Date: _____

CHAPTER 5

Graphical Models

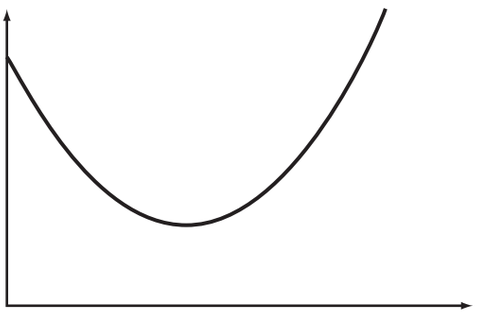
Get Set

Answer these questions to check your understanding of the Prerequisite Skills concepts on pages 266–267 of the *Foundations for College Mathematics 12* textbook.

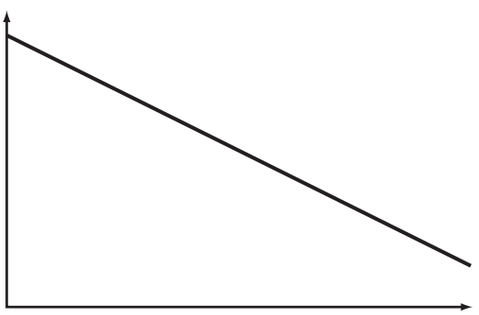
Interpret Graphs

1. Is each graph always increasing, always decreasing, constant, or none of these?

a)



b)



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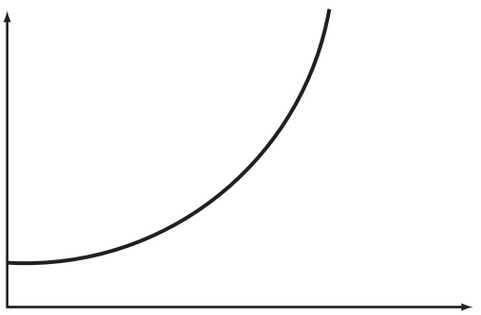
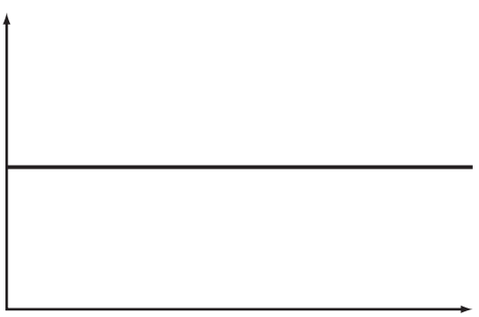


d)



Calculate Finite Differences

2. Calculate the first and second differences.

x	*y*	First Differences	Second Differences
–3	–15		
–2	–12		
–1	–5		
0	–4		
1	–3		
2	4		
3	7		

Date: _____

Draw Scatter Plots

3. The owner of a coffee shop near a beach recorded the daily sale of hot beverages and the daily high temperature. Create a scatter plot with temperature on the horizontal axis and hot beverage sales on the vertical axis.

Temperature (°C)	Hot Beverage Sales
5	45
7	40
10	32
12	25
14	20
20	7
24	3

Linear Regression

4. The table shows how the mass of a liquid is related to its volume.

Volume (mL)	Mass (g)
0	90
25	110
50	129
75	148
100	168
125	188
150	207

a) Use technology to graph the data.

b) Use linear regression to determine the equation of the line of best fit.

Evaluate Exponential Expressions

5. A liquid evaporates at a rate of 10% per hour. The equation $V = 100(0.9)^x$ models the volume of liquid, V, in millilitres, remaining after x hours.

a) Determine the volume of liquid after 30 min.

b) Determine the volume of liquid after 4 h.

c) Use systematic trial to determine when less than 50% of the liquid remains.

Simple and Compound Interest

Recall the formulas for simple and compound interest.

Simple Interest: $I = Prt$ Compound Interest: $A = P(1 + i)^n$

6. Determine the value of each investment after 7 years.

a) Jean bought a $5000 bond that pays 4.2% simple interest.

b) Carmindy deposited $3500 into an account that pays 3.9% per year, compounded annually.

