10.3 Images in Concave Mirrors

Key Terms
concave mirror
principal axis
focal point
focal length
real image
magnification
spherical aberration

The giant mirror in **Figure 10.18** is called *Sky Mirror*. It is a piece of art created by artist Anish Kapoor that was on display in New York City in 2006. *Sky Mirror* is more than 10 m in diameter, which makes it nearly three storeys tall. Its reflective surface is made from polished stainless steel, and its mass is about 20 t. As a work of art on the streets of New York, it fascinated crowds of onlookers. The image in the mirror changes as you walk by it.

To people who are interested in the science of optics, *Sky Mirror* is more than a piece of art. You have learned that a plane mirror produces an image that is the same size, shape, and orientation as the object. In a curved mirror, the size of the image is not identical to the size of the object, as demonstrated by *Sky Mirror*. Look carefully at the city scene captured in the photograph of *Sky Mirror*. Look for clues in the image that convince you that *Sky Mirror* cannot be a plane mirror.

By the end of Section 10.3, you will learn that the ray tracing skills you learned earlier can also be used to study the characteristics of images formed by curved mirrors.

Sense of scale

The mass of a small car is about 1300 kg, or about 1.3 t. So the mass of *Sky Mirror* is about the same as 15 small cars.

Figure 10.18 *Sky Mirror* was on display in New York City in 2006.

Properties of Concave Mirrors

concave mirror a mirror whose reflecting surface curves inward

The portion of *Sky Mirror* you can see in **Figure 10.18** is the inside of a large sphere. One of the most common shapes for a curved mirror is a spherical shape. You can picture a spherical mirror by cutting a section out of a sphere, such as a basketball, as shown in **Figure 10.19**. If you put a mirror surface on the inner surface of the cut-out section, you would have a **concave mirror**.

When you look at objects in a concave mirror, such as the people, buildings, and cars in **Figure 10.18**, the images are distorted. The images are even more distorted toward the edges of the mirror. In the following material, you will learn how to use ray tracing to determine the characteristics of images that are formed near the *centre* of curved mirrors. The optics of images away from the centre of curved mirrors is covered in later studies.

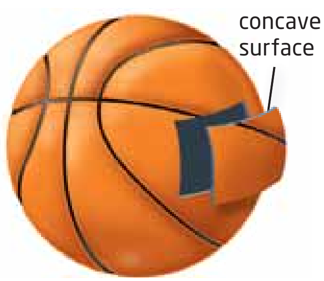


Figure 10.19 The inside surface of a sphere is a concave surface.

Drawing Ray Diagrams for Concave Mirrors

principal axis on a concave mirror, the line that passes through the centre of curvature, *C*, of the mirror and is normal to the centre of the mirror

Recall that when drawing a reflected ray for a plane mirror, you have to measure the angle of incidence between the incident ray and the normal. How do you draw a normal at a point on a curved surface, when all the normal lines will point in different directions? You can apply the same rules of reflection for a plane mirror by thinking of the curved surface as many small, flat mirrors, as shown in **Figure 10.20A**. In **Figure 10.20A**, notice how all the normals meet at a point. This point is called the *centre of curvature, C,* of the mirror. The thick, horizontal normal that touches the centre of the mirror is called the **principal axis**. The principal axis is important because it helps you locate the positions of objects that are placed in front of the mirror. The point at which the principal axis cuts the centre of the mirror is called the *vertex, V*.

In **Figure 10.20B**, the incident ray passes through the centre of curvature. Since the incident ray passes right over the normal, the angle of incidence and the angle of reflection are zero. This means that the incident ray reflects right back on itself.

focal point the point on the principal axis through which reflected rays pass when the incident rays are parallel to and near the principal axis

focal length the distance between the vertex of a mirror and the focal point

In **Figure 10.20C**, the incident ray is near and parallel to the principal axis. It also reflects according to the laws of reflection. Notice where it intersects on the principal axis. This point is called the **focal point**, *F*. The geometry of the curved mirror produces two special situations. (1) All rays that are near and parallel to the principal axis will reflect through *F*. The reverse is also true. (2) Any rays that are incident through *F* will reflect off and away from the mirror, parallel to the principal axis. The distance between *F* and the mirror at *V* is called the **focal length**.

