Key Term
thin lens equation
magnification equation

12.2 Images Formed by Lenses

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In **Figure 12.11**, you can see images through two different lenses. In **Figure 12.11A**, the lens has magnified the object and the image appears to be closer than the actual object. In **Figure 12.11B**, the image through the lens is smaller than the object and appears to be farther away than the object.

Using ray diagrams, you can predict the location, orientation, size, and type of image as it appears through a lens. As with mirrors, once the focal point has been identified, three key rays, chosen close to the principal axis, simplify the task of locating an image point produced by the lens.

As shown in **Table 12.1**, a ray that leaves an object parallel to the principal axis converges through the focus on the image side. A second ray that passes through the focus on the object side leaves the lens parallel to the principal axis. It intersects the first ray and forms an image point. If drawn neatly, a third ray through the centre of the lens should pass through the same image point. Note that in **Table 12.1**, the ray diagrams are simplified in two ways. First, partial reflection and refraction is ignored. Second, the refraction that should occur at each surface is replaced with only one bend at the axis of symmetry of the lens, which is represented by a vertical line through the centre of the lens.

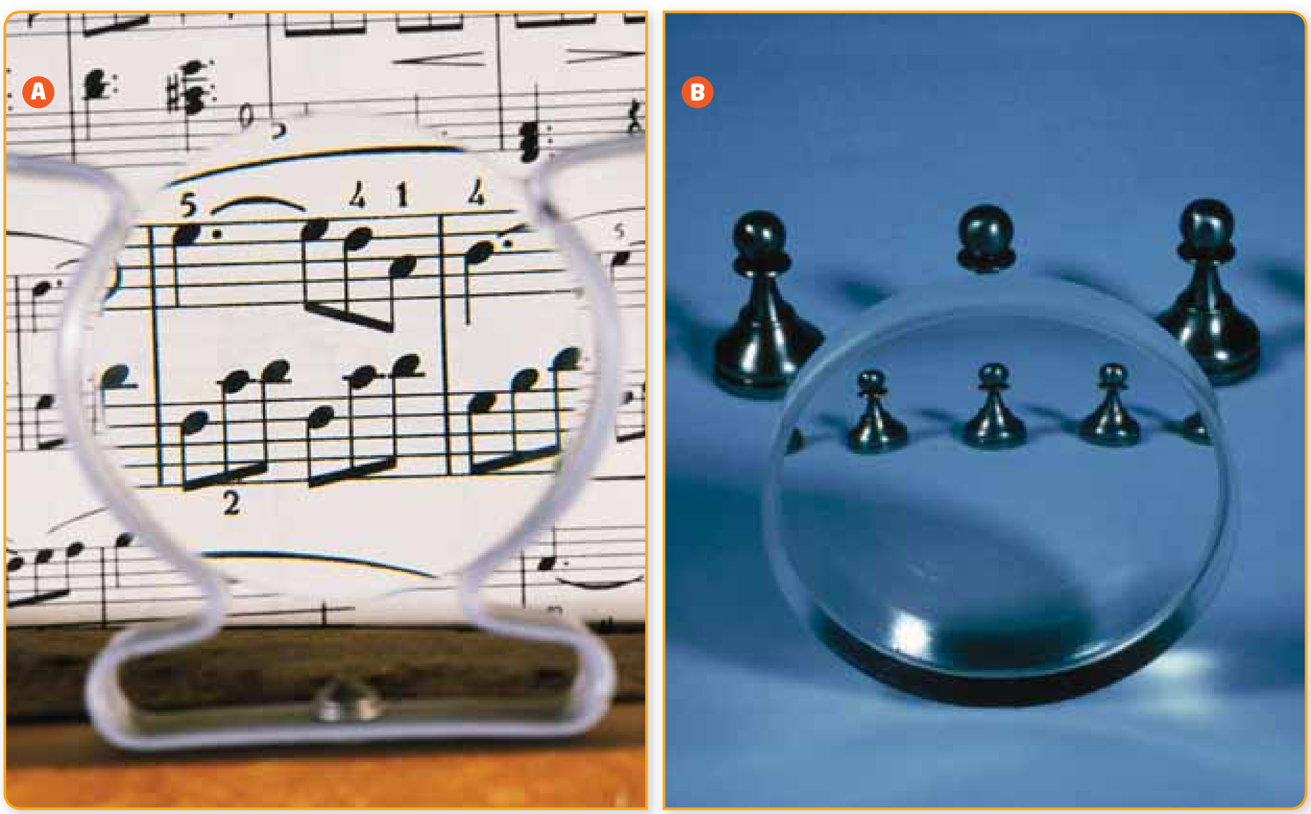


Figure 12.11 **A** The image is larger than the object and appears to be closer. **B** The image is smaller than the object and appears to be farther away.

Table 12.1 **Ray Diagrams for Converging Lenses**

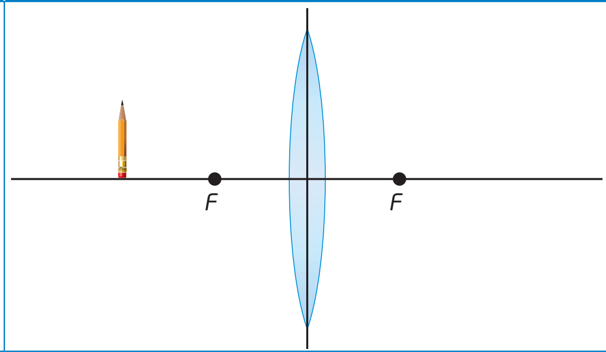