Date: _____

5.1 Linear Models

Textbook pp. 268–281

Warm-Up

1. Number Skills

Evaluate.

a) $25 - 36 \div 2^2 \times (10 - 8)$

b) $7 + 4 \times 3 \div 2 - 1$

2. Algebra

Expand and simplify.

a) $2(x + 2) + 3(x - 5)$

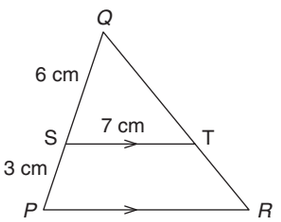
b) $4x(x - 1) - (8x - 6y)$

3. Relations

Determine coordinates of the vertex of the parabola $y = -\frac{1}{2}(x + 2)^2 - 5$.

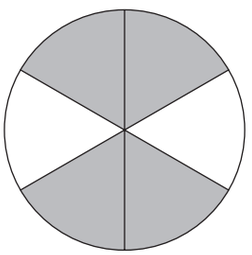
4. Geometry

Determine the length of PR.



5. Probability

Determine the probability of the pointer landing on a shaded sector.



6. Problem Solving

Determine the surface area of a cube that has a volume of 1815.8 cm^3.

7. Math Literacy

What is the name of a triangle with one right angle and two equal sides?

8. Previous Section

The number of bacteria in a colony doubles every hour. The equation $P = 1000(2)^t$ models the population, P, of bacteria after t hours.

a) What was the initial population of bacteria?

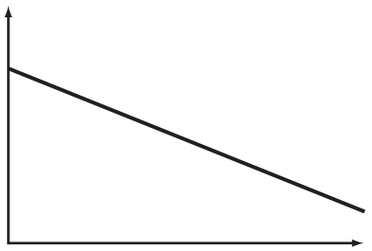
b) Determine the population after 6 h.

Date: _____

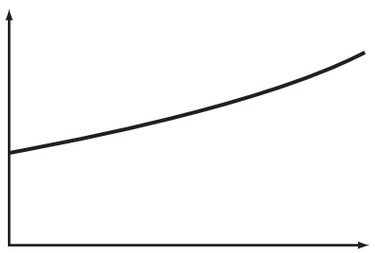
Practise

1. Classify each relation as linear or non-linear.

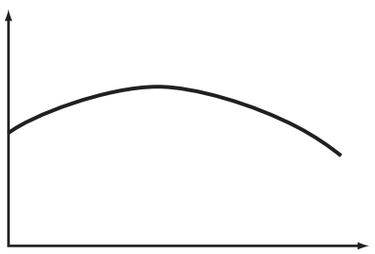
a)



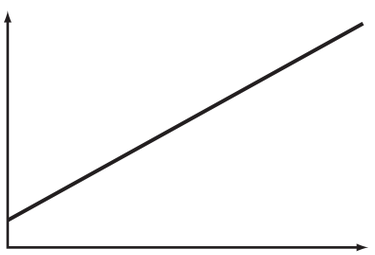
b)



c)



d)



2. Refer to question 1. For each graph that is linear, identify the rate of change as positive, zero, or negative.

3. Last week Jon worked 18 h. His gross earnings were $173.70.

a) What is the rate of change of Jon's gross earnings with respect to hours worked?

b) How much would Jon's gross earnings be for a week where he worked 11 h?

Date: _____

4. Danika drove at a constant speed from Peterborough to Ottawa. The table shows the distance she travelled over time.

Time (h)	Distance (km)
0.0	0
0.5	42
1.0	84
1.5	126
2.0	168
2.5	210
3.0	252
3.5	294

a) Draw a graph of the data. Describe the shape of the graph.

b) Does the rate of change appear to be increasing, constant, or decreasing?

c) Determine the rate of change. Include appropriate units.

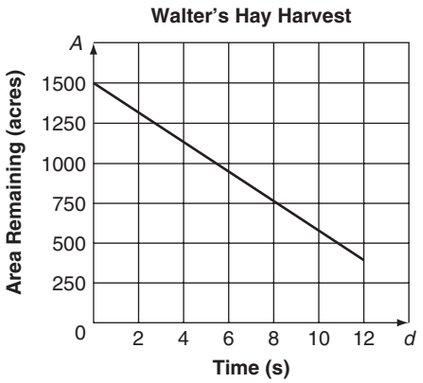
5. Describe an example of each type of linear relationship.

a) The rate of change between the two quantities is positive.

b) The rate of change between the two quantities is zero.

c) The rate of change between the two quantities is negative.