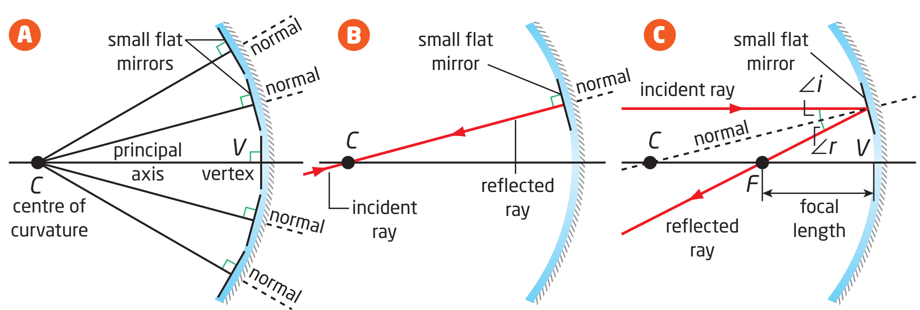


Figure 10.20 **A** Think of a curved mirror as a series of small flat mirrors. **B** The incident ray passes through the centre of curvature. **C** The incident ray is near and parallel to the principal axis.

Activity 10-3

Reflection from the Concave Surface of a Spoon

In this activity, you will use a simple kitchen tablespoon as a concave mirror.

Materials

- kitchen tablespoon with two shiny, reflective surfaces

Procedure

1. Hold up the inside of the spoon in front of your face. Look at the image of your face.
2. Bring the spoon as close to your face as you can and still see your image. Describe the characteristics of your image in the spoon.
3. Slowly move the spoon away from your face. Observe any changes in your image as the spoon gets farther away. If you can still see your image when the spoon is at arm's length, have someone else move the spoon even farther away. Describe any changes in the image of your face.

Questions

1. How is the image of your face on the inside of the spoon different from your image in a plane mirror?
2. Compare and contrast how lateral inversion happens in a plane mirror and a concave mirror.

Ray Diagrams for Concave Mirrors

When drawing a ray diagram to predict the position of an image, it is helpful to draw the object so that the bottom is on the principal axis. Because the principal axis is normal to the mirror, any ray going toward the mirror along the principal axis will reflect back on itself along the principal axis. Therefore, the bottom of the image will be on the principal axis. You only need to find the image point for the top of the object in order to draw the entire image. By using the special situations described earlier, you can use the laws of reflection to draw two incident rays without measuring angles. Then you can trace back the reflected rays as you did with plane mirrors to locate the image point for the top of the object.

The first ray travels from the top of the object to the mirror, parallel to the principal axis. As you saw in **Figure 10.20C**, the reflected ray will pass through the focal point. If you draw a second ray from the top of the object through the focal point, then the reflected ray will be parallel to the principal axis. A third ray can be drawn through *C*, the centre of curvature, to the mirror. As mentioned before, any incident ray through *C* will reflect back on itself because it is directed along a normal to the mirror.

Go to **scienceontario** to find out more

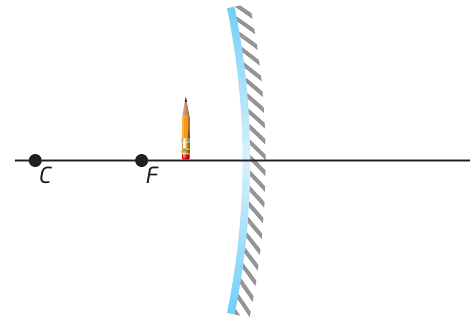
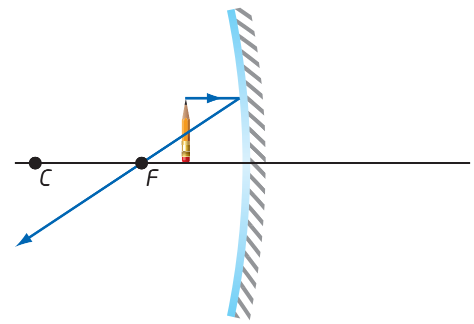
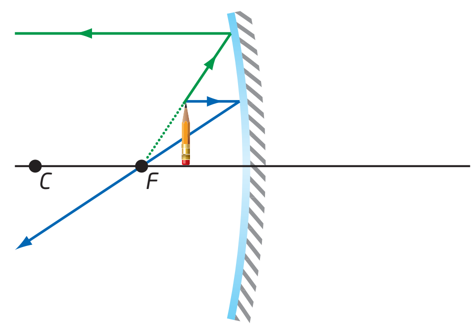
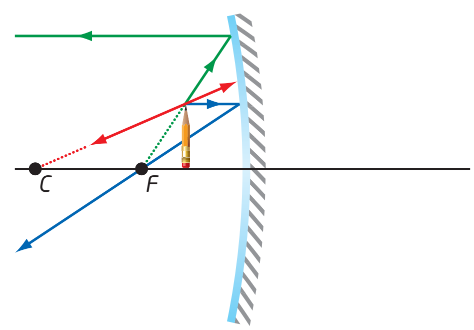
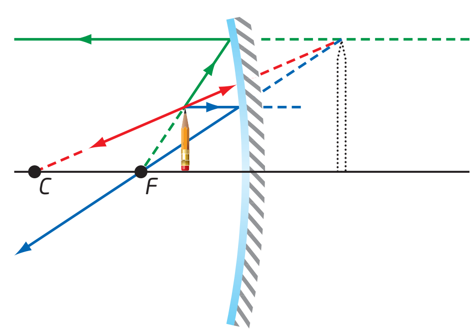
Study Toolkit