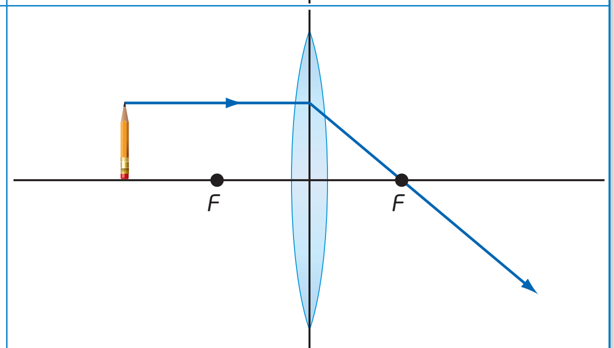
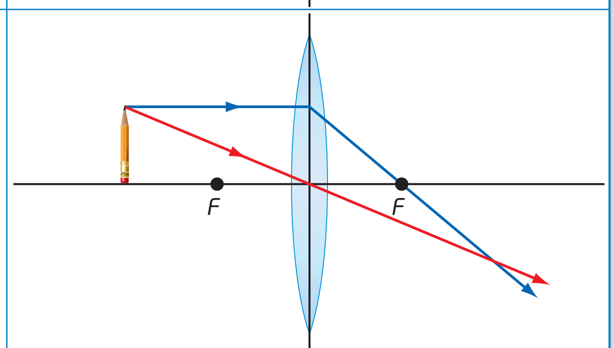
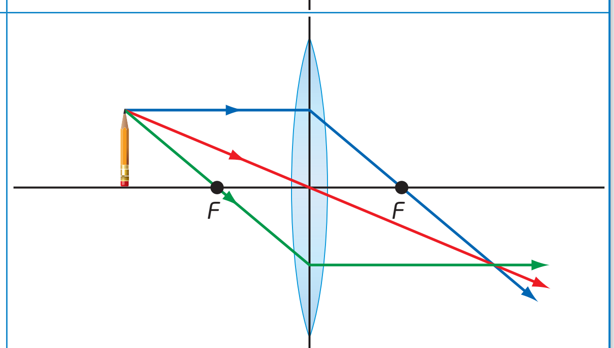
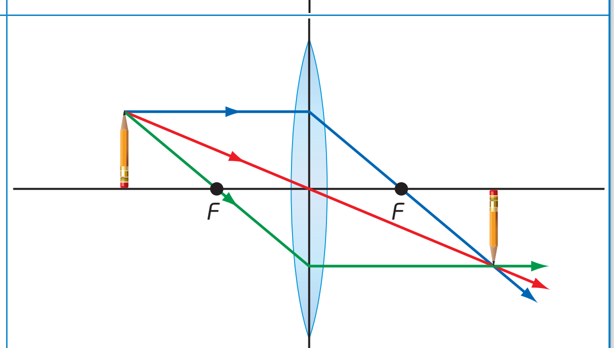
Directions	Diagram
1. Draw the principal axis and a vertical line through the centre of the axis representing the axis of symmetry of the converging lens. • Draw focal points on both sides of the lens at the same distance from the centre of the lens. • Add an object that is farther from the lens than the focal point.	
2. Draw the first ray (shown in blue). It starts at the top of the object and runs parallel to the principal axis until it reaches the axis of symmetry. *All rays that enter a converging lens parallel to the principal axis leave through the focal point.* Therefore, draw the ray from the lens through the focal point on the opposite side of the lens.	
3. Draw the second ray (shown in red). It goes from the top of the object directly through the centre of the lens. *The centre of the lens acts like a flat piece of glass, so rays leave in the same direction that they entered.* In reality, the ray would shift to the side. (Review **Figure 12.4**.) However, for thin lenses, the shift is not noticeable. Therefore, continue the ray with no change in direction.	
4. Draw the third ray (shown in green). It goes from the top of the object, through the focal point on the same side of the lens as the object, to the axis of symmetry. *Any ray that enters a converging lens from the focal point leaves the lens parallel to the principal axis.* Therefore, continue the ray horizontally until it meets the other rays.	
5. Draw the real image. The top of the image is at the point where the three rays meet. The bottom of the image is on the principal axis.	

