

## CHAPTER

## 3

## Two-Variable Statistics

**Get Set**

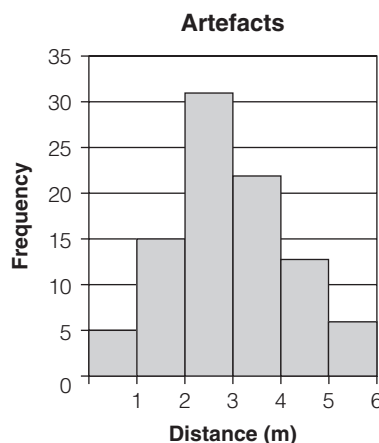
Answer these questions to check your understanding of the Prerequisite Skills concepts on pages 140–141 of the *Foundations for College Mathematics 12* textbook.

**Statistics Terminology**

1. Define each term.
  - a) mean
  - b) median
  - c) mode
  
2. Describe the difference between a sample and a population.
  
3. Explain how a random sample is different from a biased sample.

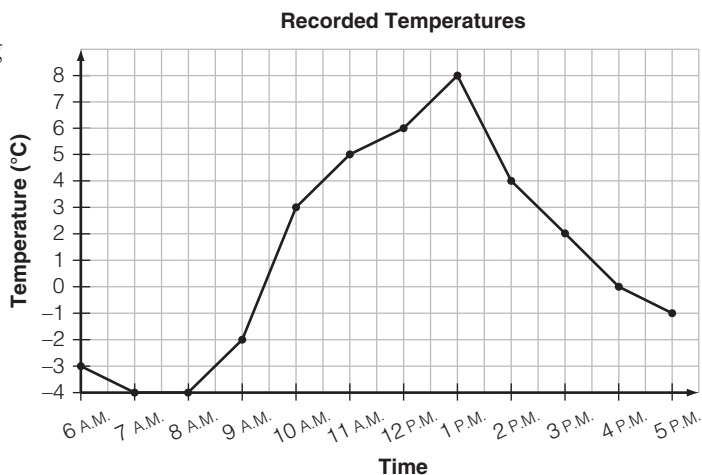
**Interpret Graphs**

4. The graph shows the distance from the shoreline of First Nation artefacts discovered in a recent archeological dig.
  - a) Identify the type of graph.
  - b) How many artefacts were found more than 4 m from shore?
  - c) How many artefacts were found within 3 m of shore?
  - d) How many artefacts were found altogether?



5. This graph shows the temperatures recorded at a weather station during one day.

- a) Identify the type of graph.
- b) When was the temperature increasing?
- c) When was the temperature decreasing?
- d) At what time was the temperature greatest?



### Use Technology

6. Holly recorded the 25 fastest times, in minutes, for a 6 km Nordic ski race.

- 16.15, 16.21, 16.94, 16.99, 17.02,  
 17.06, 17.11, 17.12, 17.24, 17.35,  
 17.38, 17.49, 17.52, 18.02, 18.04,  
 18.06, 18.07, 18.09, 18.12, 18.17,  
 18.18, 18.27, 18.52, 18.64, 18.90

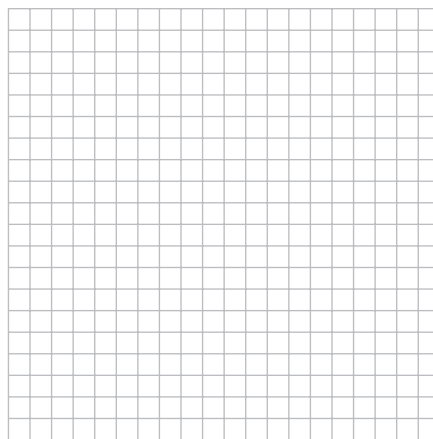
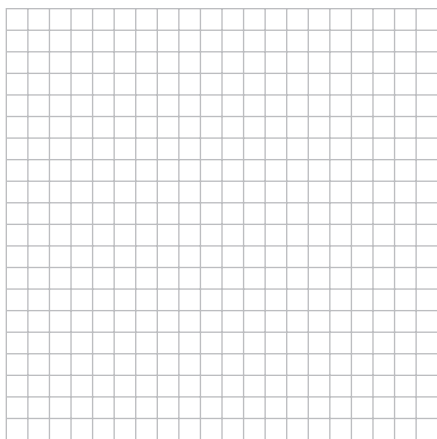
Use a graphing calculator or dynamic statistical software to make a histogram of the data. Use intervals of 0.5 min.

### Linear Graphs

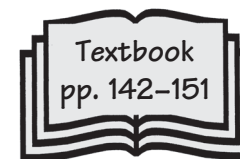
7. Graph each equation.