Base Words Knowing that the base word for *curvature* is *curve* can help you understand the meaning of the word *curvature*.

An Object between the Focal Point and the Mirror

Images in concave mirrors can be very different, depending on where the object is located relative to the focal point. Follow the steps in **Table 10.2** on the next page to draw the image of an object when it is between the focal point and the concave mirror.

Table 10.2 Drawing a Ray Diagram for an Object between *F* and a Concave Mirror

<table>
<tr><th>Directions</th><th>Diagram</th></tr>
<tr><td>1. Draw the principal axis and a curve to represent the concave mirror.
• Mark a focal point.
• Draw the object so that the bottom is on the principal axis between the focal point and the mirror.</td><td>
</td></tr>
<tr><td>2. Draw a ray (shown in blue) from the top of the object toward the mirror and parallel to the principal axis. Draw the reflected ray back through the focal point.</td><td>
</td></tr>
<tr><td>3. If you draw a ray from the top of the object to the focal point, the ray will be going away from the mirror. Instead, start at the focal point and draw a dotted line (shown in green) going toward the top of the object. The dotted line represents the ray coming from the focal point. The actual ray starts at the top of the object and goes toward the mirror. Draw the reflected ray travelling backward, parallel to the principal axis.</td><td>
</td></tr>
<tr><td>4. Starting at C, draw a dotted line (shown in red) to the top of the object. This dotted line represents the ray coming from C. The actual ray starts at the top of the object and goes toward the mirror. Draw the reflected ray travelling backward, along the incident ray.</td><td>
</td></tr>
<tr><td>5. As you can see, the reflected rays are travelling away from each other and will never intersect. Therefore, extend the reflected rays behind the mirror with dashed lines. The point where the reflected rays meet is the top of the image. The bottom of the image is on the principal axis. Draw the image. Notice that the image is larger than the object.</td><td>
</td></tr>
</table>

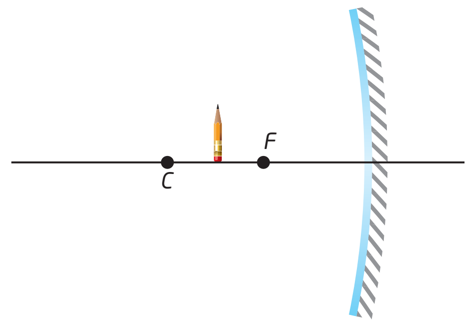
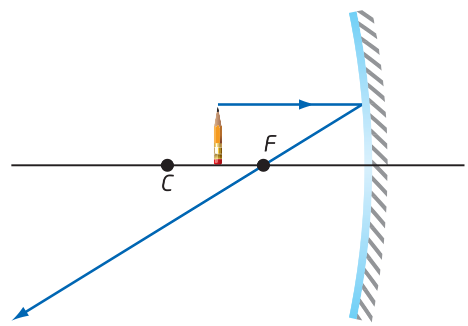
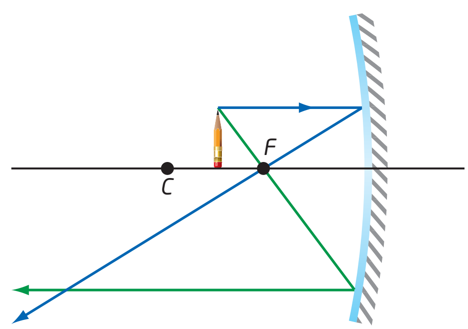
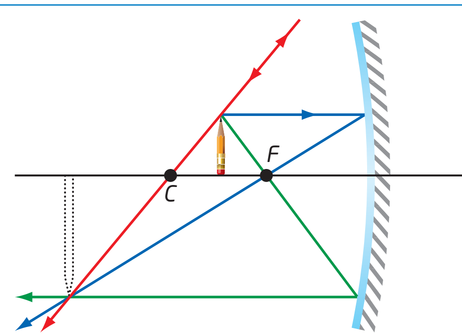
An Object between the Focal Point and the Centre of Curvature