Image Characteristics in Converging Lenses

The photographs in **Figure 12.11**, on page 494, are upright, but the image developed in **Table 12.1**, on page 495, is inverted. Some images formed by converging lenses are inverted, while others are upright. You draw ray diagrams with the object in different positions relative to the lens and the focal point to determine the orientation of the image. **Figure 12.12** shows ray diagrams with objects in locations similar to those used when analyzing images in mirrors. The characteristics of images formed by lenses are the same as those formed by mirrors.

At first glance, some of the features in **Figure 12.12A** might appear to be incorrect for a converging lens. For example, the three rays spread out after passing through the converging lens. But the diagram is correct.

When *parallel* rays pass through a converging lens, they converge, or come together. Notice that the incident rays in **Figure 12.12A** are not parallel. Because the refracted rays are spreading out, you extend them backward to find the image. As a result, there are no actual rays meeting at the image. If you placed a screen in the position of the image, nothing would appear on the screen. Therefore, the image is virtual. This combination of a converging lens and an object between the lens and the focal point is the basis of a magnifying glass like the one shown in **Figure 12.11A**, on page 494.

A The object is between the focal point and the converging lens.

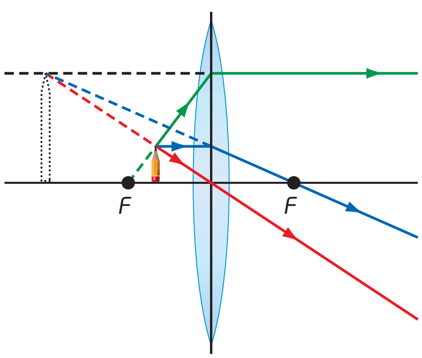


Image Characteristics:
- farther from lens than object
- upright
- larger than object
- virtual

B The object is between one and two focal lengths from the converging lens.

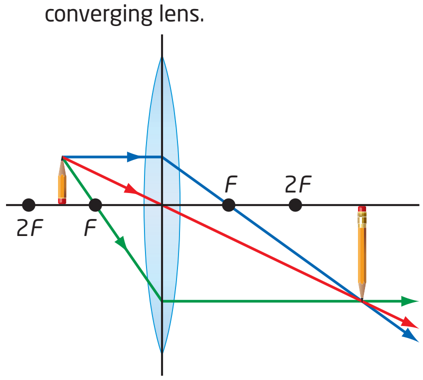


Image Characteristics:
- farther from lens than object
- inverted
- larger than object
- real

C The object is beyond two focal lengths from the converging lens.