a)  $y = -4x - 3$

b)  $y = \frac{1}{2}x + \frac{3}{8}$



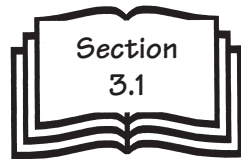
## 3.1 Two-Variable Data Sets



### Warm-Up

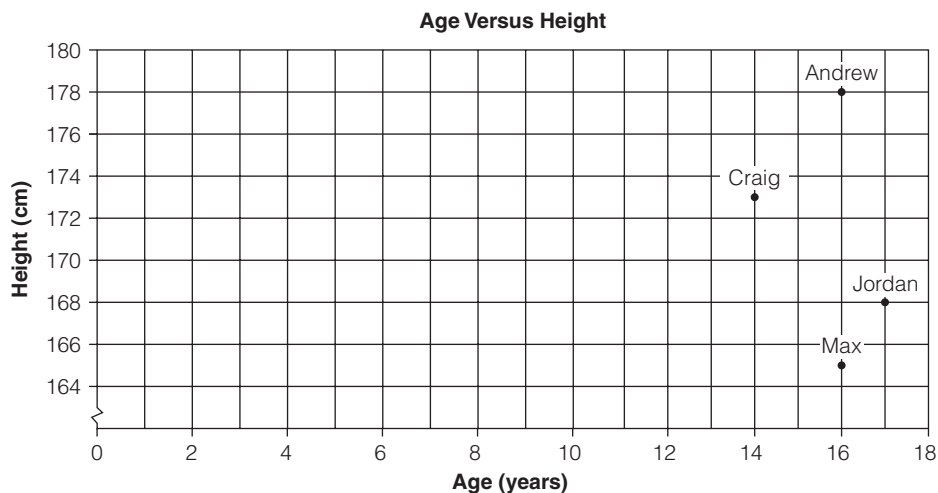
<p><b>1. Number Skills</b></p> <p>Order the numbers from least to greatest.</p> <p>1.95, -7.51, 0.12, -0.11, -4.02, 16.5</p>	<p><b>2. Algebra</b></p> <p>Simplify.</p> <p>a) <math>6x + 2y + 3x - 6y</math></p> <p>b) <math>5s + 3t - 4s + 8t</math></p> <p>c) <math>8a + 9b - 1 + 2a - 4a</math></p>
<p><b>3. Relations</b></p> <p>Line L has the equation <math>y = -\frac{4}{5}x - 14</math>.</p> <p>a) Determine the coordinates of the y-intercept.</p> <p>b) Determine the slope of a parallel line.</p> <p>c) Determine the slope of a perpendicular line.</p>	<p><b>4. Geometry/Measurement</b></p> <p>Find the volume of each cylinder. Round your answers to one decimal place. Express your answer to part b) in cubic metres.</p> <p>a) <math>r = 2</math> cm, <math>h = 12</math> cm</p> <p>b) <math>r = 10.1</math> m, <math>h = 15.5</math> cm</p>
<p><b>5. Data/Probability</b></p> <p>Calculate the probability of each event.</p> <p>a) rolling an odd number on one die</p> <p>b) rolling an odd sum with two dice</p>	<p><b>6. Modelling</b></p> <p>The side length in a regular pentagon is increased by <math>x</math> units. The perimeter of the new pentagon is 30 units. Write an equation to model this situation.</p>
<p><b>7. Math Literacy</b></p> <p>Which polygon has the greatest number of sides?</p> <p>A square</p> <p>B rhombus</p> <p>C trapezoid</p> <p>D irregular pentagon</p>	<p><b>8. Previous Section</b></p> <p>Determine the median value.</p> <p>2.12, 2.02, 2.51, 2.09, 2.66, 2.15</p>

# Practise

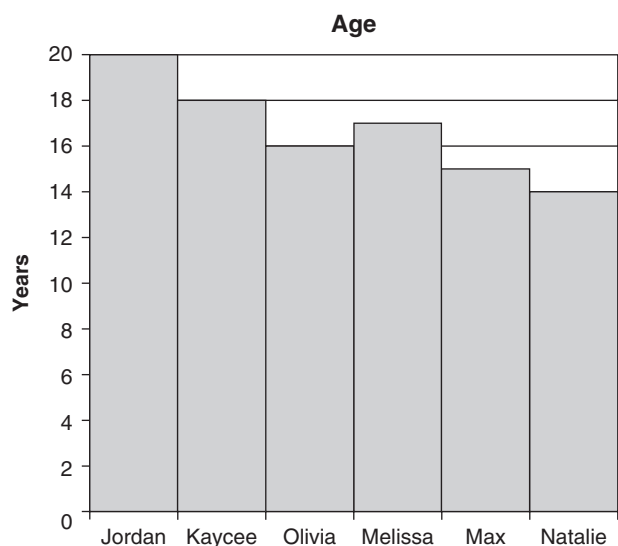


1. For each graph, indicate how many facts you know about each person and list the facts.

a)

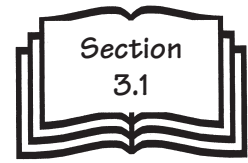


b)



2. For each situation, determine whether the data represent one-variable or two-variable data. Justify your answer.

- a) A government health agency compiles data on the number of hours students spend on-line and the number of hours they exercise each week.
- b) The guidance office surveys grade 12 students about the number of schools they are applying to for next year.
- c) A market research company surveys consumers about the number of greeting cards they purchased in the past six months.
- d) A taxi driver records the distance travelled and the corresponding fare.

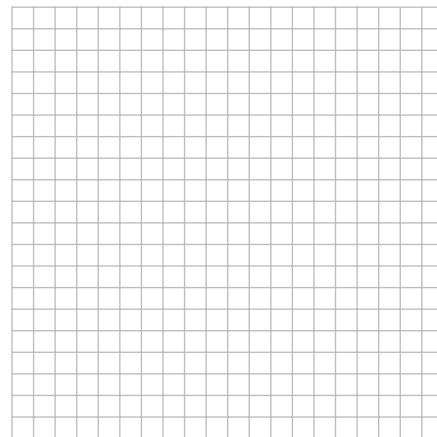


Section  
3.1

3. Jordan plays music at a figure skating club to meet his community service requirement. He made a table to record the number of times each skater's music was played.

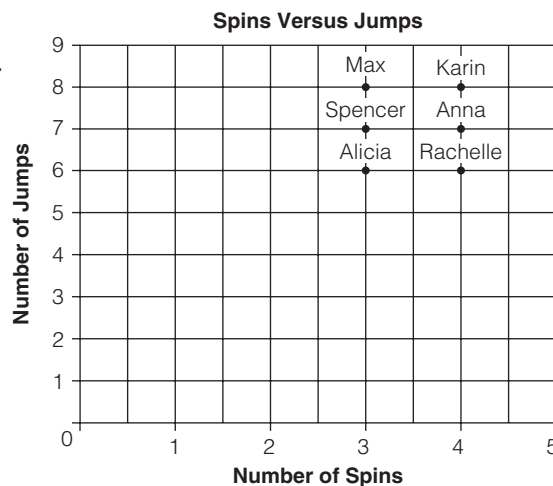
Skater	Tally	Frequency
Anna		4
Max		
Spencer	-	
Karin		
Rachelle	-	
Alicia		

- Complete the table.
- What do you know about each skater?
- Is this a one-variable or a two-variable data set? Explain.
- Identify the variable(s).
- Draw an appropriate graph.