What happens when you put an object between the focal point and *C*? Find out by drawing a ray diagram as described in **Table 10.3**. Note that you use the same three rays, but you get different results than you did when the object was between the mirror and *F*.

When you complete the ray diagram, you will discover that the rays do not have to be extended back behind the mirror. The rays actually meet at the image. Therefore, the image is a **real image** because it would appear on a screen if one were placed at the position of the image.

real image an image that is formed when reflected rays meet

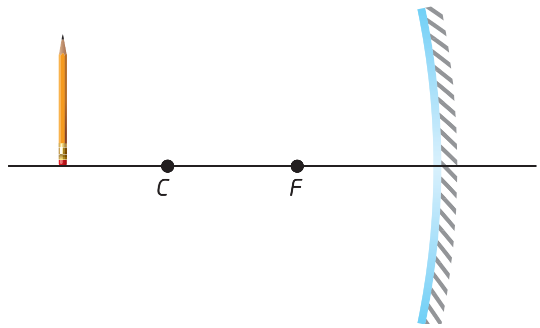
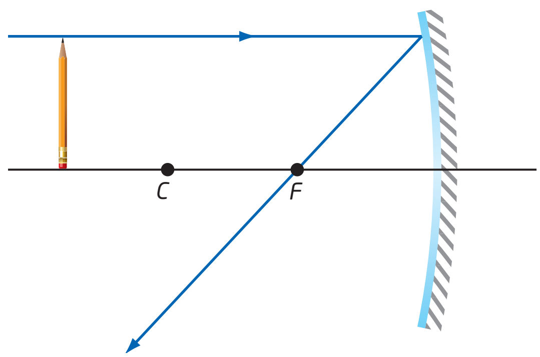
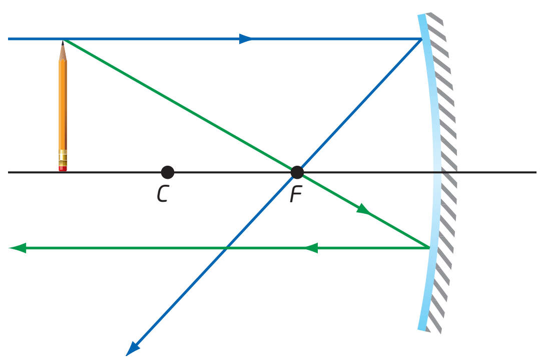
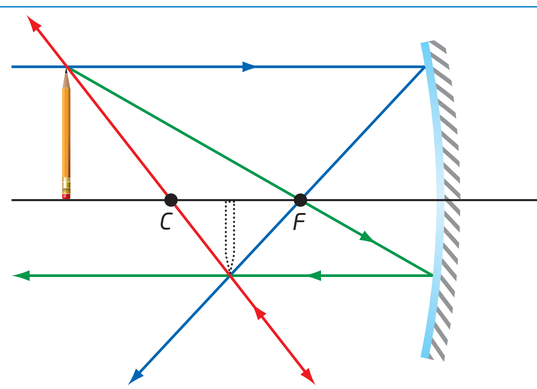
Table 10.3 Drawing a Ray Diagram for an Object between *F* and *C* in Front of a Concave Mirror

Directions	Diagram
1. Draw the principal axis and a curve to represent the concave mirror. • Mark a focal point. • Then mark the point *C* so that it is twice as far from the mirror as the focal point is. • Draw the object so that the bottom is on the principal axis between the focal point and *C*.	
2. Draw a ray (shown in blue) from the top of the object toward the mirror and parallel to the principal axis. Draw the reflected ray back through the focal point.	
3. Draw a ray (shown in green) from the top of the object through the focal point, continuing to the mirror. The reflected ray will travel backward, parallel to the principal axis.	
4. Draw a line (shown in red) from the top of the object toward the mirror, as though it is coming from *C*. The ray will not reach the mirror in the drawing. You know that the reflected ray will travel back along the incident ray. The point where the reflected rays meet is the top of the image. The bottom of the image is on the principal axis. Draw the image.	