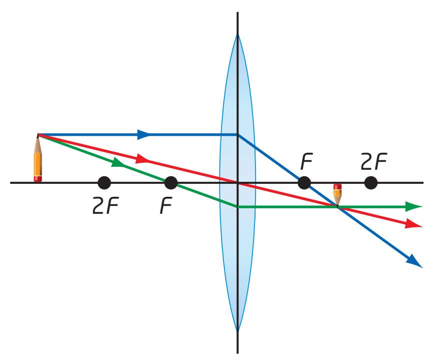


Image Characteristics:
- closer to lens than object
- inverted
- smaller than object
- real

Figure 12.12 **A** When the object is between the focal point and the converging lens, the virtual image is larger and farther from the lens than the object is. **B** When the object is between one and two focal lengths from the converging lens, the real image is larger and farther away from the lens than the object is. **C** When the object is farther than two focal lengths from the lens, the real image is smaller and closer to the lens than the object is.

Drawing Ray Diagrams for Diverging Lenses

Examine **Table 12.2** to find out how to draw ray diagrams for diverging lenses.

Table 12.2 Ray Diagrams for Diverging Lenses

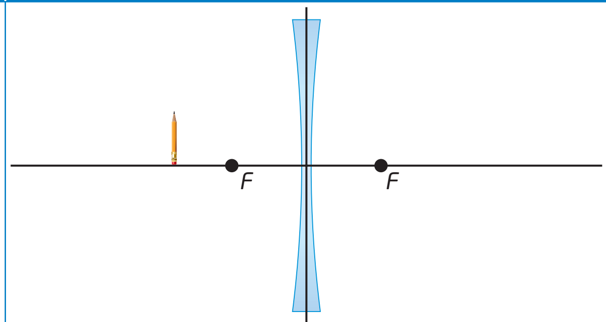
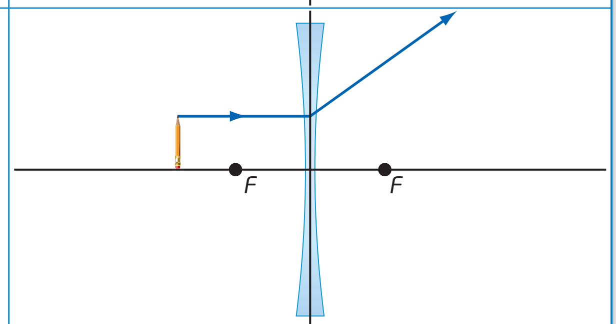
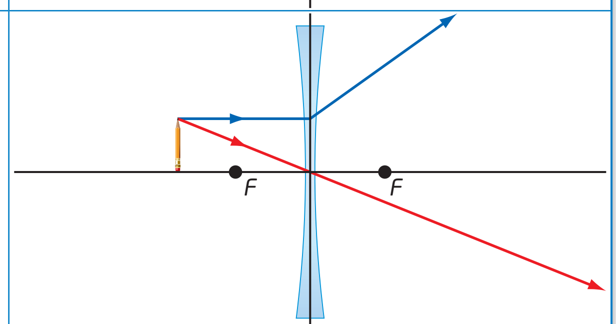
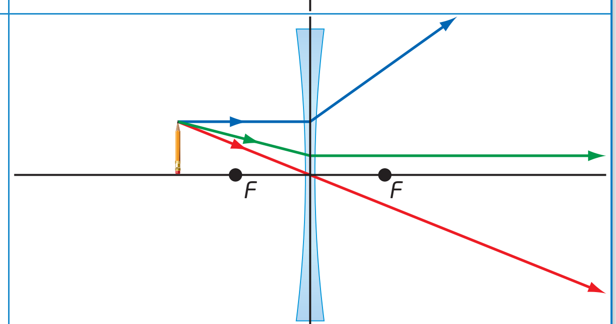
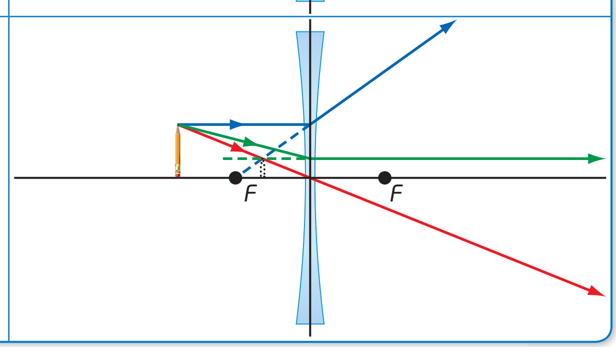
Directions	Diagram
1. Draw the principal axis and the axis of symmetry of the diverging lens. • Draw focal points on both sides of the lens at the same distance from the centre of the lens. • Add an object that is farther from the lens than the focal point.	
2. Draw the first ray (shown in blue) from the top of the object parallel to the principal axis until it reaches the axis of symmetry. *All rays that enter a diverging lens parallel to the principal axis leave as though they were coming from the virtual focal point on the object side.*	
3. Draw the second ray (shown in red) from the top of the object directly through the centre of the lens. *The centre of the lens acts like a flat piece of glass, so rays leave in the same direction that they entered.* Therefore, continue the ray with no change in direction.	
4. Draw the third ray (shown in green). It goes from the top of the object as though it were going to the focal point on the opposite side of the lens. Stop at the axis of symmetry. *Any ray that is directed toward the focal point on the opposite side of the diverging lens leaves the lens parallel to the principal axis.* Therefore, continue the ray horizontally.	
5. Notice that the rays diverge after leaving the lens. Because the rays do not meet, you extend the blue and green rays backward with dashed lines until they meet to identify the location of the image point. The red line that runs through the centre of the lens is a straight line, so you do not have to draw it backward. Draw the image, outlined with dots, with the top of the image at the point where the three rays meet. The bottom of the image is on the principal axis.	