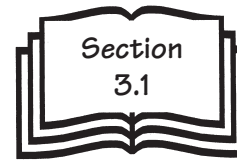


4. The scatter plot shows the number of jumps and spins in each skater's program.

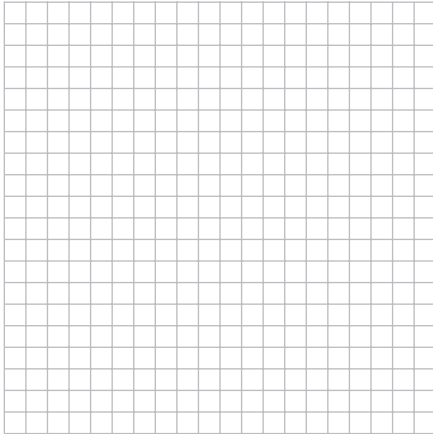
- Explain why a scatter plot was used to display the data.
- Identify the variables.
- Give two descriptive statements about this set of data.



Date: \_\_\_\_\_

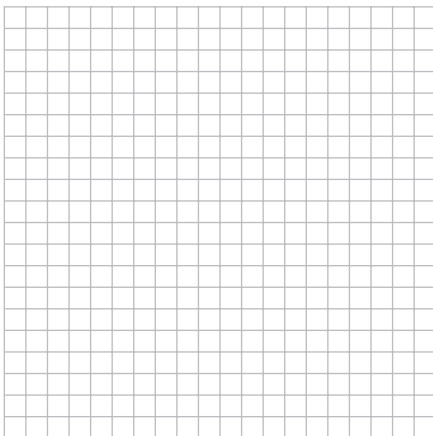


5. The table shows data about smog advisories issued for Ontario from 1995 to 2008.
- a) What do you know about each year?
  - b) Pose a question that would require one-variable data analysis.
  - c) Pose a question that would require two-variable data analysis.
  - d) Make a double bar graph to show each year's data side by side.



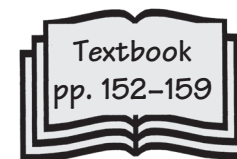
Year	Number of Smog Advisories	Total Number of Smog Days
1995	6	14
1996	3	5
1997	3	6
1998	3	8
1999	5	9
2000	3	4
2001	7	23
2002	10	27
2003	7	19
2004	8	20
2005	15	53
2006	6	17
2007	13	39
2008	8	17

- e) Make a scatter plot of the number of smog advisories versus the total number of smog days.

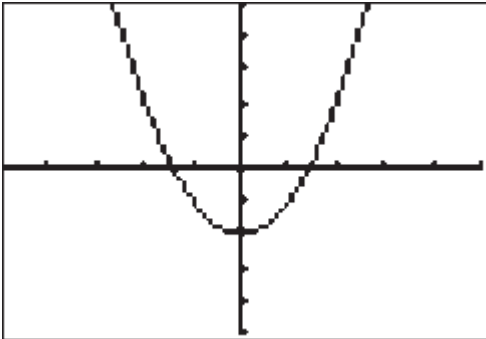
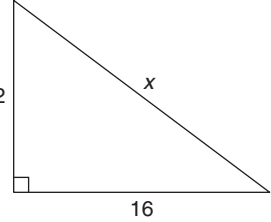
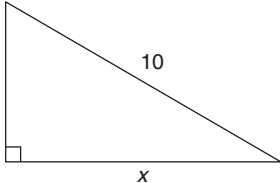


- f) Which graph will assist you in answering the question you posed in part c)? Explain.

## 3.2 Effective Surveys

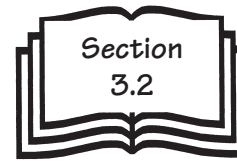


### Warm-Up

<p><b>1. Number Skills</b></p> <p>Round each number to the nearest tenth.</p> <p>a) 1.96</p> <p>b) 2.72</p> <p>c) 40.51</p>	<p><b>2. Algebra</b></p> <p>Simplify.</p> <p>a) <math>6xy - 5x + 5y - 2xy</math></p> <p>b) <math>4a + 10b - 4 - 6a + 1</math></p> <p>c) <math>(3x + 5y) - (2x + 4y)</math></p>
<p><b>3. Relations</b></p> <p>Indicate the number of roots for the equation that is modelled by the graph.</p> 	<p><b>4. Geometry/Measurement</b></p> <p>Determine the value of <math>x</math> for each triangle. Round your answer to one decimal place where necessary.</p> <p>a) </p> <p>b) </p>
<p><b>5. Data/Probability</b></p> <p>Give the probability of each event.</p> <p>a) drawing a red ace from a deck of 52 cards</p> <p>b) drawing a red card from a deck of 52 cards</p>	<p><b>6. Problem Solving</b></p> <p>Look at the pattern in the numbers. Then, find the next two numbers.</p> <p>1, 4, 5, 9, 14, ...</p>
<p><b>7. Math Literacy</b></p> <p>a) What is the name of an eight-sided figure?</p> <p>b) What is the name of a triangle with two sides of equal measure?</p>	<p><b>8. Previous Section</b></p> <p>A shopkeeper records the price charged and the method of payment for each customer. Is this a one-variable or a two-variable data set? Explain.</p>

Date: \_\_\_\_\_

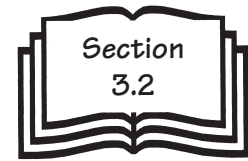
## Practise



1. What are the three principles of proper surveying?  
\_\_\_\_\_
  
2. Identify which principles of surveying are not being followed in each example. Explain your reasoning.
  - a) A survey included this question:  
“How did you hear about our Web site?  
Search Engine \_\_\_\_\_  
Friend \_\_\_\_\_  
Flyer Insert \_\_\_\_\_”
  - b) A multiple-choice survey about MP3 players listed iPod as the first choice in every question.
  - c) A group of people were given a survey about different brands of pain medication. The group was never told that the survey was being conducted by the manufacturer of one of the brands of medication.
  