6. Walter grows hay on 1500 acres of land. Hay is harvested in the summer. The graph shows the number of acres Walter has left to harvest.



a) Describe the relationship between the area Walter has left to harvest and the number of days.

b) Use the graph to estimate the area remaining after four days.

c) By how much does the area decrease each day?

d) What are suitable units for the rate of change of area remaining with respect to the number of days?

e) Is the rate of change from part d) increasing, constant, or decreasing? Explain.

f) Predict the number of days required to complete the harvest.

Date: _____

5.2 Quadratic Models

Textbook pp. 282–293

Warm-Up

1. Number Skills

Simplify.

a) $15 - (-3) + 4$

b) $\dfrac{-8 + 50}{-7}$

2. Algebra

Simplify.

$(2x - 1)(3x + 2) - 4(x - 7)$

3. Relations

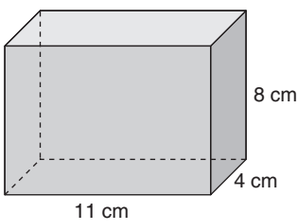
Determine if each relation is linear, quadratic, or neither.

a) $y = 5(x - 4)(x + 1)$

b) $y = \dfrac{x + 9}{2}$

4. Measurement

Determine the surface area of this rectangular prism.



5. Probability

A card is randomly selected from a deck of 52 cards, and then a coin is tossed. What is the probability of getting a face card and heads?

6. Modelling

Joe earns $10.2/h. Adriana earns a flat fee of $150 per week plus $4.65/h. Write an equation to model each person's earnings, E, in terms of the number of hours, h, worked.

7. Math Literacy

What is the name for an angle that is greater than 180° but less than 360°?

A obtuse angle

B straight angle

C acute angle

D reflex angle

8. Previous Section

Each table of values represents a line. Determine if the lines are parallel. Explain how you know.

x	y
−3	−13
−2	−11
−1	−9
0	−7
1	−5

x	y
0	1
1	3
2	5
3	7
4	9

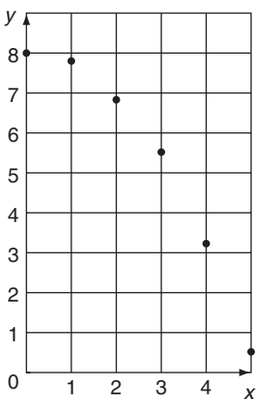
Date: _____

Practise

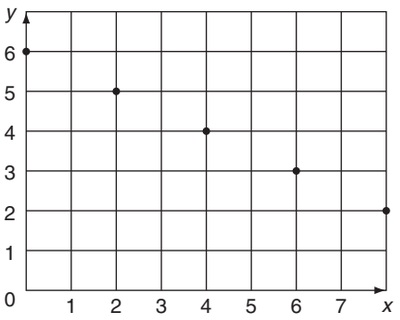
Section 5.2

1. Determine if each relation is linear, quadratic, or neither.

a)



b)



2. Calculate the first and second differences. Then, determine if each relation is linear, quadratic, or neither.

a)

x	*y*	First Differences	Second Differences
–1	16		
0	14		
1	8		
2	–2		
3	–16		

b)

x	*y*	First Differences	Second Differences
0	1		
1	2		
2	4		
3	8		
4	16		

c)

x	*y*	First Differences	Second Differences
–2	3		
–1	0		
0	–2		
1	–3		
2	–3		

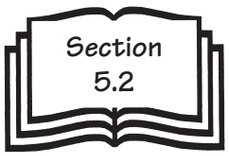
d)

x	*y*	First Differences	Second Differences
–4	–1		
0	2		
4	5		
8	8		
12	11		

3. For each relation in question 2 that is linear or quadratic, use technology to determine the equation of the line or curve of best fit.

Hint: Enter the *x*-values into **L1** and the *y*-values into **L2**.
Press STAT. Select the **CALC** menu, then
4:LinReg(ax+b) or **5:QuadReg.**
Press ENTER twice.