Image Characteristics in Diverging Lenses

As you can see in **Figure 12.11B**, on page 494, the image is upright and smaller than the object. This observation agrees with the image developed by the ray diagram for a diverging lens in **Table 12.2**, on page 497. In fact, for diverging lenses, the image is *always* upright, virtual, closer to the lens than the object, and smaller than the object, regardless of the location of the object. As the object moves farther from the lens, the image becomes smaller.

Learning Check

For each situation below, draw a ray diagram and describe the four characteristics of the image.

1. An object 1.5 cm high is placed 4 cm in front of a converging lens with a focal length of 3 cm.
2. An object 1 cm high is placed 5 cm from a converging lens with a focal length of 2 cm.
3. An object 1 cm high is placed 4 cm in front of a diverging lens with a focal length of 3 cm.
4. An object 1 cm high is placed 5 cm in front of a diverging lens with a focal length of 2 cm.

The Thin Lens and Magnification Equations

As with mirrors, you can use algebraic equations to predict the position and size of the images formed by lenses. You will use these equations for converging lenses only. The symbols used for lens diagrams and equations, illustrated in **Figure 12.13**, are as follows: f is the focal length of the lens, d_o is the distance from the lens to the object, d_i is the distance from the lens to the image, h_o is the height of the object, and h_i is the height of the image. The equations are in the shaded box on the left.

Thin Lens Equation

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

Magnification Equation

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

The negative sign means that real images are inverted. For virtual images, the image distance is negative. So the negative sign ensures that the image distance will be positive and the image will be upright.

