

# CHAPTER 1

## Measurement and Geometry

### Get Set

Answer these questions to check your understanding of the Prerequisite Skills concepts on pages 4–5 of the *Foundations for College Mathematics 12* textbook.

#### Algebra

1. Solve for  $x$ . If necessary, round answers to two decimal places.

a)  $22 = \frac{1}{4}x$

b)  $4x - 12 = 3$

c)  $2x^2 = 50$

d)  $x^2 - 7 = 72$

e)  $4x^3 = 120$

f)  $96 = 3\pi x$

2. Rearrange each formula to isolate the indicated variable.

a)  $P = 7s$ , for  $s$

b) S.A. =  $b^2 + 2bh$ , for  $h$

c)  $V = \pi r^2 h$ , for  $h$

#### Converting Measures

3. Convert each measure to the indicated units.

Metric		Imperial
1 cm	10 mm	0.3937 in.
1 m	100 cm	1.0936 yd
1 km	1000 m	0.6214 mile

Imperial	Metric	
1 in.		2.54 cm
1 ft	12 in.	0.3048 m
1 yd	3 ft	0.9144 m

a) 5 in. in centimetres

b) 3 yd in metres

c) 2 ft in centimetres

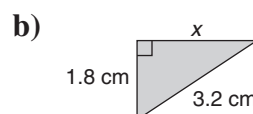
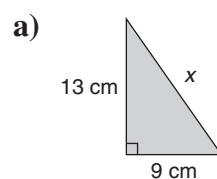
d) 8 cm in inches

e) 4 km in miles

f) 68 in. in metres

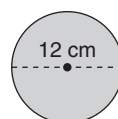
#### Pythagorean Theorem

4. Determine the length of  $x$  to the nearest tenth of a centimetre.



#### Perimeter, Circumference, and Area

5. Determine the circumference and the area.  
Round answers to one decimal place.



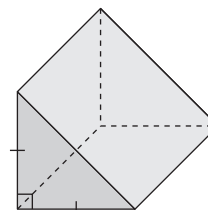
#### Three-Dimensional Figures

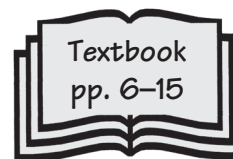
6. A box is in the shape of a triangle-based prism.

a) Draw the top, front, and side view of the prism.

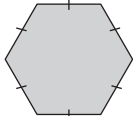
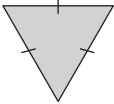
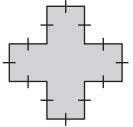
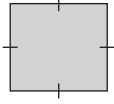
b) How many faces does the prism have?  
Name the faces by shape.

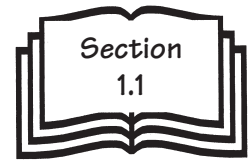
c) Which faces are congruent?





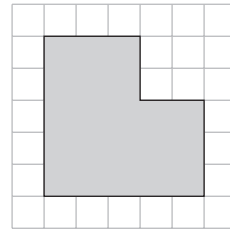
## Warm-Up

<p><b>1. Number Skills</b></p> <p>Write each decimal number as a fraction in lowest terms.</p> <p>a) 0.8</p> <p>b) 0.65</p> <p>c) 0.375</p> <p>d) 0.12</p>	<p><b>2. Algebra</b></p> <p>Factor each trinomial fully.</p> <p>a) <math>x^2 + 2x - 15</math></p> <p>b) <math>2x^2 - 18x + 16</math></p>
<p><b>3. Relations</b></p> <p>Determine the coordinates of the vertex of <math>y = 2(x - 1)^2 + 3</math>.</p>	<p><b>4. Geometry/Masurement</b></p> <p>Which polygon is irregular?</p> <p>A. </p> <p>B. </p> <p>C. </p> <p>D. </p>
<p><b>5. Data/Probability</b></p> <p>Find the mean, median, and mode of the data.</p> <p>18, 27, 9, 8, 14, 25, 21, 25, 21, 14</p>	<p><b>6. Problem Solving</b></p> <p>A cube has a volume of <math>125 \text{ cm}^3</math>. What is its surface area?</p>
<p><b>7. Math Literacy</b></p> <p>a) A three-dimensional object with three pairs of congruent rectangular faces is called a _____.</p> <p>b) A polygon with five congruent sides is called a _____.</p>	<p><b>8. Previous Section</b></p> <p>Solve.</p> $7 = -\frac{1}{4}x + 3$

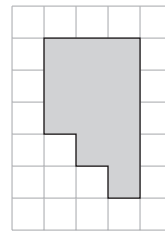


## Practise

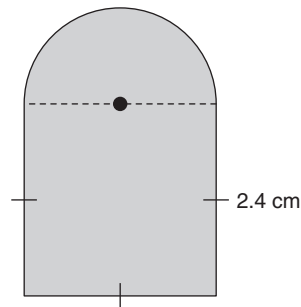
1. a) Draw a line to divide the composite shape into component shapes.  
b) Calculate the area of the composite shape using components.