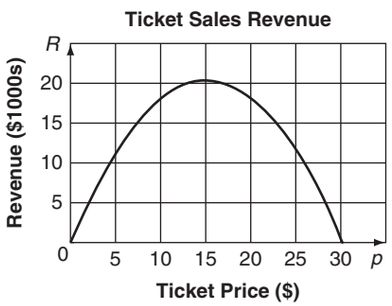
Date: _____



4. Can finite differences be used to classify the data in this table as linear, quadratic, or neither? If yes, classify the data. If not, explain why not.

x	*y*
−2	2.0
−1	5.5
1	9.5
2	10.0
4	8.0
5	5.5

5. Tickets at a hockey arena currently sell for $10 each. At this price, all 1600 seats are sold for each game. According to market research, the number of seats sold will decrease if the ticket price is increased. The graph shows the expected revenue at different ticket prices.



a) Describe the relationship between ticket price and revenue.

b) Estimate the revenue for each ticket price.
i) $14 **ii)** $19

c) Is the rate of change of revenue with respect to ticket price increasing, constant, or decreasing? Explain.

6. The table shows the edge length and surface area of different-sized cubes.

Edge Length (cm)	Surface Area (m^2)
15	0.135
30	0.540
45	1.215
60	2.160
75	3.375
90	4.860

a) Calculate the finite differences. Does a linear or a quadratic model best fit the data?

b) Is the rate of change of surface area with respect to edge length increasing, constant, or decreasing? Explain.

c) Use technology to determine an equation that models the data.

d) Use your equation from part c) to determine the surface area of a cube with an edge length of 105 cm.

Date: _____

5.3 Exponential Models

Textbook pp. 294–305

Warm-Up

1. Number Skills

Evaluate.

a) $(-3)^3$ **b)** $(-10)^4$

c) -5^2 **d)** $(-2)^4$

2. Algebra

Factor.

a) $2x^2 + 6y$

b) $7x^3 - 14x^2 + 21x$

3. Relations

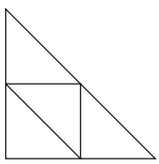
Consider the parabola $y = (x - 6)^2 - 4$.

a) Determine the coordinates of the x- and y-intercepts.

b) Determine the coordinates of the vertex. Is the vertex a maximum or a minimum?

4. Measurement

Four congruent right isosceles triangles are arranged to form a larger right isosceles triangle. The larger triangle has an area of 8 cm^2.



Determine the length of the legs in each smaller triangle.

5. Data/Probability

Find the mean, the median, and the mode of the set of data.

41, 28, 27, 30, 27, 24, 36, 52, 24, 54, 25, 30, 37, 30, 40, 24

6. Problem Solving

Mandy is making 10 kg of a new blend of tea. She mixes tea that costs \$16/kg with tea that costs \$20/kg. The blend costs \$17.60/kg. How much of each type of tea did she use?

7. Math Literacy

What is the name of the vertical line that passes through the vertex of a parabola?

8. Previous Section

Determine the equation of a parabola that opens downward, has vertex (4, 2), and passes through (–2, –16).

Date: _____

Practise

Section 5.3

1. According to a historical account, a servant asks to be rewarded by receiving one grain of rice for the first square of a checkerboard, two grains for the second square, four grains for the third square; the amount doubles with each square on the checkerboard.

a) How many grains of rice would the servant receive for the tenth square on the checkerboard?

b) Explain why an exponential model can be used for the relationship between the square on the checkerboard and the number of grains of rice.

2. Does each number sequence represent exponential growth? Explain.

a) 10, 100, 1000, 10 000

b) 3, 6, 9, 12

c) $\frac{1}{4}, \frac{1}{2}$, 1, 2, 4

3. Calculate the finite differences and the ratios. Is each relation linear, quadratic, or exponential?

a)

x	*y*	First Differences	Second Differences	Ratios
–3	–64			
–2	–16			
–1	–4			
0	–1			
1	$-\frac{1}{4}$			
2	$-\frac{1}{16}$			

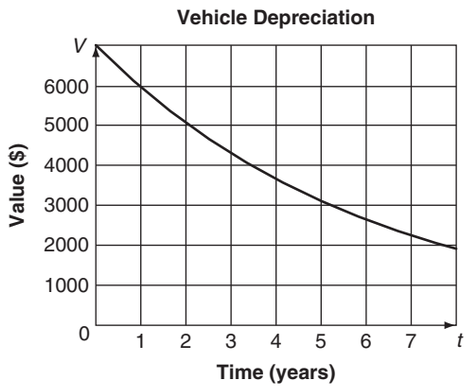
b)

x	*y*	First Differences	Second Differences	Ratios
–2	6.0			
–1	4.9			
0	3.8			
1	2.7			
2	1.6			
3	0.5			

Date: _____

Section 5.3

4. The graph shows the value of a vehicle over time.



a) Describe the relationship between time and the value of the vehicle.

b) Estimate the value of the vehicle after 4 years.

c) What are suitable units for the rate of change of the value of the vehicle with respect to time?

d) Is the rate of change of the value of the vehicle with respect to time increasing, constant, or decreasing? Explain.