An Object beyond the Centre of Curvature

What happens if you put an object farther away from the mirror than *C*? Find out by drawing a ray diagram, as described in **Table 10.4**. Look for the differences in the characteristics of the image relative to the object compared with previous ray diagrams.

Table 10.4 Drawing a Ray Diagram for an Object beyond *C* in Front of a Concave Mirror

Directions	Diagram
1. Draw the principal axis and a curve to represent the concave mirror. • Mark a focal point. • Then mark *C* so that it is twice as far from the mirror as the focal point. • Draw the object so that the bottom is on the principal axis beyond *C*.	
2. Draw a ray (shown in blue) from the top of the object toward the mirror, parallel to the principal axis. Draw the reflected ray back, through the focal point.	
3. Draw a ray (shown in green) from the top of the object through the focal point, continuing to the mirror. The reflected ray will travel backward, parallel to the principal axis.	
4. Draw a ray (shown in red) from the top of the object through *C*, continuing toward the mirror. Although the line does not reach the mirror in the diagram, draw the reflected ray back along the incident ray. The point where the reflected rays meet is the top of the image. The bottom of the image is on the principal axis. Draw the image.	

Learning Check

1. An object is between F and a concave mirror. Draw a ray diagram to show the characteristics of the image.
2. An object is in front of a concave mirror between F and C. Draw a ray diagram to show the characteristics of the image.
3. The distance from a make-up mirror to C (the radius of curvature) is 70 cm. How far from the mirror can a person be and still see an upright, magnified image?
4. An object is in front of a concave mirror and beyond C. Draw a ray diagram to show the characteristics of the image.

Mirror and Magnification Equations

You can also predict the characteristics of an image using two equations: the mirror equation and the magnification equation. The mirror equation allows you to calculate the location of the image. In the magnification equation, the **magnification**, m, tells you the size, or height, of the image relative to the object using object and image distances. The magnification equation allows you to find the magnification from the object and image distances. **Figure 10.21** illustrates the variables in the equations.

magnification the change in size of an optically produced image

Mirror Equation

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

The image distance, d_i, is negative if the image is behind the mirror (a virtual image).

Magnification Equation

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

The image height, h_i, is negative if the image is inverted relative to the object.

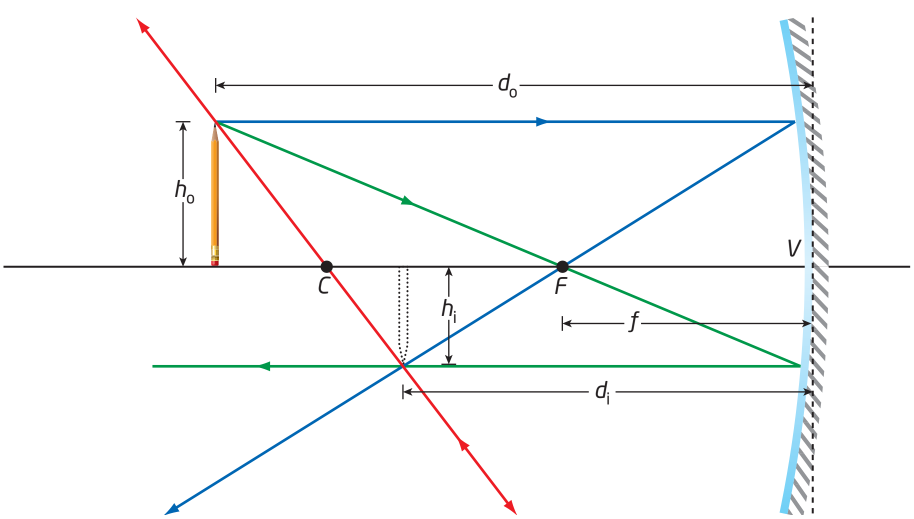
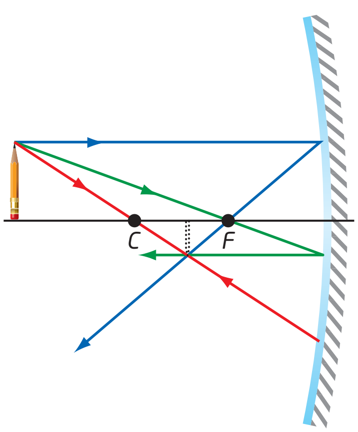


Figure 10.21 Note that d represents distance and h represents height. The subscripts "o" and "i" indicate whether the symbol represents the object or the image, respectively.