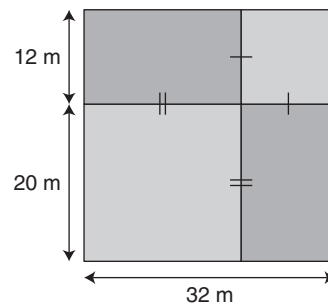
2. Calculate the area of the composite shape using net area.



3. This figure is in the shape of a square with a semi-circle on top. Determine the area of the figure to the nearest tenth of a square centimetre.

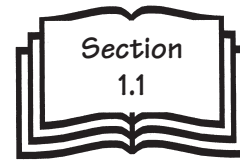
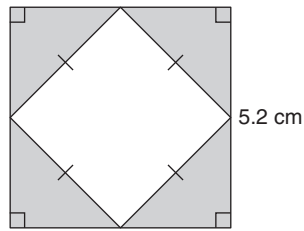


4. This design is to be painted on the floor of a convention centre.  
a) Determine the amount of dark grey paint that is required.  
b) Determine the amount of light grey paint that is required.

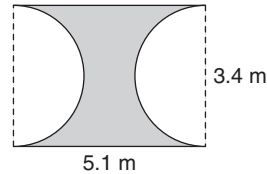


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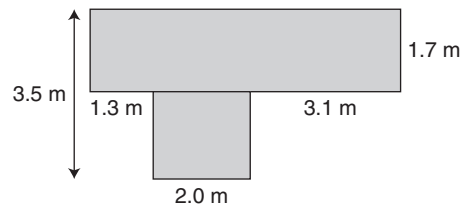
5. a) Describe two different ways to determine the area of the shaded region of this square.  
 b) Calculate the area of the shaded region.



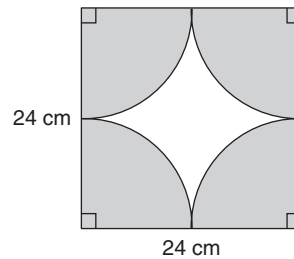
6. Calculate the net area of this shape to the nearest tenth of a square metre.



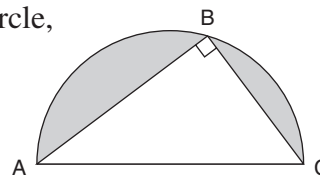
7. Calculate the total area of this patio.



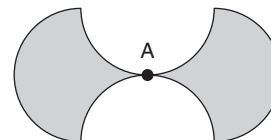
8. Find the total area of the shaded regions.



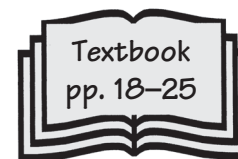
9. In the diagram, AC is the diameter of a semi-circle, and  $AC = 10$  cm,  $AB = 8$  cm,  $BC = 6$  cm, and  $\angle ABC = 90^\circ$ . What is the total area of the shaded regions to one decimal place?



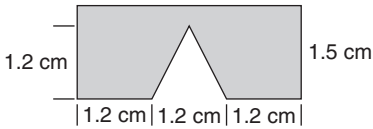
10. The figure is bordered by four semi-circles, each with a radius of 1 cm. What is the total area of the shaded regions?



# 1.2 Volume

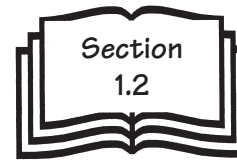


## Warm-Up

<p><b>1. Number Skills</b></p> <p>Round each number to the nearest hundredth.</p> <p>a) 7.3066</p> <p>b) 224.0632</p> <p>c) 55.997</p> <p>d) 16.735</p>	<p><b>2. Algebra</b></p> <p>Write each relation in standard form.</p> <p>a) <math>y = -(x + 2)^2 + 7</math></p> <p>b) <math>y = (x - 3)^2 - 29</math></p>
<p><b>3. Relations</b></p> <p>Determine the <math>x</math>- and <math>y</math>-intercepts of the parabola.</p> $y = 2(x - 1)(x + 3)$	<p><b>4. Geometry/Measurement</b></p> <p>What is the sum of the interior angles in a pentagon?</p>
<p><b>5. Data/Probability</b></p> <p>The faces of a hexagonal-based prism are numbered from 1 to 8. What is the theoretical probability of rolling a number on a hexagonal face?</p>	<p><b>6. Modelling</b></p> <p>The length of a rectangle is 3 cm longer than its width. Write an equation to model the perimeter of the rectangle.</p>
<p><b>7. Math Literacy</b></p> <p>What is the name for a polygon with two pairs of parallel sides and two pairs of equal opposite angles?</p>	<p><b>8. Previous Section</b></p> <p>Calculate the area of this composite shape.</p> 