5. Christine had 72 six-sided dice. She tossed all the dice, removed the dice that showed 2, and then tossed the remaining dice. Christine repeated this experiment many times and averaged her results.

Toss	Dice Remaining
0	72
1	61
2	50
3	42
4	34
5	28
6	24
7	20
8	16
9	14
10	11
11	10
12	8

Use the following window settings:
Xmin = 0
Xmax = 20
Xscl = 2
Ymin = 0
Ymax = 75
Yscl = 5

a) Calculate the finite differences and the ratios.

b) Which model appears to be the most suitable: linear, quadratic, or exponential? Explain.

c) Use technology to determine the equation of the line or curve of best fit.

d) Use the equation to estimate the number of dice remaining after 16 tosses.

e) Graph the data in the table and the line of best fit in the same window.

Date: _____

5.4 Analyse Graphical Models

Textbook pp. 310–319

Warm-Up

1. Number Skills

Evaluate.

a) $\frac{3}{8}+\frac{1}{3}$

b) $\frac{4}{5}-\frac{1}{2}$

c) $\frac{2}{9}\times\frac{3}{4}$

2. Algebra

Factor.

a) $x^2-2x-35$

b) $9x^2+27x+18$

3. Relations

Determine the value of x when $y = 4$ in the relation $\frac{3x}{2}-\frac{y}{8}=1$.

4. Geometry

Which of these regular polygons cannot be used to tile a plane? Circle all that apply.

square	triangle
pentagon	hexagon
octagon	decagon

5. Data

Sara recorded the times it took her to run different distances. Draw a scatter plot of the data in the table.

Distance (m)	Time (s)
100	14
200	27
300	43
400	56
500	70
600	85

6. Problem Solving

Jameer drove 460 km in 5 h. For part of the trip, he travelled 100 km/h and for the remainder he travelled 80 km/h. How far did Jameer travel at each speed?

7. Math Literacy

Which term means the least distance from the edge of the base of a pyramid to the vertex?

A height

B slant height

C perpendicular bisector

D median

8. Previous Section

The relation $A=600\left(\frac{1}{2}\right)^{\frac{n}{13.81}}$ models the amount, A, in grams, of beryllium-11 that remains after n seconds.

a) What is the half-life of beryllium-11?

b) What is the initial amount of beryllium-11?

Date: _____

Practise

Section 5.4

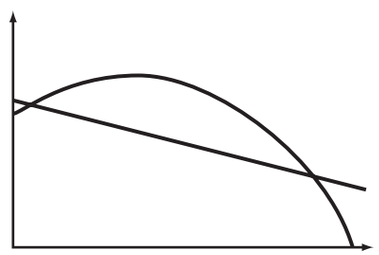
1. Match each situation with its graph.

a) The number of bacteria in Colony X remained the same over time. Colony Y started with 50 bacteria and doubled every half-hour.

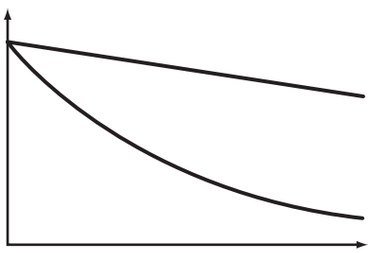
b) Two cups of water were cooled in different controlled environments. Cup X cooled at a constant rate. The temperature of Cup Y decreased by one-half every 20 min.

c) Ball X rolled down a ramp. Ball Y was thrown from a point above the ground.

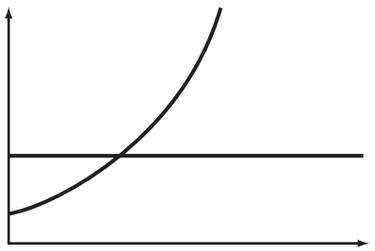
A



B



C



2. The population of Town X started at 90 000 and increased by 25 000 every year. The population of Town Y started at 4000 and doubled every year. Which statement is true?