GRASP
Go to **Science Skills Toolkit 11** to learn about an alternative problem solving method.

Hint: Starting with the equation
$$\frac{1}{d_i} = \frac{1}{12\text{ cm}} - \frac{1}{40.0\text{ cm}},$$
use the [1/x] button to evaluate the second two terms. Put 12 in your calculator, and press the [1/x] button. You will get 0.8333. Now put 40 in your calculator, and press the [1/x] button. You will get 0.025. Your equation has become
$$\frac{1}{d_i} = 0.083\,33 - 0.025$$
$$= 0.058\,333$$
Press the [1/x] button again, and the result is d_i = 17.14 cm, which rounds to 17 cm.



Sample Problem: Mirror Equations and Concave Surfaces

Problem

A concave mirror has a focal length of 12 cm. An object with a height of 2.5 cm is placed 40.0 cm in front of the mirror.

a. Calculate the image distance.
b. Calculate the image height.

Solution

a. Use the mirror equation to find the image distance.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$
$$= \frac{1}{12\text{ cm}} - \frac{1}{40.0\text{ cm}}$$
$$= \frac{10}{120\text{ cm}} - \frac{3}{120\text{ cm}}$$
$$= \frac{7}{120\text{ cm}}$$
$$d_i = \frac{120\text{ cm}}{7}$$
$$= 17.14\text{ cm}$$

The image is 17 cm (after rounding) from the mirror. The sign is positive, so the image is in front of the mirror.

b. Use the magnification equation to find h_i.

$$\frac{h_i}{h_o} = \frac{-d_i}{d_o}$$
$$\frac{h_i}{2.5\text{ cm}} = \frac{-17.14}{40.0}$$
$$h_i = 2.5\text{ cm}\left(\frac{-17.14}{40.0}\right)$$
$$h_i = -1.07\text{ cm}$$

The height of the image is 1.1 cm (after rounding). The image height is negative, so the image is inverted.

Check Your Solution

The value of C is twice the value of F, so C is 2 × 12 cm = 24 cm. The object is at 40 cm, so it is beyond C. Therefore, the image should be closer to the mirror than the object, smaller than the object, and inverted. All of these characteristics agree with the answers. The ray diagram on the left verifies the solution.

Practice Problems

1. A concave mirror has a focal length of 6.0 cm. An object with a height of 0.60 cm is placed 10.0 cm in front of the mirror.

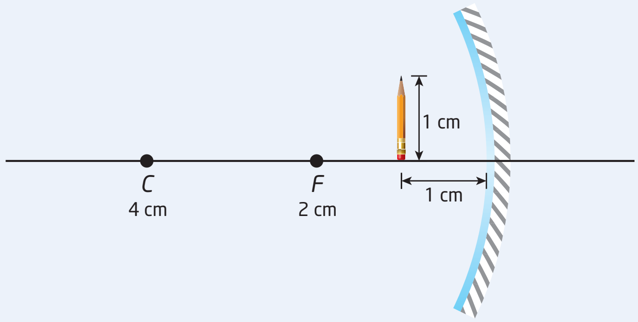
a. Calculate the image distance.

b. Calculate the image height.

2. In the diagram below, the object is between the mirror and *F*. Use the data in the diagram to answer the questions below.

a. Calculate the image distance.

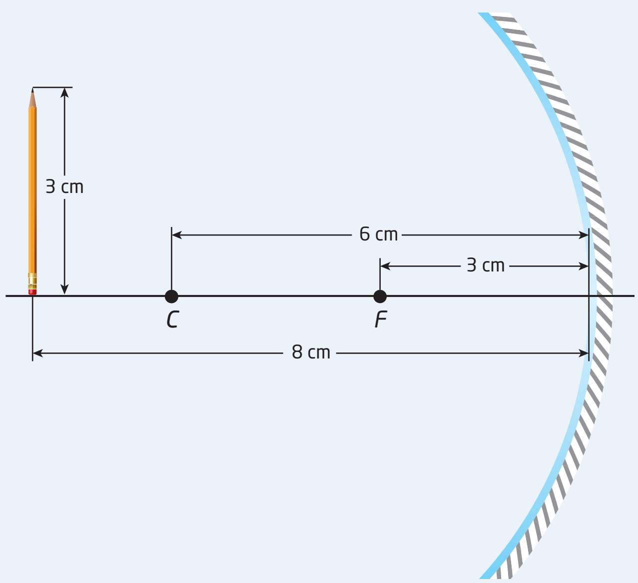
b. Calculate the image height.