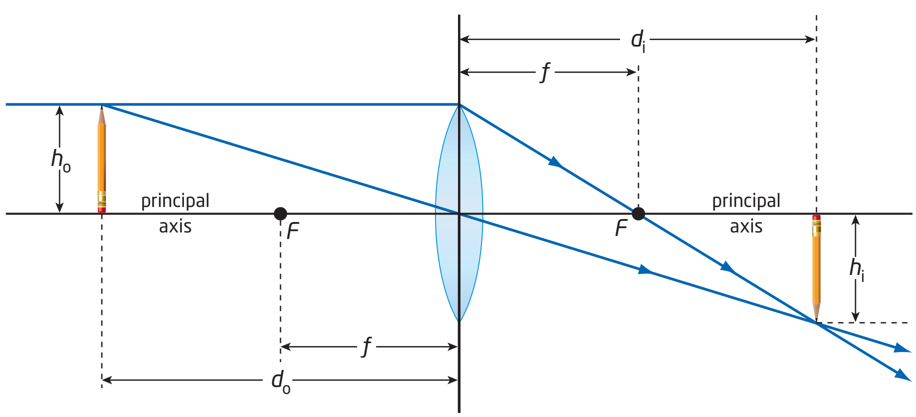


Figure 12.13 Notice that d represents the distance from the lens and h represents height. The subscripts "o" and "i" indicate whether the symbol applies to the object or to the image.

Sample Problem: Using the Thin Lens and Magnification Equations for Converging Lenses

Problem

An object 8.5 cm high is placed 28 cm from a converging lens. The focal length of the lens is 12 cm.

a. Calculate the image distance, d_i.

b. Calculate the image height, h_i.

Solution

a. You know the object distance, $d_o = 28$ cm; the focal length of the lens, $f = 12$ cm; and the object height, $h_o = 8.5$ cm.

Arrange the thin lens equation to solve for d_i. Substitute the known values into the equation, and solve for d_i.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$= \frac{1}{12\text{ cm}} - \frac{1}{28\text{ cm}} = \frac{7}{84\text{ cm}} - \frac{3}{84\text{ cm}}$$

$$= \frac{4}{84\text{ cm}} = \frac{1}{21\text{ cm}}$$

$$d_i = 21\text{ cm}$$

The image is 21 cm from the lens.

b. You know the object distance, $d_o = 28$ cm; the focal length of the lens, $f = 12$ cm; and the object height, $h_o = 8.5$ cm. From part a, you know that the image distance is $d_i = 21$ cm.

Arrange the magnification equation to solve for h_i. Substitute the known values, and solve for h_i.

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

$$\frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

$$h_i = \frac{(-d_i)(h_o)}{d_o} = \frac{(-21\text{ cm})(8.5\text{ cm})}{28\text{ cm}} = -6.375\text{ cm}$$

The image height is 6.4 cm (after rounding). The negative sign means that the image is inverted.

Check Your Solution

Because the distance from the object to the converging lens is more than two times the focal length, the image should be smaller than the object, inverted, and closer to the lens than the object. The solution confirms these characteristics.

GRASP

Go to **Science Skills Toolkit 11** to learn about an alternative problem solving method.

Hint: Starting with the equation

$$\frac{1}{d_i} = \frac{1}{12\text{ cm}} - \frac{1}{28\text{ cm}},$$

use the 1/x button to evaluate the second two terms. Put 12 in your calculator, and press the 1/x button. You will get 0.8333. Now put 28 in your calculator, and press the 1/x button. You will get 0.0357. Your equation has become

$$\frac{1}{d_i} = 0.083\,33 - 0.0357$$
$$= 0.04762$$

Press the 1/x button again, and the result is $d_i = 21$ cm.

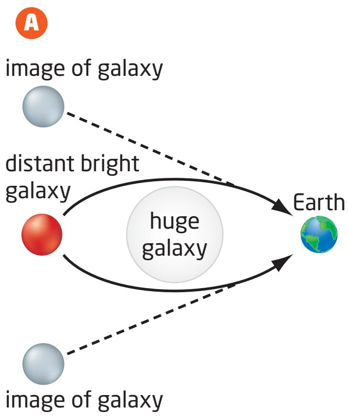


Figure 12.14 **A** The solid lines show the actual path of the light from the bright galaxy. The dashed lines are the path that the observer on Earth perceives that the light is following. **B** The bright area in the centre is a huge galaxy, and the blue ring is the light from a bright galaxy behind the huge one.