A The population of Town X is always greater than the population of Town Y.

B The rate of change of the population of Town X is increasing.

C The rate of change of the population of Town Y is increasing.

D The population of Town Y is greater than the population of Town X after 4 years.

Date: _____

3. Refer to question 2.

a) Complete the table of values.

Year	Town X Population	Town Y Population
0		
1		
2		
3		
4		
5		

b) Determine an equation to model the population of each town.

c) In what year is the population of Town Y greater than the population of Town X?

4. Ing has the choice of two payment options for her new job.

Option A: Starting salary of $48 000, with a $1000 raise every following year.

Option B: Starting salary of $45 000, with a 2.5% raise every following year.

a) Complete the table of values.

Year	Option A Salary ($)	Option B Salary ($)
0		
1		
2		
3		
4		
5		
6		
7		

b) Which option should Ing choose? Why?

Date: _____

5. The bank is offering a new savings account that pays 5% per year, compounded annually. Curtis and Linnea each opened a new savings account. Curtis invested $2000 and Linnea invested $4000.

a) Complete each table.

Curtis				
Year	**Amount ($)**	**First Differences**	**Second Differences**	**Ratios**
0				
1				
2				
3				
4				
5				

Linnea				
Year	**Amount ($)**	**First Differences**	**Second Differences**	**Ratios**
0				
1				
2				
3				
4				
5				

b) What are suitable units for the rate of change of each amount?

c) Determine an equation for the amount in each account. Graph each equation on the same set of axes.

d) The ratio of the amounts Curtis and Linnea initially invested was 2000:4000 or 1:2. Does this ratio increase, remain constant, or decrease over time? Explain.

6. Refer to question 5. Suppose the new savings account paid 7% simple interest. Would your answer to part d) be different? If yes, explain how. If not, explain why not.

Date: _____

5.5 Select a Mathematical Model

Textbook pp. 320–331

Warm-Up

1. Number Skills

Write each decimal as a fraction in lowest terms.

a) 0.82

b) 0.35

c) 0.5625

1. Number Skills

Factor.

a) $9x^2 - 1$

b) $4x^2 + 4x + 1$

3. Relations

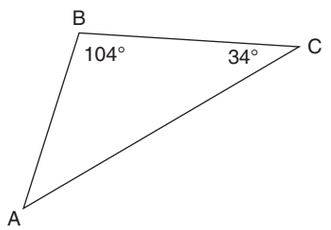
Voltage, V, in an electrical circuit varies directly with the current, A.

a) Write an equation that relates the voltage to the current if the voltage is 12 V when the current is 8 A.

b) Determine the voltage when the current is 14 A.

4. Geometry

Determine the measure of $\angle A$.



5. Data

The table shows the test marks for Mr. Wong's class.

Mark	Frequency
<50	1
50–59	3
60–69	5
70–79	11
80–89	6
90–99	2

a) How many students are in the class?

b) How many students have a mark less than 70%?

6. Modelling

At a golf academy, Tracy measured the horizontal distance, d, and the height, h, of the ball after she hit it. Determine the equation of the line or curve of best fit.

d (m)	h (m)
0	0
25	10.6
50	17.5
75	20.7
100	20.0
125	15.6
150	7.5

7. Math Literacy

What is the name of the point at which a parabola changes from decreasing to increasing or from increasing to decreasing?

8. Previous Section

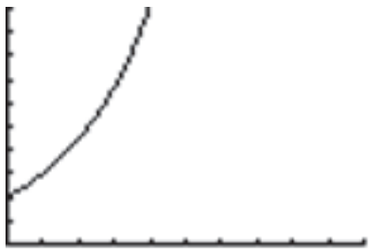
Sven invested $2500 in an account that pays 4.5% simple interest per year. Nori invested $2400 in an account that pays 4.2% per year, compounded annually. When will Nori's investment be worth more than Sven's?