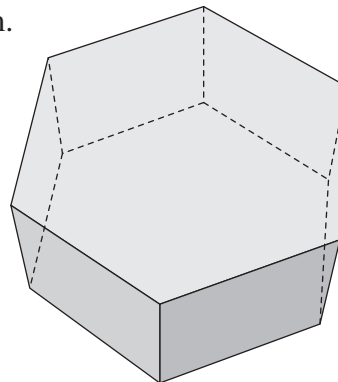
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## Practise



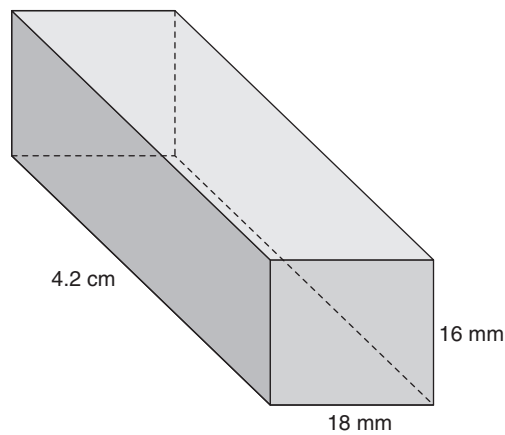
1. a) Identify the shape of the base of this prism.

b) The prism has a base area of  $32 \text{ cm}^2$  and a height of  $5 \text{ cm}$ . Determine the volume of the prism.

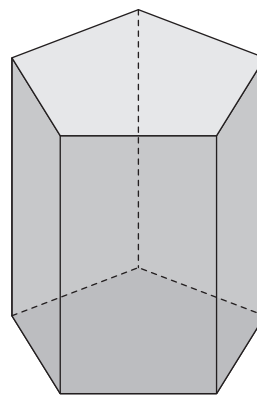


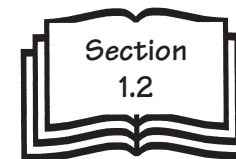
2. a) Which units would you use for the volume of this prism? Explain.

b) Calculate the volume of the prism to one decimal place.



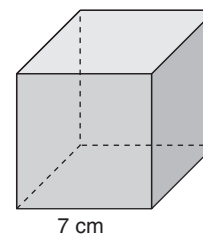
3. Could this prism be described as a rectangular-based prism? Explain.



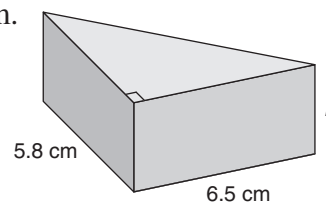


4. A cylinder has a radius of 4.5 cm and a height of 10.2 cm.
- Sketch and label a diagram of the cylinder.
  - Calculate the volume of the cylinder to the nearest tenth of a cubic centimetre.
  - Determine the volume of liquid this cylinder will hold in litres.  
Recall  $1 \text{ L} = 1000 \text{ cm}^3$ .

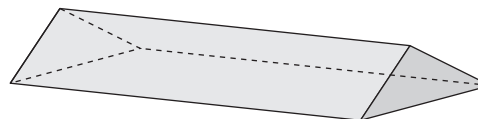
5. A cube has a side length of 7 cm.  
Suppose three cubes are to be packaged together.  
What is the minimum volume of the package?



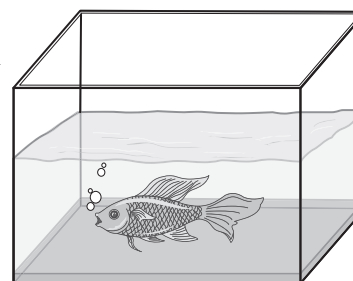
6. This box is in the shape of a right-triangular prism.  
It has a volume of  $60.32 \text{ cm}^3$ .
- Determine the area of the base of the box.
  - Determine the height of the box.



7. A lead triangular prism has a length of 1 m, a height of 4 cm and a width of 6 cm. The mass of  $1 \text{ cm}^3$  of lead is 11.4 g. What is the mass of the lead prism?