3. Give an example of each survey question type below that would relate to a landscaping company.
  - a) Dichotomous
  - b) Multiple Choice
  - c) Rating Scale
  - d) Completion
  - e) Open-ended
  
4. A survey of grade 12 students included this question:  
“Do you agree that the amount of scholarship money available is the most important factor to consider when choosing a post-secondary school?”
  - a) Describe the bias in the question.
  - b) Rewrite the question to eliminate the bias.
  
5. Describe one advantage and one disadvantage of each mode of survey delivery.
  - a) personal interview
  - b) telephone
  - c) Internet
  - d) mail



6. A market research company plans to present this survey to a sample group. Examine the questionnaire.



### Cellular Telephone Survey

1. Do you own a cellular telephone? Yes \_\_\_\_\_ No \_\_\_\_\_
2. Which features do you feel are important on a cellular telephone?  
(Select all that apply.)
 

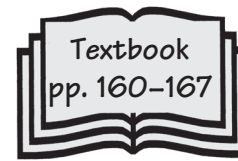
Camera _____	MP3 Player _____
QWERTY Keyboard _____	Bluetooth _____
Touch Screen _____	none of these _____
3. Do you agree with this statement?  
"I get good value from my cellular telephone contract."  

Agree strongly _____	Agree _____
Neutral _____	Disagree _____
Disagree strongly _____	
4. What is the main purpose of using your cellular telephone?  
\_\_\_\_\_
5. What could your cellular telephone company do to improve service for you?  
\_\_\_\_\_  
\_\_\_\_\_

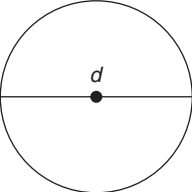
- a) Who would you administer the survey to and why?
  - b) What results would you hope to find?
  - c) What other question(s) should you include to improve the survey?
7. The student council has a surplus of \$500 and is considering how the money should be spent.
- a) Write a three-question *unbiased* survey designed to evaluate what the student population would like to do with the money.
  - b) Write a three-question *biased* survey designed to suggest the student population would rather invest the money than spend it.

## 3.3

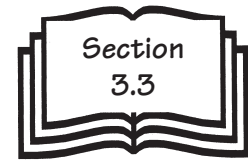
## Collect and Organize Data



## Warm-Up

<p><b>1. Number Skills</b></p> <p>Evaluate.</p> <p>a) <math>4 + [3 - (7 + 4)]</math></p> <p>b) <math>18 - [16 - (10 - 11)]</math></p>	<p><b>2. Algebra</b></p> <p>Simplify.</p> <p>a) <math>(2x^2 + 3x + 4) + (5x^2 + 4x + 9)</math></p> <p>b) <math>(x^2 + 2) + (6x^2 + 3x - 2)</math></p> <p>c) <math>(2x + 2y) - (3y + 6x) - (y + x)</math></p>
<p><b>3. Relations</b></p> <p>Give the point of intersection of lines <math>L_1</math> and <math>L_2</math>.</p> <p><math>L_1: y = x - 4</math>      <math>L_2: y = 2x + 4</math></p>	<p><b>4. Geometry/Measurement</b></p> <p>A can has a radius of 10.1 cm and a height of 24.2 cm. What is the volume of the can, to the nearest tenth of a millilitre? Recall that <math>1 \text{ cm}^3 = 1 \text{ mL}</math>.</p>
<p><b>5. Data/Probability</b></p> <p>Eileen tossed a coin 40 times and got heads 22 times.</p> <p>a) What is the experimental probability?</p> <p>b) What is the theoretical probability of flipping heads on 40 tosses?</p>	<p><b>6. Modelling</b></p> <p>Write an equation to model the area, <math>A</math>, of the circle.</p> 
<p><b>7. Math Literacy</b></p> <p>Similar figures have which characteristic(s)?</p> <p>A proportional corresponding sides</p> <p>B equal corresponding angles</p> <p>C both A and B</p> <p>D none of the above</p>	<p><b>8. Previous Section</b></p> <p>A survey includes this question:</p> <p>“Do you agree that banning smoking in cars will cut down on toxic second-hand smoke for passengers?”</p> <p>Identify which principle of surveying is not being followed in this question. Explain your reasoning.</p>

## Practise



For help with question 1, refer to Example 1.

1. a) Create a plan for an experiment to measure the accuracy of a player shooting baskets based on their distance from the hoop.

**Step 1.**

**Step 2.**

**Step 3.**

**Step 4.**

**Step 5.**

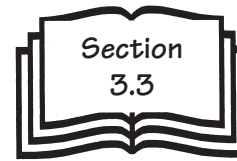
- b) What are the variables?
- c) What equipment do you need for your experiment?
- d) Describe the number and type of participants you need to have an accurate sample.
- e) What steps would you take to make sure the conditions were the same for all participants in your experiment?
- f) Set up a data-recording sheet for your experiment. Use the sample table as a template.

Distance from Hoop (m)	Accuracy (%)

- g) Describe how you would analyse and present your data.

Date: \_\_\_\_\_

2. The table shows the land area of the provinces and territories, and the unemployment rate for people 15 years and older from the 2006 Census.