Date: _____

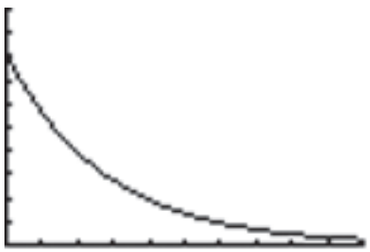
Practise

1. Determine if each relation is linear, quadratic, or exponential.

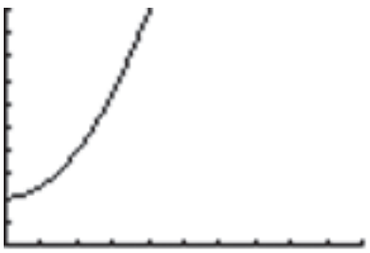
a)



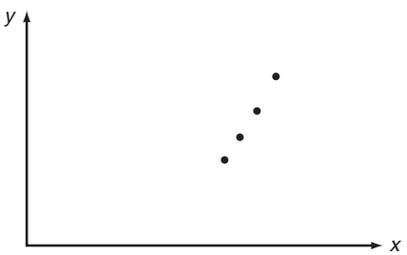
b)



c)



2. Consider this scatter plot.



a) Which model—linear, quadratic, or exponential—seems most appropriate for this scatter plot? Why?

b) Can this scatter plot be used to develop a reasonable mathematical model? If yes, explain how. If not, explain why not.

Date: _____

3. The table shows data collected from an experiment.

x	*y*
10.0	760.40
12.5	659.78
15.0	509.15
17.5	305.53

a) Create a scatter plot of the data.

b) Which model—linear, quadratic, or exponential—appears to be most appropriate? Why?

c) Generate a model for the data. Display the model graphically and algebraically.

d) Use your model to predict the value of y when $x = 20$.

4. Refer to question 2. The table shows the coordinates of the points on the scatter plot.

a) Calculate the finite differences and the ratios.

x	*y*	First Differences	Second Differences	Ratios
2.2	11.2			
2.4	14.0			
2.6	17.4			
2.8	21.7			

b) Does your answer to part a) support using one model over the other two models? Explain.

c) The table and the scatter plot show part of a relation. The coordinates of four other points on the relation are (1.6, 5.8), (1.8, 7.2), (3.2, 33.6), and (3.4, 41.9). Draw a scatter plot showing all eight points.

d) Which model seems most appropriate now?

e) Determine the equation of the line or curve of best fit for your model.

Date: _____

5. The table shows the average value of a single-family home in the 1970s in a particular community.

Year	Average Value ($)
1971	42 000
1973	51 000
1975	63 000
1977	77 000
1979	93 000

a) Construct a scatter plot of the data.

b) Describe any trends you see in the graph.

c) Calculate the finite differences and the ratios. Which type of model appears most suitable?

Year	Average Value ($)	First Differences	Second Differences	Ratios
1971	42 000			
1973	51 000			
1975	63 000			
1977	77 000			
1979	93 000			

d) Generate a linear, a quadratic, and an exponential model.

e) Graph each model from 1930 to 2020.

f) Use your models to estimate the average value of a single-family home in each year.

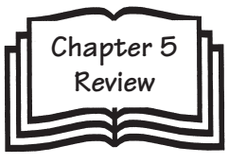
i) 1940

ii) 2010

g) Consider your answers for parts c) to f). Which model best represents the data? Explain.

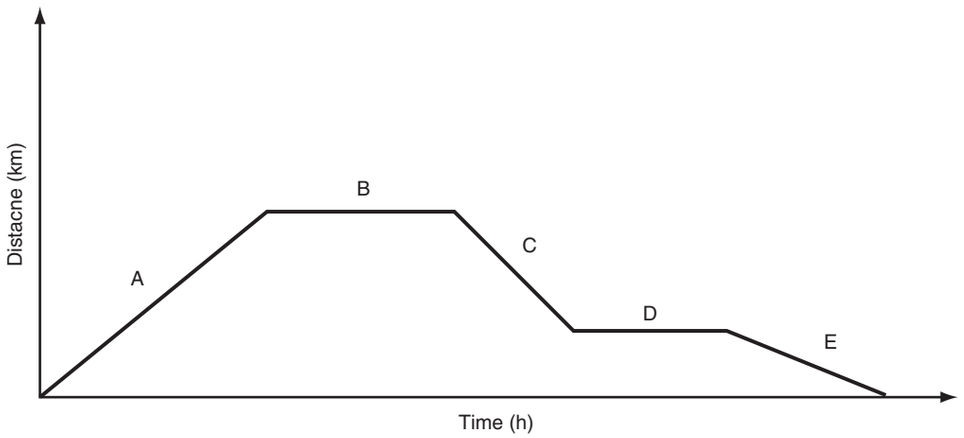
Date: _____

Chapter 5 Review



5.1 Linear Models, textbook pages 268–281

1. This graph shows Marney's distance from home over time.



a) For which parts of the graph is the rate of change of Marney's distance from home with respect to time increasing?

b) For which parts of the graph is the rate of change decreasing?

c) For which parts of the graph is the rate of change constant?

d) What are appropriate units for the rate of change of Marney's distance from home with respect to time?

