion, which involves a combination of a sinusoidal function and an exponential function. Some students may need to review the characteristics of these component functions. Use **T–3 *Fathom*™** to support this activity.
- Although *Fathom*™ is recommended for the **Investigate**, *The Geometer's Sketchpad*® (GSP) could also be used. If needed, use **T–2 *The Geometer's Sketchpad*® 4** to support this activity.
  - To plot the points in the table using GSP, from the **Graph** menu, choose **Plot Points**. Select **Rectangular (x, y)**. Enter the coordinates and click on **Plot**. Repeat. When all points are plotted, click on **Done**.
  - To create an adjustable parameter, from the **Graph** menu, choose **New Parameter**. Give it a name, such as *k*, and click **OK**. Adjust its value, as needed, using one of the following methods:
    - From the **Edit** menu, choose **Edit Parameter** and enter a value.
    - While the parameter is selected, click on the + or − buttons.
    - Right click on the parameter and choose **Animate Parameter** to enable the **Motion Controller**.
- The context of **Example 1** involves the combining of sinusoidal functions to construct musical chords. If possible, demonstrate the chords and discordant combination of notes used in this example on a guitar, piano, or other suitable instrument.
- Technology tip for **Example 1**:
  - Students may want to revisit a window setting for a similar problem by storing the window setting. To do this, press ZOOM, cursor over to the **MEMORY** menu, and select **2:ZoomSto**. To revisit the window setting, press ZOOM, cursor over to the **MEMORY** menu, and select **3:ZoomRcl**.
- In **Example 2**, a linear function is combined with a sinusoidal function to model the path of a skier. Some students may need to review the characteristics of sinusoidal functions, particularly the concepts of amplitude, frequency, and period.
- **Example 2** gives students the opportunity to apply their abilities to reason and problem solve when trying to develop the algebraic and graphical representations of the height of the skier versus time. They will use their connecting skills along with a selection of the appropriate tools to make these representations.
- The **Communicate Your Understanding** questions are designed to see if students can identify the components of a combined function, given graphical or contextual information. This chapter, and in particular this section, can be thought of as a culmination of sorts, of the major concepts of this course.
- There are connections to physics in **questions 1** to **3** and **question 7** that students interested in the physical sciences will appreciate. A possible extension is to ask students to explain how the bungee jumper's path would change if he were on the moon.
- **Questions 4** to **6** really lend themselves to demonstration with a guitar. Fans of heavy metal music will find the Connections involving power chords interesting.
- To illustrate the sound of a flat note using a guitar for **question 6**, strike the note and then gradually detune it by adjusting the corresponding machine head at the end of the guitar's neck.

- **Questions 8** and **9** provide opportunities to assess students' ability to represent given contextual information in a graphical format. Consider having students discuss these scenarios in pairs before transcribing their individual responses.
- The scenarios posed in **questions 10** to **12** illustrate how combined functions can be used to analyse performance trends in sports. Hockey fans, or students interested in pursuing a career in sports analysis, may find these problems particularly interesting. It should be noted that while the mathematical models that are developed can certainly provide insight, a situation as complex as this one will undoubtedly involve many other factors, all of which makes the analysis controversial and engaging. A possible extension or performance task could involve assigning students to perform a similar analysis for a sport of their choice using secondary data available from the Internet or other sources. This sort of activity is a key element of the Data Management course.
- **Question 12** allows students to problem solve and use reasoning skills to consider an adjustment to the given function. They will communicate the effect that this alteration has on the functions. It will be necessary for them to select tools and make connections with different strands in mathematics to discuss the significance of the *t*-intercept and the slight increase in slope.
- For **question 13**, students could also use a CBR™ (Calculator Based Ranger) when capturing the motion of a person on a swing with the CBL/CBR or Vernier EasyData™ App on the graphing calculator. **Use T–5 Using the CBR™ (Calculator Based Ranger)** to support this activity.
- For **question 14**, encourage students to consider various types of functions studied throughout the course.
- For **question 15**, a computer lab is recommended.
- **Question 15** requires research be done. Students will have to reflect upon and reason through different material found on the Internet to solve the problem of finding a real-world situation in which a combined function could be used to model a relationship. They will select tools needed to solve this problem and two other problems based on the research. They will need to use connecting skills to determine the component functions that are needed and a combination of functions that are required. An explanation of what was found while doing the research will be a very important communication component of this question.
- Use **BLM 8–7 Section 8.5 Practice** for remediation or extra practice.

Investigate Answers (pages 461–462)

2. a)



b) The points appear to form a sinusoidal curve that decreases in amplitude over time (damped oscillation).

Differentiated Instruction

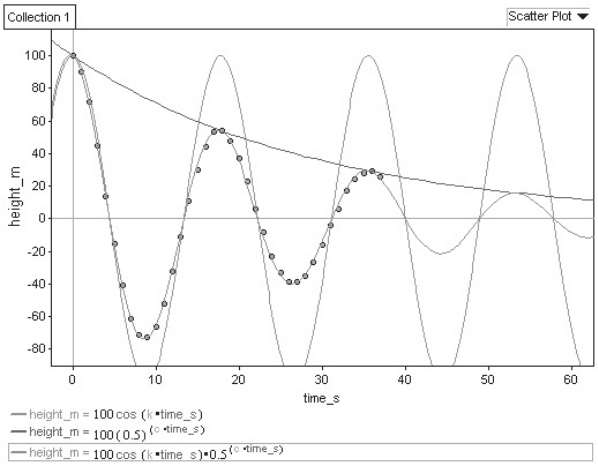
Use **what-so what double entry** to teach this section.

Use a **journal entry**. Give the topic as "Convince Me That You Understand Composite Functions."

3. a) It graph exhibits sinusoidal features in its shape and the fact that is periodic.

b) It graph exhibits exponential features in that it is decreasing and approaching 0 with asymptote $y = 0$.

4. a), 5. a), 6.



4. b) $k = 0.35$

5. b) $c = 0.05$

7. a) 15.5 m

b) approximately 63.5 s

8. a) It is the bungee jumpers height above the ground when he jumps.

b) The cosine function is at its maximum value at $t = 0$.

c) Answers may vary. Sample answer: To maintain the correct starting height of 100 m at $t = 0$.

d) The magnitude of the rate of change is the greatest the first time the bungee jumper is at the rest position ($h = 0$).

Communicate Your Understanding Responses (page 468)

C1. Answers may vary. Sample answers:

a) Simple harmonic motion is represented by a sinusoidal function.
Damped harmonic motion is represented by a combined sinusoidal and exponential functions.

b) Yes. The motion of the bungee jumpers is cyclic, but his amplitude decreases over time.

c) Answers may vary. Sample answer:
A girl's displacement on a swing after receiving a push from her big brother.
A released pendulum.

C2. a) trigonometric and linear

b) exponential and linear

C3. Answers may vary. Sample answers:

a) a rotating fan located a fixed distance beneath a warehouse roof

b) decay of a quantity that will always be greater than 4

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	12, 15
Reasoning and Proving	8, 10–13, 15
Reflecting	11, 15
Selecting Tools and Computational Strategies	4–13, 15
Connecting	8, 12, 13, 15
Representing	1–11, 13, 14
Communicating	6, 8, 11–13, 15