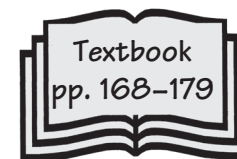


Region	Land Area (km <sup>2</sup> )	Unemployment Rate (%)
Newfoundland and Labrador	370 494.89	18.6
Prince Edward Island	5 683.91	11.1
Nova Scotia	52 917.46	9.1
New Brunswick	71 355.12	10.0
Quebec	1 356 366.78	7.0
Ontario	907 573.82	6.4
Manitoba	552 369.96	5.5
Saskatchewan	588 276.09	5.6
Alberta	640 044.57	4.3
British Columbia	924 815.43	6.0
Yukon	474 711.02	9.4
Northwest Territories	1 140 834.90	10.4
Nunavut	1 932 254.97	15.6

Source: Statistics Canada, E-STAT Table, *Cumulative Profile, 2006 - Provinces and Territories in Canada*, 2006 Census of Population (Provinces, Census Divisions, Municipalities) (database), Using E-STAT (distributor).

- a) Use technology to make a scatter plot of the data.
- b) Is the data primary or secondary data?
- c) Identify any outliers in the data.
- d) Is there a relationship between the land area of a province or territory and its unemployment rate? Explain.
3. a) Use the 2006 Canadian Census in the E-STAT Web site to research the land area of a region in Ontario and the unemployment rate for people 15 years and older. Download the data, present it in a summary table, and make a scatter plot. Print or make a sketch of the scatter plot.
- b) Is the data primary or secondary data?
- c) Identify any outliers in the data.
- d) Is there a relationship between the land area of a region in Ontario and its unemployment rate? Explain.
- e) How do these results compare with the results of question 2?

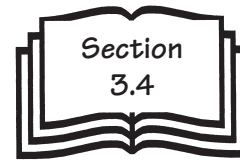
## 3.4 The Line of Best Fit



### Warm-Up

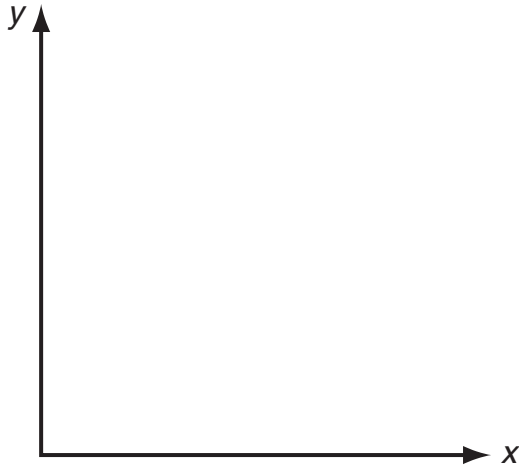
<p><b>1. Number Skills</b></p> <p>Evaluate.</p> <p>a) <math>\frac{142 + 10}{24 \div 3} - 5</math></p> <p>b) <math>\frac{12}{8 \times 6}</math></p>	<p><b>2. Algebra</b></p> <p>Solve.</p> <p>a) <math>4x + 1 = 7</math></p> <p>b) <math>3(x - 1) = 6</math></p> <p>c) <math>(2x + 4) + (2x + 2) = 4</math></p>
<p><b>3. Relations</b></p> <p>What translation maps the parabola <math>y = 3x^2</math> onto the parabola <math>y = 3x^2 - 3</math>?</p>	<p><b>4. Geometry/Measurement</b></p> <p>At a point 2.3 m away from a flagpole, the angle of elevation to the top of the flagpole is <math>62^\circ</math>. How tall is the flagpole, to the nearest tenth of a metre?</p>
<p><b>5. Data/Probability</b></p> <p>Determine the mean, the median, and the mode of the set of data.</p> <p>44, 23, 88, 22, 99, 66, 84, 87, 70</p>	<p><b>6. Modelling</b></p> <p>Twice Alan's age is 10 years older than his sister. The sum of their ages is 40 years. Write an equation to model this situation.</p>
<p><b>7. Math Literacy</b></p> <p>Which set describes the set of whole numbers?</p> <p>A <math>\{1, 2, 3, 4, 5, \dots\}</math></p> <p>B <math>\{0, 1, 2, 3, 4, \dots\}</math></p> <p>C <math>\{\dots, -3, -2, -1, 0, 1, 2, \dots\}</math></p> <p>D none of the above</p>	<p><b>8. Previous Section</b></p> <p>Explain the meaning of the term <i>outlier</i>.</p>