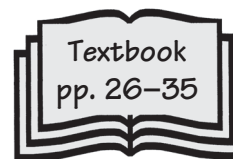
8. When a mass is placed in water, the water that is displaced when the mass is submerged is equal to the volume of the mass.  
A solid of irregular shape is submerged in water in a rectangular container with a base of 20 cm by 25 cm. The solid causes the water level to rise 1.5 cm.  
What is the volume of the solid?




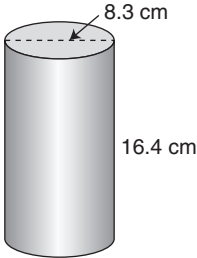
9. A copper pipe with an inside radius of 2.5 cm and an outside radius of 3 cm is 0.5 m long. If the mass of  $1 \text{ cm}^3$  of copper is 8.96 g, what is the mass of the copper pipe to two decimal places? Hint:  $1 \text{ g} = 1 \text{ cm}^3$ .



## 1.3 Surface Area

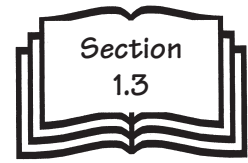


### Warm-Up

<p><b>1. Number Skills</b></p> <p>Express in lowest terms.</p> <p>a) <math>\frac{16}{68}</math>                      b) <math>\frac{21}{35}</math></p> <p>c) <math>\frac{45}{100}</math>                      d) <math>\frac{36}{50}</math></p>	<p><b>2. Algebra</b></p> <p>Expand and simplify.</p> <p>a) <math>(2x - 5)(3x - 1)</math></p> <p>b) <math>(8x + 3)(x - 10)</math></p>
<p><b>3. Relations</b></p> <p>Determine the point of intersection of <math>y = x + 3</math> and <math>y = -x^2 + 3</math>.</p>	<p><b>4. Geometry/Measurement</b></p> <p>Six congruent right isosceles triangles are arranged to form a rectangle. Each triangle has an area of <math>8 \text{ cm}^2</math>. What is the perimeter of the rectangle?</p> 
<p><b>5. Data/Probability</b></p> <p>Find the mean, median, and mode of the data.</p> <p>2, 24, 9, 16, 24, 24, 20, 25, 4, 15, 7, 11, 7, 4</p>	<p><b>6. Problem Solving</b></p> <p>A rectangle has an area of <math>28 \text{ cm}^2</math> and a perimeter of <math>22 \text{ cm}</math>. What are the dimensions of the rectangle?</p>
<p><b>7. Math Literacy</b></p> <p>What is the measure of the amount of space contained in a three-dimensional object?</p>	<p><b>8. Previous Section</b></p> <p>Determine the volume of this cylinder to the nearest tenth of a cubic centimetre.</p> 

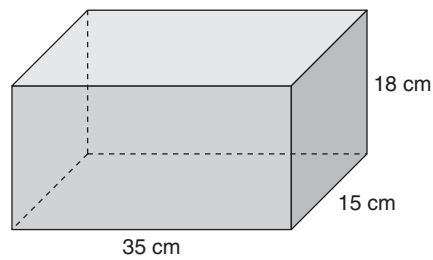


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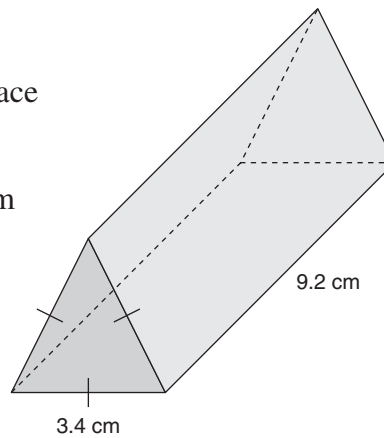


## Practise

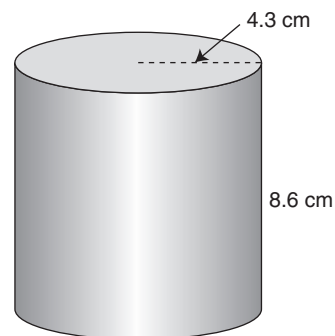
- Draw a net for this box.
  - Determine the surface area of the box.

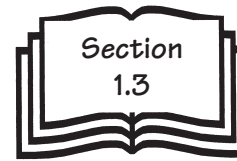


- Draw a net for this triangular prism.
  - Determine the area of one triangular face to two decimal places.
  - Determine the surface area of the prism to one decimal place.

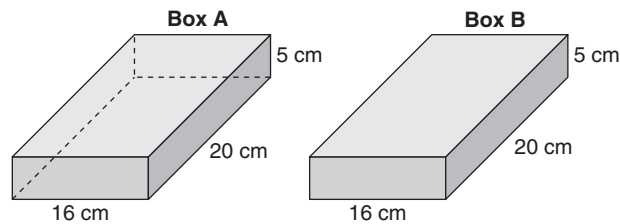


- Draw a net for this cylinder.
  - Determine the surface area of the cylinder to the nearest tenth of a square centimetre.