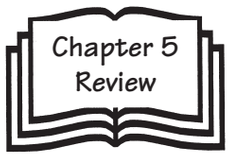
5.2 Quadratic Models, textbook pages 282–293

2. a) Calculate the finite differences.

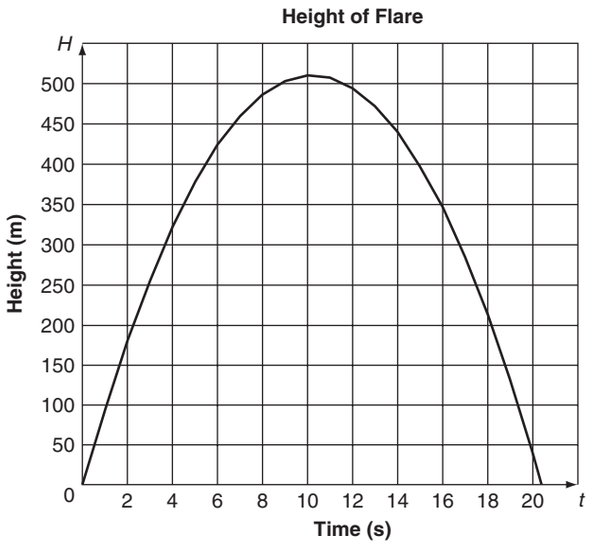
x	*y*	First Differences	Second Differences
–1	–0.4		
1	0.8		
3	3.6		
5	8.0		
7	14.0		
9	21.6		

b) Is this relation linear, quadratic, or neither? Explain.

Date: _____



3. The graph shows the height of a flare after it is shot into the air.



a) Estimate the height of the flare after 5 s.

b) Estimate the time it takes the flare to reach its maximum height.

c) What are appropriate units for the rate of change of height with respect to time?

d) Is the rate of change of height with respect to time increasing, constant, or decreasing?

5.3 Exponential Models, textbook pages 294–305

4. The table shows the percent of a sample of strontium-90 that remains every 5 years.

a) Calculate the finite differences and the ratios.

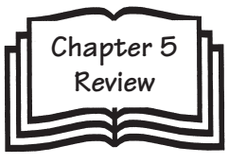
Year	Percent Remaining	First Differences	Second Differences	Ratios
0	100.0			
5	88.7			
10	78.7			
15	69.9			
20	62.0			
25	55.0			
30	48.8			

b) Would you use a linear, quadratic, or exponential model to represent the percent of strontium-90 that remains over time? Explain.

c) Use technology to determine the equation of the line or curve of best fit for your chosen model.

d) Use your equation to estimate the half-life of strontium-90.

Date: _____



5.4 Analyse Graphical Models, textbook pages 310–319

5. Kartik has $4000 to invest. He has a choice of two accounts. Account A pays 4.2% simple interest. Account B pays 3.8% per year, compounded annually.

a) Complete the table of values for the two accounts.

Time (years)	Amount in Account A ($)	Amount in Account B ($)
0		
1		
2		
3		
4		
5		
6		
7		

b) Which account would you advise Kartik to choose? Why?

5.5 Select a Mathematical Model, textbook pages 320–331

6. The table shows the average amount households spend annually on utilities in a particular community.

a) Calculate the finite differences and the ratios.

Year	Average Amount ($)	First Differences	Second Differences	Ratios
2000	6214.00			
2001	6648.98			
2002	7083.92			
2003	7518.89			
2004	7953.90			

b) Which model appears most suitable: linear, quadratic, exponential, or none of these?

c) Generate a model. Display the model graphically and as an equation.

d) Use your model to estimate the average amount spent on utilities in each year.

i) 1995

ii) 2010