## Practise

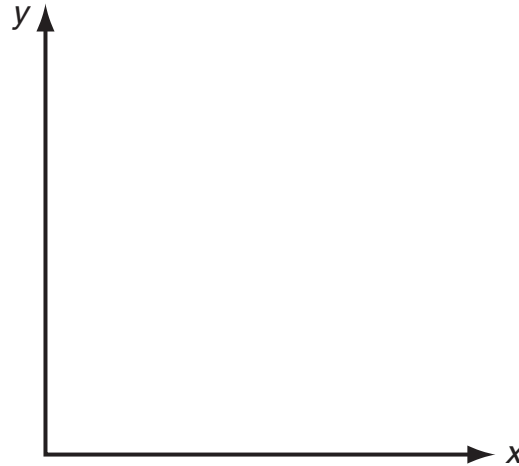


1. Sketch the general appearance of a scatter plot associated with each value of the correlation coefficient.

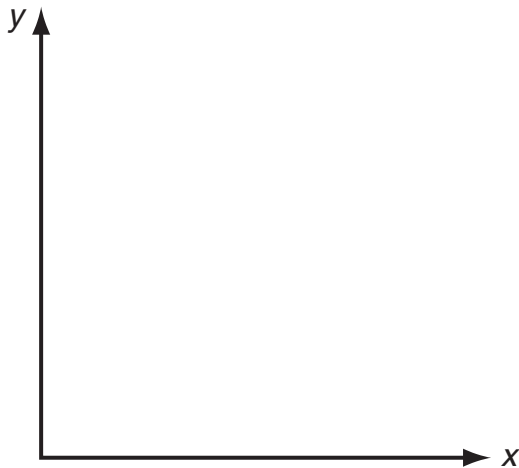
a)  $r = 0.5$



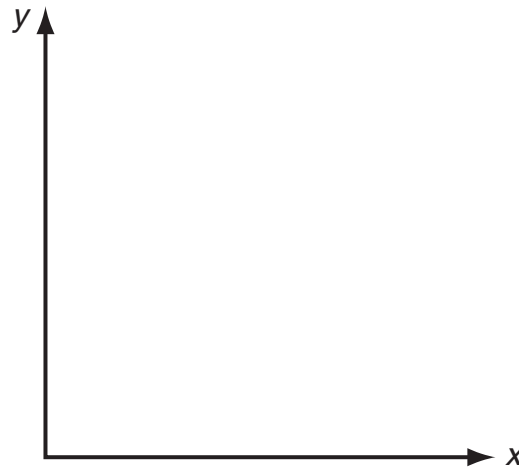
b)  $r = -0.2$



c)  $r = 0.90$



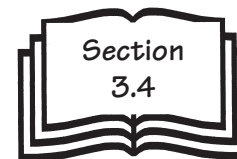
d)  $r = -0.75$



2. Indicate whether the correlation would be positive, negative, or zero for each situation. Explain your reasoning.

- income versus amount of charitable donations
- muscle flexibility versus Intelligence Quotient (IQ)
- essay mark versus number of hours spent preparing and writing the essay
- price of gasoline versus number of litres sold

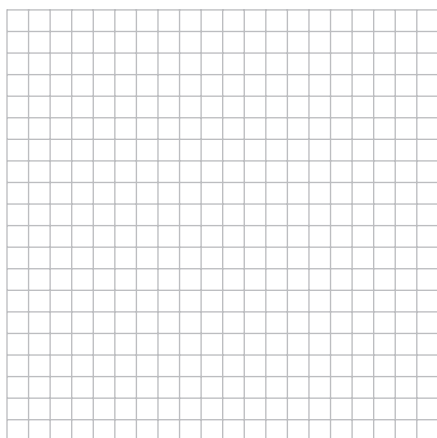
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3. The data in the table was collected from a random sample of mathematics students. Marks are out of 100.

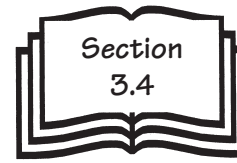
Student	Math Mark	Overall Average
A	74	70
B	64	71
C	87	85
D	92	86
E	77	80
F	85	75
G	58	63
H	80	81
I	73	65
J	69	74

- a) Make a scatter plot of the data.



- b) Describe the relationship between the variables.
- c) Use algebra to determine the equation of the line of best fit.  
Check your answer using technology.
- d) Use the line of best fit to predict the overall average of a student who has a mathematics mark of 90.
- e) How accurate do you think your prediction is? Explain.

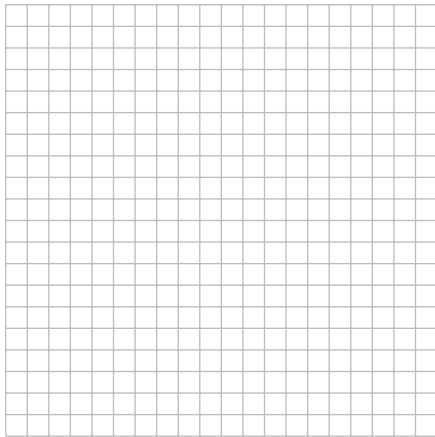
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4. Ravi is training for a canoeing expedition. He recorded the distances he could paddle in different lengths of time.

Time (min)	Distance Travelled (km)
10	0.5
20	1.6
30	2.5
40	3.7
50	4.5
60	5.7
70	6.9
80	8.1
90	9.4

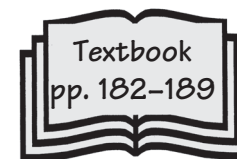
- a) Make a scatter plot of the data.



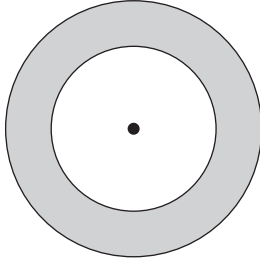
- b) Use linear regression to determine the equation of the line of best fit. Express  $d$  in terms of  $t$ . Give the slope and vertical intercept.
- c) How well does the regression equation fit the data? Explain.
- d) How far might Ravi expect to paddle in 65 min? Round your answer to the nearest tenth of a kilometre.
- e) Is it reasonable to use the regression equation to predict the distance Ravi could paddle in 8 h? Explain.