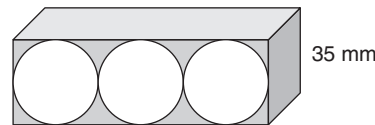




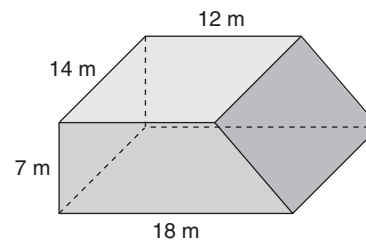
4. Two boxes have the same dimensions. Both are in the shape of a rectangular prism, but Box A has no lid.



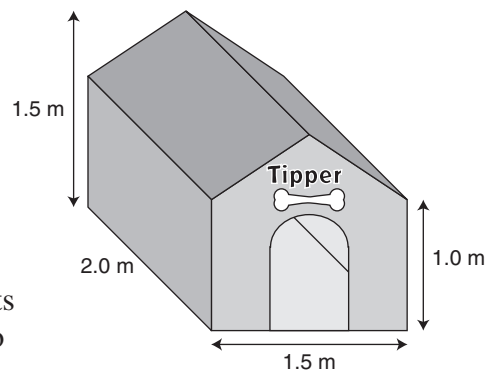
- Draw a net for each box.
  - Calculate the surface area of each box. How do the surface areas compare?
5. A rubber ball has a diameter of 35 mm. Three balls are to be packaged together as shown.
- Determine the minimum length and width of the package.
  - Determine the minimum surface area of the package.



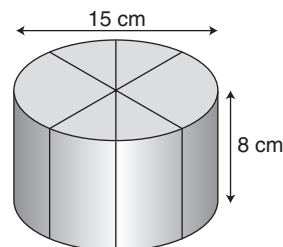
6. A raised platform has the dimensions shown.
- Determine the surface area of the platform of the platform if the bottom is not included. Round your answer to the nearest square metre.
  - The platform is to be painted with three coats of paint. One can of paint will cover  $10 \text{ m}^2$  of surface area. How many cans of paint are needed?



7. John is making a doghouse for his dog, Tipper.
- What is the surface area of the exterior of the doghouse before the doorway is cut? Include the floor. Round your answer to the nearest tenth of a square metre.
  - The exterior walls and roof of Tipper's house are to be painted. A 50-cm wide doorway has been cut as shown. The doorway is 55 cm at its highest point. What is the area to be painted to the nearest tenth of a square metre?



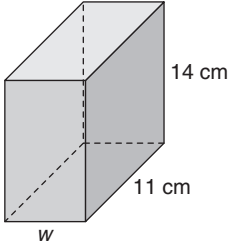
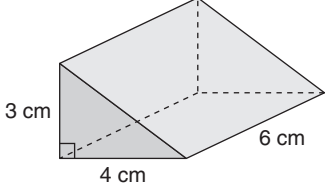
8. A cylindrical wheel of cheese is divided into six congruent sector-based prisms. What is the least amount of wrapping required for each wedge of cheese? Round your answer to the nearest square centimetre.



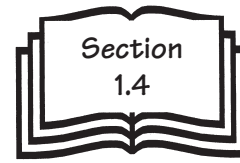
# 1.4 Optimize Perimeter and Area

Textbook  
pp. 36–45

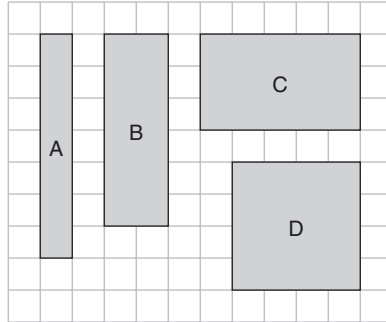
## Warm-Up

<p><b>1. Number Skills</b></p> <p>Calculate each percent.</p> <p>a) 26% of 48</p> <p>b) 31% of 67</p> <p>c) 18% of 24</p>	<p><b>2. Algebra</b></p> <p>Factor each trinomial fully.</p> <p>a) <math>3x^2 - 9x - 12</math></p> <p>b) <math>-2x^2 - 18x - 28</math></p>
<p><b>3. Relations</b></p> <p>What is the maximum value of <math>y = -x^2 - 2x + 3</math>?</p>	<p><b>4. Geometry/Measurement</b></p> <p>Which of these words apply to any rectangle? Circle all that apply.</p> <p><i>right angles</i>      <i>quadrilateral</i>  <i>rhombus</i>          <i>regular</i>  <i>square</i>            <i>parallelogram</i></p>
<p><b>5. Data/Probability</b></p> <p>Amir tosses three coins. What is the probability he will get at least two heads?</p>	<p><b>6. Problem Solving</b></p> <p>This prism has a volume of <math>616 \text{ cm}^3</math>. Determine the width of the prism.</p> 
<p><b>7. Math Literacy</b></p> <p>Which term means a six-sided polygon with all sides equal and all angles equal?</p> <p>A an irregular hexagon</p> <p>B a quadrilateral</p> <p>C a parallelogram</p> <p>D none of the above</p>	<p><b>8. Previous Section</b></p> <p>Calculate the surface area of this prism.</p> 