3. In the diagram below, the object is beyond *C*. Use the data in the diagram to answer the questions below.

a. Calculate the image distance.

b. Calculate the height of the image.



4. A dancer is applying make-up using a concave mirror. The dancer's face is 35 cm in front of the mirror, and the image is 72 cm behind the mirror. Use the mirror equation to calculate the focal length of the mirror.

5. A concave mirror magnifies an object placed 30.0 cm from the mirror by a factor of +3.0. Calculate the radius of curvature of the mirror.

Distortion of Images in Curved Mirrors

spherical aberration irregularities in an image in a curved mirror that result when reflected rays from the outer parts of the mirror do not go through the focal point

Figure 10.22 shows what happens when light rays that are parallel to the principal axis hit a spherical mirror at points that are not within the small centre region of the mirror. In this case, the reflected rays do not meet at the same point. The focal point becomes spread out over a larger area than a point. The same thing happens if you use these rays to try to locate the image point of an object by tracing the rays behind the mirror. The rays will not meet at a point, and the image becomes spread out. This effect is called **spherical aberration**. Scientists have discovered that a concave mirror in the shape of a parabola eliminates spherical aberration.

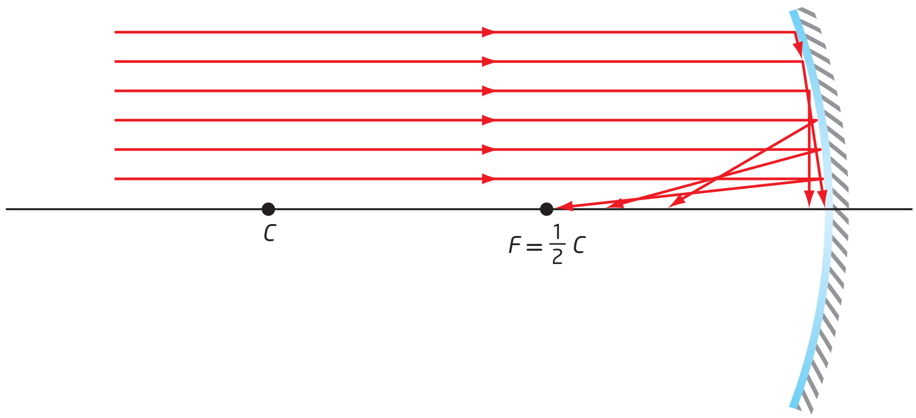


Figure 10.22 As the rays get farther from the principal axis, the point at which the reflected rays cross the principal axis moves toward the mirror.

STSE Case Study

Saved by the Sun

In many parts of the world, people burn wood or kerosene to cook their food. But increasingly, people who live in areas that are hot and sunny are using the Sun. Solar ovens use heat from the Sun, which is free, to cook food. So solar ovens can be a big help in countries such as Somalia and Tanzania, where most people live on just a few hundred dollars a year. Solar ovens also work in Canada, and they do not use up electricity or natural gas the way that conventional ovens do.

Why Use a Solar Oven?

In Tanzania, an average family lives on less than $600 a year. Stoves and cooking fuels, such as kerosene, ethanol, and propane, are expensive. Therefore, most people collect wood to make cooking fires. But the smoke from these fires is a leading cause of respiratory diseases. Collecting wood is also a leading cause of deforestation in equatorial areas.

Burning kerosene and propane fuel emits dangerous gases. Solar cooking requires no fuel, emits no greenhouse gases, and is smoke-free. However, while solar ovens cost nothing to operate, they can be expensive to build and ship to the people who need them.