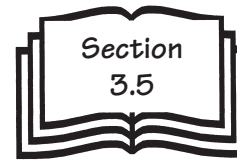
## 3.5 Analysis and Conclusions



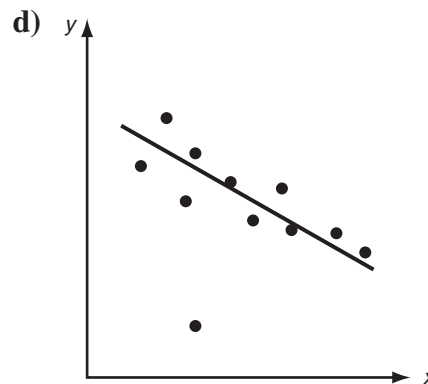
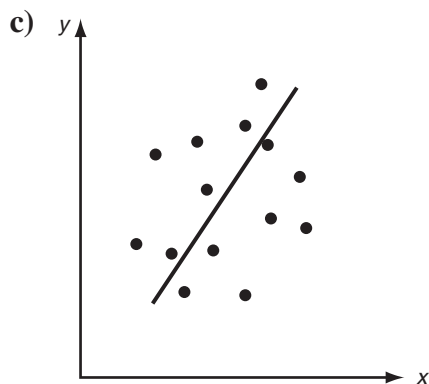
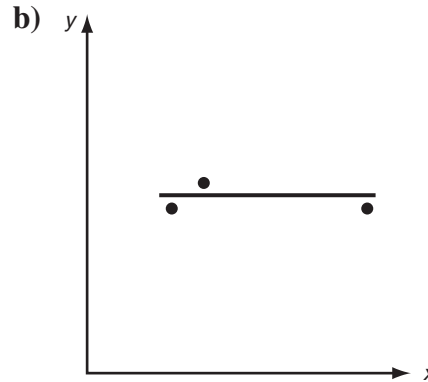
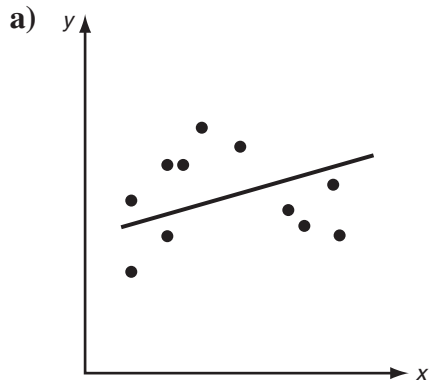
### Warm-Up

<p><b>1. Number Skills</b></p> <p>Evaluate.</p> <p>a) <math>5^2 + \sqrt{25} + 4</math></p> <p>b) <math>\sqrt{144} - \sqrt{6^2}</math></p>	<p><b>2. Algebra</b></p> <p>Solve.</p> <p>a) <math>-3x + x - 2 = 3x + 8</math></p> <p>b) <math>2(m + 3) + 2(m - 1) = 3(m - 2) + 2</math></p> <p>c) <math>8(2m - 7) + 3(m + 2) = 20m - 50</math></p>
<p><b>3. Relations</b></p> <p>Give the maximum or minimum value for the parabola.</p> <p><math>y = -2x^2 - 5</math></p>	<p><b>4. Geometry/Measurement</b></p> <p>The outer circle has a radius of 12 m. The inner circle has a radius of 9 m. Find the area of the shaded region, to the nearest square metre.</p> 
<p><b>5. Data/Probability</b></p> <p>The mean of a set of data is 82.5. The sum of the data values is 990. How many values are in the set of data?</p>	<p><b>6. Modelling</b></p> <p>The width of a rectangle is 4 cm less than its length. The area of the rectangle is 32 cm. Write an equation to model the area of the rectangle.</p>
<p><b>7. Math Literacy</b></p> <p>a) What is the term for a triangle with a <math>90^\circ</math> angle?</p> <p>b) What is the term for a triangle with an angle greater than <math>90^\circ</math>?</p>	<p><b>8. Previous Section</b></p> <p>Karin performed regression on a set of data and found the correlation coefficient to be <math>-0.3</math>.</p> <p>Describe what the data might look like when displayed in a scatter plot.</p>

## Practise

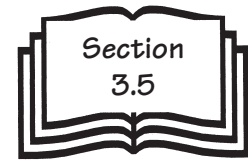


1. Describe the error in regression for each situation.



2. Describe the error in analysis for each situation.

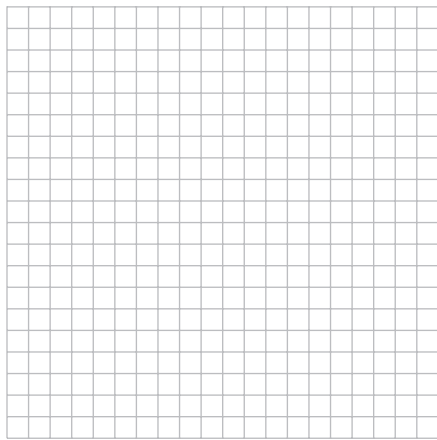
- The student council was deciding which types of fundraising activities would be used to pay for special events in the spring semester. Based on a survey of eight students, they decided to sell magazine subscriptions.
- A study showed a strong correlation between the number of times a person exercises each week and their overall level of health. The researchers concluded that being healthy influences you to exercise more.
- A study showed a strong correlation between the size of a city and the median income of city employees. A student used the study to decide where to look for summer employment.



3. A librarian randomly chose twelve popular books and recorded the number of weeks each book was on the bestseller list and the number of times each book was checked out during the past year.

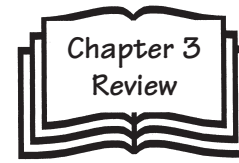
Book	Number of Weeks on Bestseller List	Number of Times Checked Out
A	28	36
B	24	6
C	25	26
D	16	17
E	20	26
F	12	20
G	9	10
H	35	38
I	20	21
J	60	57
K	26	33
L	32	36

- a) Make a scatter plot of the data.



- b) Identify any outliers or influential points.
- c) Is there a cause and effect relationship? If so, describe it.
- d) Use linear regression to model the relationship. Record the slope and the vertical intercept.
- e) Is the line of best fit appropriate? Explain.