# Practise

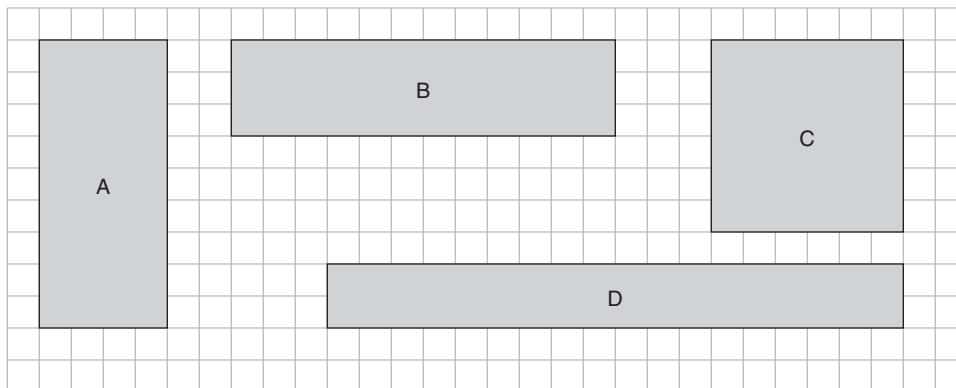


1. These rectangles all have a perimeter of 16 units.



- Calculate the area of each rectangle.
- Order the rectangles from least to greatest area.
- Compare the dimensions of the rectangles. What do you notice about the dimensions of the rectangle with greatest area?

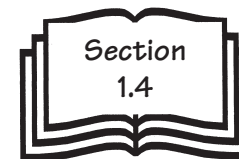
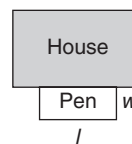
2. These rectangles all have the same area.



- Calculate the perimeter of each rectangle.
- Order the rectangles from least to greatest perimeter.

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3. Ngaio has 28 m of fencing to build a pen for her dog. She plans to build the pen along one wall of her house as shown.



- a) Complete the table to find the possible areas of the pen Ngaio can build. (Fence perimeter =  $2w + l$ .)

$w$ (m)	$l$ (m)	$A$ (m <sup>2</sup> )
1	26	26
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		

- b) What are the dimensions of the pen with the greatest area?
- c) How is the length of the pen with the greatest area related to its width?
4. Jorge has 18 segments of 2-m fence rails to build a pen.

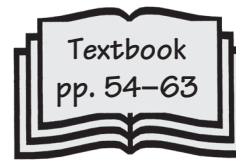
- a) Suppose the fence rails cannot be cut. Complete the table to find the possible areas of the pen.

Length (segments)	Length (m)	Width (segments)	Width (m)	Area (m <sup>2</sup> )
1	2	8	16	32
2				
3				
4				
5				
6				
7				
8				

- b) What are the dimensions of the pen with the maximum possible area?
- c) Suppose the fence rails can be cut. What are the dimensions of the pen with the maximum possible area?