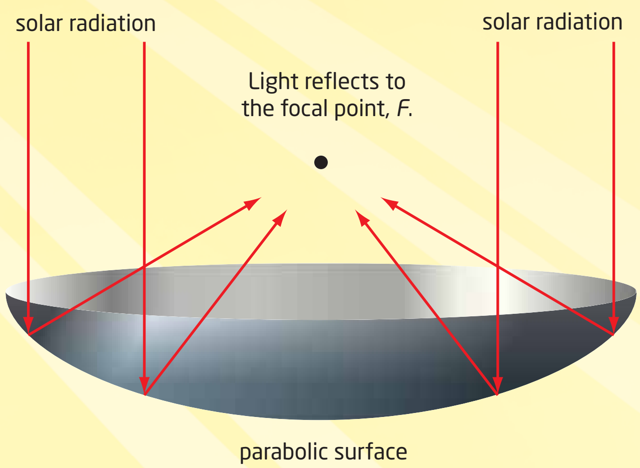
To overcome these drawbacks, solar ovens must be manufactured closer to where they are needed. Solar Circle, a volunteer group based in the United States, is helping to set up a solar oven industry in Tanzania that will use local materials. This local industry will reduce the cost of solar ovens and make it easier for more people to make the switch to solar cooking. Solar cooking, in turn, will protect not only the environment but also people's health.

Solar ovens can help the environment while improving life for people. Not surprisingly, solar ovens work best in areas that are hot and sunny. These areas are home to most of the world's populations, many of whom are very poor.

Radar Technology and Concave Surfaces

In Section 10.2, you read that radar technology is used to detect aircraft (except the aircraft like the stealth, which need to avoid detection). **Figure 10.23** shows a radar antenna. A radar antenna is basically a concave mirror in the shape of a parabola that can send and receive radio waves. A radio wave generator and detector are located at the focal point of the antenna. A pulse of radio waves that lasts a few thousandths of a second hits the antenna and is sent out toward the sky. For the next few seconds, the antenna acts as a receiver. Any returning radio waves that reach the antenna are directed to the detector at the focal point. Then another pulse is sent out.

Figure 10.23 A radar antenna acts as a concave mirror for radio waves.



The Sun's light rays reflect off the shiny metal (aluminum) dish, similar in shape to the radar dish in **Figure 10.23**. The rays concentrate at the focal point. The light that is concentrated at the focal point is converted to heat. When food is placed at the focal point, it is cooked.

Your Turn

1. List the advantages and disadvantages of using a solar oven.
2. If a \$4 canister of propane cooks food for a family for three days, how many days would it take for a \$400 solar oven to pay for itself through savings on fuel?
3. Conduct research to find out whether any organizations in Canada are helping to provide people in developing nations with solar ovens. If so, prepare a brief report about these organizations and present it to the class. If not, prepare a brief report that you could submit to a non-governmental organization, promoting the idea of a project to help provide solar ovens. Suggest some ways that students in your school could raise some funds for the project.

Section 10.3 Review

Section Summary

- The reflecting surface of a concave mirror curves inward.
- Rays that travel toward a concave mirror, parallel to and near the principal axis, will reflect and pass through the principal axis at the focal point, *F*.
- For an object between the focal point and the concave mirror, the virtual, upright image is larger than the object, and the image distance is larger than the object distance.
- For an object between the focal point and the centre of curvature, the real, inverted image is larger than the object, and the image distance is larger than the object distance.
- For an object beyond *C*, the real, inverted image is smaller than the object, and the image distance is smaller than the object distance.
- You can calculate the image distance and size using the mirror equation, $\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$, and the magnification equation, $m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$.
- Spherical aberration is the distortion of an image in a curved mirror that results when reflected rays from the outer parts of the mirror do not go through the focal point.

Review Questions

C **1.** In a diagram, show the relationship between *C*, the centre of curvature of a concave mirror, and the focal point, *F*, of the mirror.

K/U **2.** Explain how to draw the three rays that allow you to locate the image of an object in a concave mirror.

K/U **3.** What information does the sign (+ or –) of the image distance give you?

T/I **4.** If an image is inverted, smaller than the object, and closer to the mirror than the object, where is the object located?

K/U **5.** Draw a ray diagram, given the following data: $f = 5$ cm, $d_o = 4$ cm, and $h_o = 3$ cm. Use **Table 10.2** as a guide.

K/U **6.** A concave mirror has a focal length of 5 cm. An object 2 cm high is 11 cm from the mirror. Calculate the image height and image distance. Draw a ray diagram to confirm your solution.

A **7.** Suppose that you are holding a shaving mirror 30 cm from your face and the magnified image of your face is upright. A classmate tells you that the focal length of the mirror is 25 cm. Explain to your classmate why this focal length is not possible.

K/U **8.** Explain the difference between a real image and a virtual image.

The image in this shaving mirror is magnified and upright.