Practice Problems

1. A converging lens has a focal length of 12.0 cm. An object 6.30 cm high is placed 54.0 cm from the lens. Calculate the image distance and the image height.
2. An object 7.50 cm high is placed 150 cm from a converging lens that has a focal length of 90.0 cm. Calculate the image distance and the image height.
3. An object that is 4.20 cm high is placed 84.0 cm from a converging lens that has a focal length of 120.0 cm. Calculate the image distance and the image height.
4. A real, inverted image that is 96.0 cm high is formed 144 cm from a converging lens. The object is 36.0 cm from the lens. (**Hint:** Remember that an inverted image is indicated by a negative sign in front of the image height, h_i.) Calculate the focal length and the object height.

Gravitational Lenses

The famous scientist Albert Einstein proposed that gravity can bend light. According to Einstein's calculations, only a huge galaxy or collection of galaxies is massive enough to bend light enough to observe the effect. According to Einstein, if there were an extremely bright galaxy directly behind a huge galaxy, relative to Earth, the light from the extremely bright galaxy would be bent around the huge galaxy, as illustrated in **Figure 12.14A**. An observer would see the light as a ring around the huge galaxy, like the image in **Figure 12.14B**.

Making a Difference

Kienan Marion has participated in science projects since she was 12. In Grade 10, she consulted an astronomy professor at the University of Calgary about project ideas. The professor introduced Kienan to a demonstration that models gravitational lenses. Large masses in space, such as a galaxy or collection of galaxies, can act as a gravitational lens and bend light from another galaxy behind it. Kienan decided to test the model. She used the bases of wine glasses to simulate different gravitational lenses. She took her project, called "Gravity: Through the Looking Glass," to the 2008 Canada Wide Science Fair in Ottawa, where she won a silver medal. The project also won a gold medal from the Royal Astronomical Society of Canada at the Calgary Youth Science Fair. Kienan plans to study science and engineering at university.

What questions do you have about light? How could you test them using everyday materials?

Section 12.2 Review

Section Summary

- Ray diagrams consisting of three rays can be drawn to determine the characteristics of images formed by lenses.
- When an object is between a converging lens and the focal point, the image is always virtual, upright, and larger than the object.
- When an object is farther from a converging lens than the focal point, the image is always real and inverted.
- When an image is formed by a diverging lens, it is always upright, virtual, smaller than the object, and on the same side as the object.
- Given the focal length of the lens and the size and location of the object, you can use algebraic equations to calculate the characteristics of the image.

Review Questions

C **1.** Copy the table on the right into your notebook. The conditions are for an object in front of a converging lens. Describe and then explain how the rays travel.

Ray Diagrams for Converging Lenses

Condition	Where the Ray Goes	Explanation
a. The ray travels parallel to the principal axis until it reaches the converging lens.		
b. The ray travels toward the centre of the lens.		
c. The ray travels through the focal point (or as though it were coming from the focal point) and on to the lens.		

K/U **2.** When an object is between a converging lens and the focal point of the lens, the three rays appear to diverge. Why, then, is the lens considered to be a converging lens?

K/U **3.** Assume that an image is real, inverted, larger than the object, and farther from the lens than the object.

a. What type of lens is forming the image?

b. Where is the object located relative to the lens?

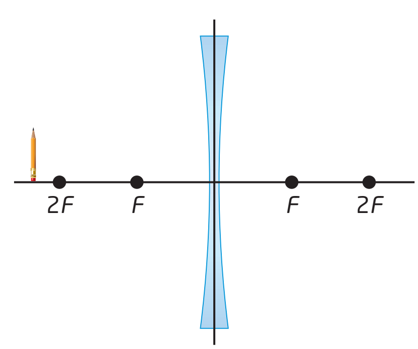
K/U **4.** Refer to **Figure 12.11B**. Describe the image size and orientation relative to the object. What type of lens formed the image?

T/I **5.** An object 5.50 cm high is placed 100 cm from a converging lens that has a focal length of 40.0 cm. Calculate the image distance and the image height.

T/I **6.** Copy the diagram on the right, and complete a ray diagram. Describe the four characteristics of the image.

T/I **7.** Solve the following problem by using both a ray diagram and algebraic equations. An object that is 3.0 cm high is placed 14 cm from a converging lens that has a focal length of 8.0 cm. Calculate the distance of the image and the image height.

K/U **8.** What is a gravitational lens?



Use this diagram for question 6.