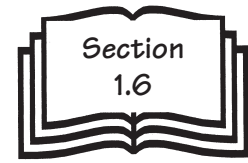
## 1.6

## Analyse Optimum Volume and Surface Area



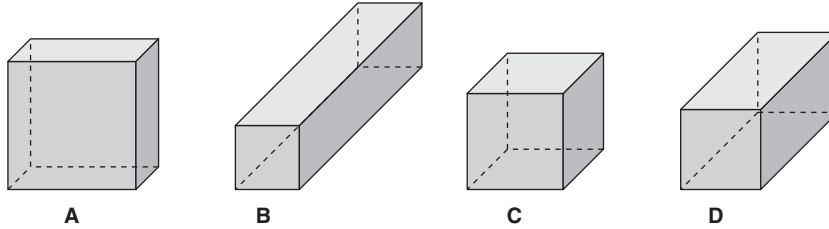
## Warm-Up

<b>1. Number Skills</b>	<b>2. Algebra</b>
Evaluate. a) $(6 - 8)^2 - 15 \div 3$ b) $9 \times 8 + 4 \times 3$ c) $150 \div 30 + 25$	Factor.  a) $2x^2 - 98$  b) $3x^2 - 6x - 45$
<b>3. Relations</b>	<b>4. Geometry/Measurement</b>
The relation $N = 500(2)^t$ models the number of bacteria, $N$ , in a dish after $t$ hours. Determine the number of bacteria after 3 h.	A regular polygon with which number of sides cannot be used to tile a plane?  A 3 B 4 C 5 D 6
<b>5. Data/Probability</b>	<b>6. Modelling</b>
Find the median, and first and third quartiles.  40, 44, 45, 39, 49, 38, 45, 39, 34, 49, 48, 31, 31, 46, 36, 44	The perimeter of a rectangle is 24 cm. Write an equation to model the area of the rectangle.
<b>7. Math Literacy</b>	<b>8. Previous Section</b>
What is the name of a quadrilateral with opposite angles equal and two pairs of congruent parallel sides?	A rectangle is to have an area of $36 \text{ cm}^2$ . a) What is the minimum perimeter? b) What dimensions produce a rectangle with a minimum perimeter?

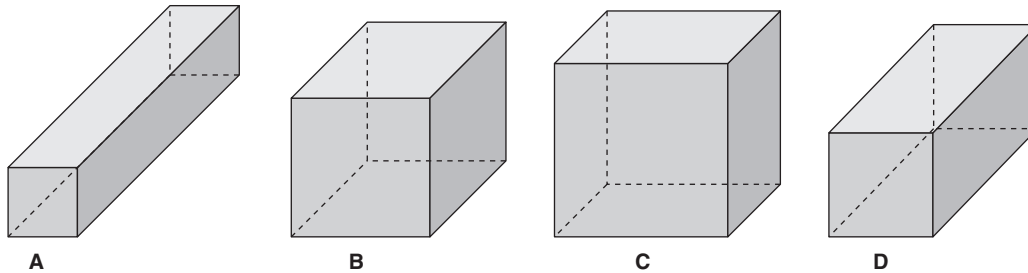


## Practise

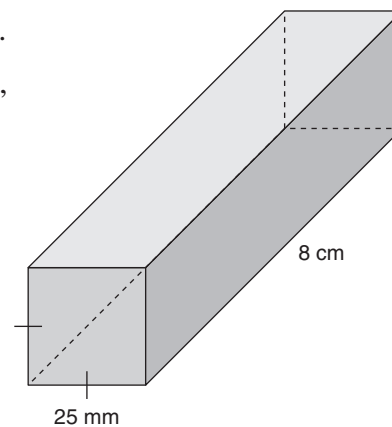
1. Each square-based prism has a volume of  $24 \text{ cm}^3$ .



- Calculate the surface area of each prism.
  - Order the prisms from minimum to maximum surface area.
2. Each square-based prism has a surface area of  $72 \text{ cm}^2$ .

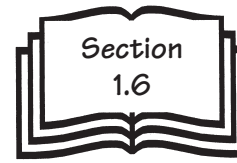
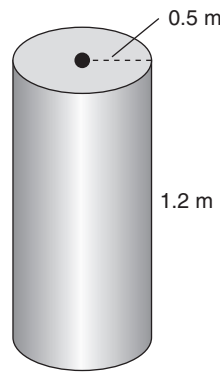


- Calculate the volume of each prism.
  - Order the prisms from minimum to maximum volume.
3. A box, in the shape of a square-based prism, is to have a volume of  $27\,000 \text{ cm}^3$  and a minimum surface area.
- What is the shape of the box with a minimum surface area?
  - Determine the dimensions of the box.
4. a) Determine the volume of this square-based prism.  
 b) Determine the dimensions, to two decimal places, of a square-based prism with the same volume but with minimum surface area.

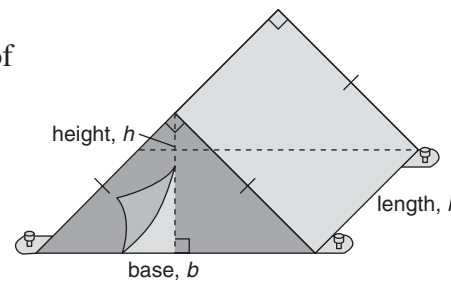


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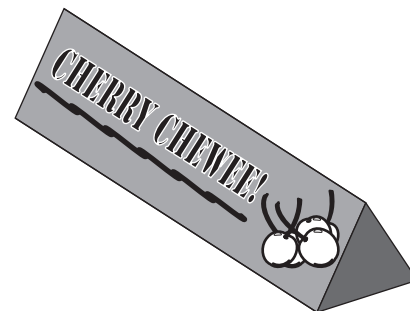
5. a) Determine the volume of this cylindrical storage tank to two decimal places.
- b) Under what conditions will a cylinder with a given volume have minimum surface area?
- c) Determine the height and radius of a cylinder with the same volume but with minimum surface area. Round your answers to the nearest hundredth of a metre.