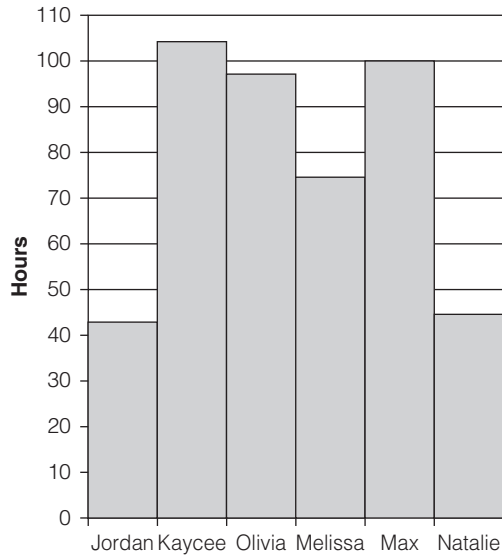
## Chapter 3 Review



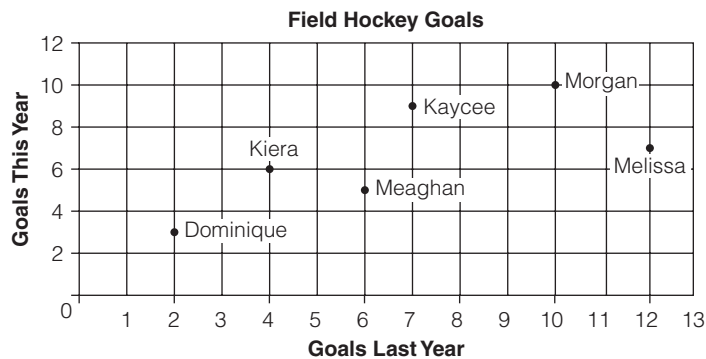
### 3.1 Two-Variable Data Sets, textbook pages 142–151

1. a) Indicate how many facts you know about each person. List the facts.

i) **High School Community Service Hours**



ii)

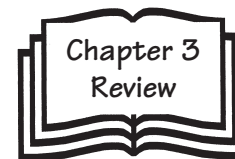


b) Pose a question about the field hockey players that would require two-variable analysis.

### 3.2 Effective Surveys, textbook pages 152–159

2. Identify which principles of surveying are not being followed in each example.

- A survey about the popularity of amusement parks was conducted at the entrance to Canada's Wonderland.
- People were asked to calculate the proportion of their grocery money they spent on baked goods last year.
- At the beginning of a survey about employee contentment, workers were told that the results of the survey would not be anonymous and answers would be shared with their immediate supervisor.



3. Give an example of each survey question type below that would relate to an organic produce farmer.
- Dichotomous
  - Multiple Choice
  - Rating Scale
  - Completion
  - Open-ended

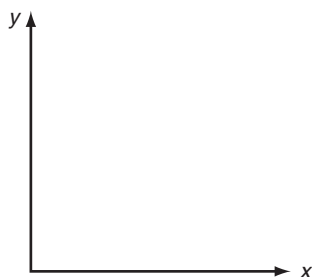
### 3.3 Collect and Organize Data, textbook pages 160–167

4. Indicate whether the data in each situation is primary or secondary data. Explain your choice.
- An advertising agency surveyed television audiences about their reaction to a commercial.
  - Kareem downloaded statistics about different countries from the United Nations Web site.
  - Heather gathered data from an experiment using plants and different types of fertilizers to see which fertilizer was most effective.
5. a) Use the 2006 Canadian Census in the E-STAT Web site to research the change in the population of a major city and the unemployment rate. Download the data, present it in a summary table, and make a scatter plot. Print or make a sketch of the scatter plot.
- b) How does the change in the population of a major city relate to the unemployment rate?

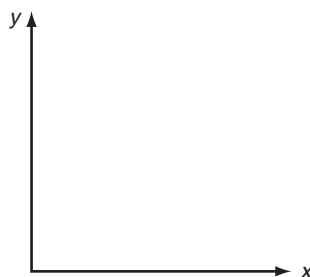
### 3.4 The Line of Best Fit, textbook pages 168–179

6. Sketch a scatter plot associated with each correlation coefficient.

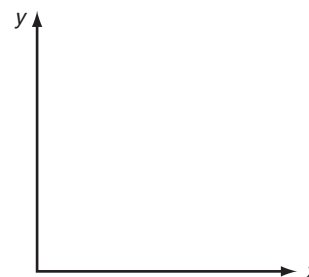
a)  $r = -1$

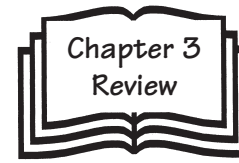


b)  $r = 0.15$



c)  $r = 0.8$

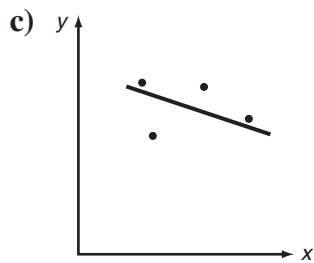
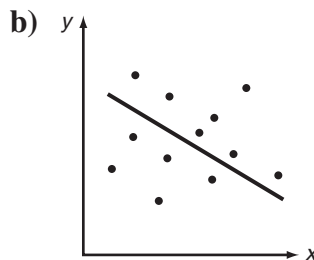
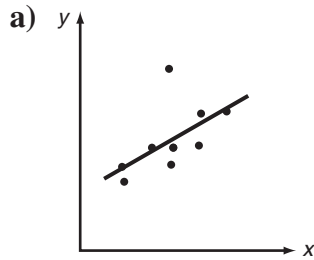




7. a) Determine an equation for the line of best fit in question 5.  
Give the slope and vertical intercept.
- b) How well does the line of best fit approximate the data? Explain.

### 3.5 Analysis and Conclusions, textbook pages 182–189

8. Describe the error in regression for each situation.



9. The table shows how many calories are burned in 1 h by a person walking at various speeds.

<b>Walking Speed (km/h)</b>	3.3	4.0	4.8	5.6	6.4	7.2	8.0
<b>Calories Burned</b>	170	204	224	258	340	428	544

- a) Perform regression analysis to determine the strength of the relationship and the equation of the line of best fit.
- b) Discuss whether a cause and effect relationship exists.