6. An sporting goods store has a tent for sale. The tent is in the shape of an isosceles right triangular prism. The base of the triangular face is twice the height of the tent. To fit 6 people comfortably, the volume of the tent must be  $700 \text{ ft}^3$ . What are the dimensions of a tent with a volume of  $750 \text{ ft}^3$  and a minimum surface area? Round your answers to the nearest tenth of a foot.



7. The producer of sweet confections is designing a cherry nougat bar that will just fit inside a package in the shape of an equilateral triangular-based prism. Determine the dimensions of 400-mL bar that requires a minimum amount of packing material. Round your answers to two decimal places.

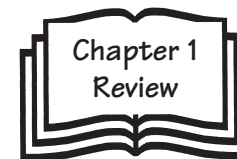


8. Joseph is going to build a tree house as a birthday present for his daughter, Jenny. It will be in the shape of a square-based prism. He has  $30 \text{ m}^2$  of wood to build the tree house. What is the maximum volume he can enclose? Round your answer to the nearest cubic metre.



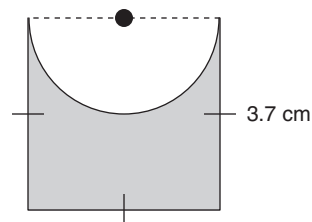


# Chapter 1 Review



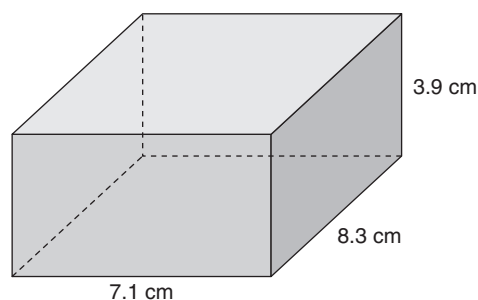
## 1.1 Area, textbook pages 6–15

1. Determine the area of this shape to the nearest tenth of a square centimetre.



## 1.2 Volume, textbook pages 18–25

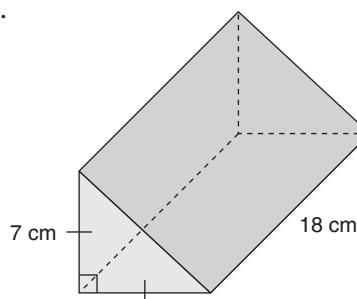
2. Calculate the volume of this prism to one decimal place.



3. A cylinder has a volume of  $450 \text{ cm}^3$  and a radius of 4.4 cm. Calculate the height of the cylinder to the nearest tenth of a centimetre.

## 1.3 Surface Area, textbook pages 26–35

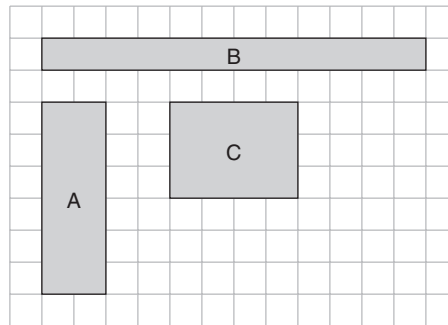
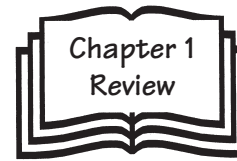
4. a) Determine the length, to one decimal place, of the hypotenuse of a triangular face of this triangular prism.  
 b) Draw a net for the prism.  
 c) Determine the surface area of the prism to one decimal place.



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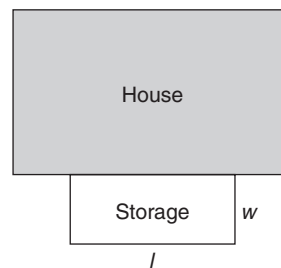
**1.4 Optimize Perimeter and Area, textbook pages 36–45**

5. These rectangles all have the same area. Order the rectangles from least to greatest perimeter.



6. Felicia has 20 m of fencing to build a storage area along one wall of her house.

What are the dimensions of the pen with the greatest area?



**1.6 Analyse Optimum Volume and Surface Area, textbook pages 54–63**

7. a) Determine the volume of this square-based prism.

- b) Determine the dimensions, to two decimal places, of a square-based prism with the same volume but with minimum surface area.

